# Basic Structural Chemistry 

## Crystalline state

Structure types

## Degree of Crystallinity



- Single Crystalline
- Polycrystalline
- Amorphous



## Crystal Structure

-The building blocks of these two are identical, but different crystal faces are developed


- Cleaving a crystal of rocksalt


## Crystals

- Crystal consist of a periodic arrangement of structural motifs = building blocks
- Building block is called a basis: an atom, a molecule, or a group of atoms or molecules
- Such a periodic arrangement must have translational symmetry such that if you move a building block by a distance:

$$
\bar{T}=n_{1} \bar{a}+n_{2} \bar{b}+n_{3} \bar{c}
$$

$$
\text { where } n_{1}, n_{2} \text {, and } n_{3} \text { are integers, and } \bar{a}, \bar{b}, \bar{c} \text { are vectors. }
$$

then it falls on another identical building block with the same orientation.

- If we remove the building blocks and replace them with points, then we have a point lattice or Bravais lattice.


## Planar Lattice 2D



## Five Planar Lattices


(a)

(d)

(c)
(b)

(e)

| Name | Number of <br> Bravais lattices | Conditions |
| :---: | :---: | :--- |
| Square | 1 | $a_{1}=a_{2}, \alpha=90^{\circ}$ |
| Rectangular | 2 | $a_{1} \neq a_{2}, \alpha=90^{\circ}$ |
| Hexagonal | 1 | $a_{1}=a_{2}, \alpha=120^{\circ}$ |
| Oblique | 1 | $a_{1} \neq a_{2}, \alpha \neq 120^{\circ}, \alpha \neq 90^{\circ}$ |




Unit Cell: An „imaginary" parallel sided region of a structure from which the entire crystal can be constructed by purely translational displacements Contents of unit cell represents chemical composition

Space Lattice: A pattern that is formed by the lattice points that have identical environment.

Coordination Number (CN): Number of direct neighbours of a given atom (first coordination sphere)

## Crystal $=$ Periodic Arrays of Atoms

Lattice point
(Atom, molecule, group of molecules, $\ldots$.)


## Primitive Cell:

- Smallest building block for the crystal lattice.
- Repetition of the primitive cell gives a crystal lattice
\(\left.$$
\begin{array}{lcc}\hline & \begin{array}{c}\text { Bravais Lattice } \\
\text { (Lattice point = Basis of } \\
\text { Spherical Symmetry) }\end{array} & \begin{array}{c}\text { Crystal Structure } \\
\text { (Structural motif = Basis of } \\
\text { Arbitrary Symmetry) }\end{array} \\
\hline \begin{array}{l}\text { Number of } \\
\text { point groups: }\end{array}
$$ \& (7 crystal systems) \& (32 crystallographic point <br>

groups)\end{array}\right]\)| $\mathbf{2 3 0}$ |  |
| :---: | :---: |
| Number of <br> space groups: | $(14$ Bravais lattices) |

## Seven Crystal Systems



## Fourteen Bravais Lattices



## Simple Cubic (SC)

$$
a=b=c \quad \text { Conventional Cell }=\text { Primitive Cell }
$$

$$
\mathrm{a} \perp \mathrm{~b} \perp \mathrm{c}
$$

Add one atom at the center of the cube


Body-Centered Cubic (BCC)
Face-Centered Cubic (FCC)
Conventional Unit Cell $\neq$ Primitive Cell

## Primitive Cell

A volume of space translated through all the vectors in a lattice just fills all of space without overlapping or leaving voids is called a primitive cell of the lattice.

A primitive cell contains just one Bravais lattice point.
The primitive cell is the smallest cell that can be translated throughout space to completely recreate the entire lattice.
There is not one unique shape to a primitive cell and many possible shapes fulfill the definition.
The primitive cell for the simple cubic lattice is equal to the simple cubic unit cell (they are identical).
A common choice for the primitive cell of the body-centered cubic lattice is shown below.


Body-Centered Cubic (I)

Unit Cell


Primitive Cell

## Nonprimitive Unit Cell vs. Primitive Cell



Face-Centered Cubic (F)

Unit Cell


Primitive Cell


Rotated $90^{\circ}$

The primitive cell is smaller or equal in size to the unit cell. The unit cells help to remind us of the symmetry (ie. Cubic).

## Primitive Cell of BCC

-Rhombohedron primitive cell

-Primitive Translation Vectors:

$$
\begin{gathered}
a_{1}=\frac{1}{2} a(\hat{k}+9-2) ; \quad a_{2}=\frac{1}{2} a(-\hat{\mathbf{x}}+\hat{y}+\bar{z}) ; \\
\mathbf{a}_{3}=\frac{1}{2} a(\hat{\mathrm{~h}}-9+2) .
\end{gathered}
$$

Primitive Cell of FCC


$$
a_{1}=\frac{1}{2} a(1+y) ; \quad a_{2}=\frac{1}{2} a(\hat{y}+2) ; \quad a_{3}=\frac{1}{4} a(2+\hat{x})
$$

## Index System for Crystal Planes (Miller Indices)

1) Find the intercepts on the axes in terms of the lattice constants $a, b, c$. The axes may be those of a primitive or nonprimitive unit cell.
2) Take the reciprocals of these numbers and then reduce to three integers having the same ratio, usually the smallest three integers. The result enclosed in parenthesis ( $h \mathrm{kl}$ ), is called the index of the plane.

## Miller Indices



## Miller Indices



## Miller Indices



## Crystals

- metallic ( $\mathrm{Cu}, \mathrm{Fe}, \mathrm{Au}, \mathrm{Ba}$, alloys ) metallic bonding
- ionic ( $\mathrm{NaCl}, \mathrm{CsCl}, \mathrm{CaF}_{2}, \ldots$ )
cations and anions, electrostatic interactions
- covalent (diamond, graphite, $\mathrm{SiO}_{2}, \mathrm{AlN}, \ldots$ ) atoms, covalent bonding
- molecular (Ar, $\mathrm{C}_{60}, \mathrm{HF}, \mathrm{H}_{2} \mathrm{O}$, organics, proteins ) molecules, van der Waals and hydrogen bonding


## Crystal Bonding

- van der Waals bonds
- Ionic bonds
- Hydrogen bonds
- Metallic bonds
- Covalent bonds


## Three Cubic Cells



## Table 2 Characteristics of cubic lattices ${ }^{\text {a }}$

|  | Simple | Body-centered | Face-centered |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  | $a^{3}$ | $a^{3}$ | $a^{3}$ |
| Volume, conventional cell | $\cdots$ | 2 | 4 |  |
| Lattice points per cell | 1 | $a^{3}$ | $\frac{1}{2} a^{3}$ | $\frac{1}{4} a^{3}$ |
| Volume, primitive cell | $1 / a^{3}$ | $2 / a^{3}$ | $4 / a^{3}$ |  |
| Lattice points per unit volume | 1 | 12 |  |  |
| Number of nearest neighbors | 6 | 8 | $a / 2^{1 / 2}=0.707 a$ |  |
| Nearest-neighbor distance | $a$ | $3^{1 / 2} a / 2=0.866 a$ | 6 |  |
| Number of second neighbors | 12 | 6 | $a$ |  |
| Second neighbor distance | $2^{1 / 2} a$ | $a$ | $\frac{1}{6} \pi \sqrt{2}$ |  |
| Packing fraction | $\frac{1}{6} \pi$ | $\frac{1}{8} \pi \sqrt{3}$ | $=0.740$ |  |

## Cube


$a=$ edge
d = face diagonl

$$
\left(d^{2}=a^{2}+a^{2}=2 a^{2}\right)
$$

$\mathrm{D}=$ body diagonal

$$
\left(D^{2}=d^{2}+a^{2}=2 a^{2}+a^{2}=3 a^{2}\right)
$$

## $d=\sqrt{2} \cdot \mathrm{a}$



## $\mathbf{S C}=\mathbf{P o l o n i u m}$


(b)


Space filling $52 \%$

## $\mathbf{B C C}=\mathbf{W}$, Tungsten



Space filling 68\%
CN 8


Fe, Cr, V, Li-Cs, Ba

$\mathrm{FCC}=\mathrm{Copper}, \mathrm{Cu}=\mathbf{C C P}$


Space filling 74\%
CN 12

## Close Packing in Plane 2D



$B$ and $C$ holes cannot be occupied at the same time

## Close Packing in Space 3D




hexagonal

$\mathbf{M g}, \mathrm{Be}, \mathrm{Zn}, \mathrm{Ni}, \mathrm{Li}, \mathrm{Be}, \mathrm{Os}, \mathrm{He}$,

(a)

(b)


Sc, Ti, Co, Y, Ru

hexagonal

cubic
$\mathbf{C u}, \mathbf{C a}, \mathbf{S r}, \mathbf{A g}, \mathbf{A u}, \mathbf{N i}, \mathbf{R h}$, solid $\mathbf{N e}-\mathrm{Xe}, \mathrm{F}_{2}, \mathrm{C}_{60}$, opal ( $\mathbf{3 0 0} \mathbf{n m}$ )

## Structures with Larger Motifs





## Coordination Polyhedrons



## Coordination Polyhedrons



## Space Filling

|  | Atom Radius | Number of <br> Atoms (lattice <br> points) | Space filling |
| :--- | :---: | :---: | :---: |
| SC | $\mathrm{a} / 2$ | 1 | $52 \%$ |
| BCC | $\sqrt{ } 3 \mathrm{a} / 4$ | 2 | $68 \%$ |
| FCC | $\sqrt{ } 2 \mathrm{a} / 4$ | 4 | $74 \%$ |
| Diamond | $\sqrt{ } 3 \mathrm{a} / 8$ | 8 | $34 \%$ |


$\underline{\text { Type of Packing }} \xlongequal{$|  Packing  |
| :---: |
|  Efficiency  |$} \xlongequal{$|  Coordination  |
| :---: |
|  Number  |$}$

Simple cubic (sc)


Body-centered cubic (bcc)


52\% 6
$68 \%$
8

Hexagonal close-packed
$74 \%$12
(hcp)
74\%
12
Cubic close-packed


## $\mathrm{CCP}=\mathrm{FCC}$



Close packed layers of CCP are oriented perpendicularly to the body diagonal of the cubic cell of FCC

## Periodic Tolple of Metal Structures



## Two Types of Voids (Holes)



5 Tetrahedral hole



2 Octahedral hole


3


Tetrahedral Holes T+

N cp atoms in lattice cell

N Octahedral Holes
2N Tetrahedral Holes


Octahedral Holes


Tetrahedral Holes T-


## Two Types of Voids (Holes)



Octahedral Holes


Tetrahedral Holes

## Tetrahedral Holes (2N)


$\mathrm{Z}=\underline{4}$ number of atoms in the cell (N)
$\mathrm{N}=\underline{8}$
number of tetrahedral holes ( 2 N )

## Octahedral Holes (N)


$\mathrm{Z}=\underline{4}$ number of atoms in the cell ( N )
$\mathrm{N}=\underline{4}$
number of octahedral holes ( N )

## Different Types of Radii



1 Metallic radius


3 lonic radius

## Variation of atomic radii

## through the Periodic table



## Variation of ionic radii with coordination number



3 lonic radius

The radius of one ion was fixed to a reasonable value
( $\mathrm{r}\left(\mathrm{O}^{2-}\right)=140 \mathrm{pm}$ ) (Linus Pauling)
That value is then used to compile a set of self consistent values for all other ions.


## General trends for ionic radii

1. Ionic radii increase down a group.(Lanthanide contraction restricts the increase of heavy ions)
2. Radii of equal charge ions decrease across a period
3. Ionic radii increase with increasing coordination number the higher the CN the bigger the ion
4. The ionic radius of a given atom decreases with increasing charge

$$
\left(\mathbf{r}\left(\mathbf{F e}^{2+}\right)>\mathbf{r}\left(\mathbf{F e}^{3+}\right)\right)
$$

5. Cations are usually the smaller ions in a cation/anion combination (exceptions: $\mathbf{r}\left(\mathrm{Cs}^{+}\right)>\mathbf{r}\left(\mathbf{F}^{-}\right)$)
6. Frequently used for rationalization of structures:
„radius ratio" $\quad \mathbf{r}$ (cation)/r(anion) ( $<1$ )

## Variation of the electron density along the $\mathrm{Li}-\mathrm{F}$ axis in LiF

$\mathrm{P}-$ Pauling radius
G - Goldschmidt radius
S - Shannon radius.


## Cation/anion Radius Ratio



## Limiting Radius Ratios



Structure map:

Dependence of the structure type (coordination number) on the electronegativity difference and the average principal quantum number (size and polarizability)
AB compounds


## Lattice Enthalpy

The lattice enthalpy change $\Delta H_{L}^{0}$ is the standard molar enthalpy change for the following process:

$$
\mathrm{M}_{(\text {gas })}^{+}+\mathrm{X}_{\text {(gas) }}^{-} \rightarrow \mathrm{MX}_{\text {(solid) }} \quad \Delta H_{L}^{0}
$$

Because the formation of a solid from a „gas of ions" is always exothermic lattice enthalpies (defined in this way) are usually negative. If entropy considerations are neglected the most stable crystal structure of a given compound is the one with the highest lattice enthalpy.

## Lattice enthalpies can be determined by a thermodynamic cycle $\rightarrow$ Born-Haber cycle



A Born-Haber cycle for KCl
(all enthalpies: $\mathrm{kJ} \mathrm{mol}^{-1}$ for normal conditions $\rightarrow$ standard enthalpies)
standard enthalpies of

- formation: 438
- sublimation: +89 (K)
- ionization: + 425 (K)
- atomization: $+244\left(\mathrm{Cl}_{2}\right)$
- electron affinity: -355 (Cl)
- lattice enthalpy: x


## Born-Haber cycle


$\mathbf{0}=\mathbf{4 1 1}+\mathbf{1 0 8}+\mathbf{1 2 1}+\mathbf{5 0 2}+(-354)+\mathrm{L}$
$\mathrm{L}=-788 \mathrm{~kJ} \mathrm{~mol}^{-1}$
all enthalpies: $\mathrm{kJ} \mathrm{mol}^{-1}$ for normal conditions $\rightarrow$ standard enthalpies

## Lattice Enthalpy



$$
\mathrm{L}=\mathrm{E}_{\mathrm{coul}}+\mathrm{E}_{\mathrm{rep}}
$$

One ion pair
$\mathrm{E}_{\text {coul }}=\left(1 / 4 \pi \varepsilon_{0}\right) \mathrm{z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}} / \mathrm{d}$
$\mathrm{E}_{\text {rep }}=\mathrm{B} / \mathrm{d}^{\mathrm{n}}$
$\mathrm{n}=$ Born exponent (experimental measurement of compressibilty)

## Lattice Enthalpy

1 mol of ions

$$
\begin{aligned}
& \mathrm{E}_{\text {coul }}=\mathrm{N}_{\mathrm{A}}\left(\mathrm{e}^{2} / 4 \pi \varepsilon_{0}\right)\left(\mathrm{z}_{\mathrm{A}} \mathrm{z}_{\mathrm{B}} / \mathrm{d}\right) A \\
& \mathrm{E}_{\text {rep }}=\mathrm{N}_{\mathrm{A}} \mathrm{~B} / \mathrm{d}^{\mathrm{n}} \\
& L=N_{A} A \frac{Z_{A} Z_{B} e^{2}}{4 \pi \varepsilon_{0} d}+N_{A} \frac{B}{d^{n}} \\
& \mathrm{~L}=\mathrm{E}_{\text {coul }}+\mathrm{E}_{\text {rep }}
\end{aligned}
$$

Find minimum $\mathrm{dL} / \mathrm{d}(\mathrm{d})=0$

## Calculation of lattice enthalpies

Coulombic contributions to lattice enthalpies

$$
V_{A B}=-A\left(\frac{z_{+} z_{-} e^{2}}{4 \pi \varepsilon_{0} r_{A B}} N\right.
$$

$\mathrm{V}_{\mathrm{AB}}$ : Coulomb potential (electrostatic potential)
A: Madelung constant (depends on structure type)
N : Avogadro constant
z : charge number
e: elementary charge
$\varepsilon_{0}$ : dielectric constant (vacuum permittivity)
$\mathrm{r}_{\mathrm{AB}}$ : shortest distance between cation and anion

## Madelung Constant

Count all interactions in the crystal lattice


## Calculation of the Madelung constant



3D ionic solids:
Coulomb attraction and repulsion

Madelung constants:
CsCl: 1.763
$\mathrm{NaCl}: 1.748$
ZnS: 1.641 (wurtzite)
ZnS: 1.638 (sphalerite) ion pair: 1.0000 (!)
$A=6-\frac{12}{\sqrt{2}}+\frac{8}{\sqrt{3}}-\frac{6}{2}+\frac{24}{\sqrt{5}} \ldots \underset{\text { (infinite summation) }}{=1.748 \ldots(\mathrm{NaCl})}$

## Madelung constant for $\mathbf{N a C l}$


$\mathrm{E}_{\text {coul }}=\left(\mathrm{e}^{2} / 4 \pi \varepsilon_{0}\right) *\left(\mathrm{z}_{\mathrm{A}} \mathrm{z}_{\mathrm{B}} / \mathrm{d}\right) *[6(1 / 1)-12(1 / \sqrt{ } 2)+8(1 / \sqrt{ } 3)-6(1 / \sqrt{ } 4)+24(1 / \sqrt{ } 5) \ldots$. convergent series

$$
\mathrm{E}_{\text {coul }}=\left(\mathrm{e}^{2} / 4 \pi \varepsilon_{0}\right) *\left(\mathrm{z}_{\mathrm{A}} \mathrm{z}_{\mathrm{B}} / \mathrm{d}\right) * A
$$

## Madelung Constants for other Structural Types

| Structural Type | A |
| :---: | :---: |
| NaCl | 1.74756 |
| CsCl | 1.76267 |
| $\mathrm{CaF}_{2}$ | 2.519 |
| ZnS Sfalerite | 1.63805 |
| ZnS Wurtzite | 1.64132 |

## Born repulsion $\mathbf{V}_{\text {Born }}$



## Repulsion arising from overlap of electron clouds

Because the electron density of atoms decreases exponentially towards zero at large distances from the nucleus the Born repulsion shows the same behavior
approximation:

$$
V_{\text {Born }}=\frac{B}{r^{n}}
$$

$B$ and $n$ are constants for a given atom type; $n$ can be derived from compressibility measurements ( $\sim 8$ )

## Total lattice enthalpy from Coulomb interaction and Born repulsion

## $\Delta \mathrm{H}_{L}^{0}=\operatorname{Min} .\left(V_{A B}+V_{\text {Born }}\right)$

(set first derivative of the sum to zero)

$$
\Delta \mathrm{H}_{L}^{0}=-A \frac{z_{+} z_{-} e^{2}}{4 \pi \varepsilon_{0} r_{0}} N\left(1-\frac{1}{n}\right)
$$

Measured (calculated) lattice enthalpies ( $\mathrm{kJ} \mathrm{mol}^{-1}$ ):
$\mathrm{NaCl}:-772$ (-757); CsCl: -652 (-623) (measured from Born Haber cycle)

## The Kapustinskii equation

Kapustinskii found that if the Madelung constant for a given structure is divided by the number of ions in one formula unit $(v)$ the resulting values are almost constant:

| Structure | Madel. const.(A) | Alv | Coordination |
| :--- | :--- | :--- | :--- |
| CsCl | 1.763 | 0.88 | $8: 8$ |
| NaCl | 1.748 | 0.87 | $6: 6$ |
| $\mathrm{CaF}_{2}$ | 2.519 | 0.84 | $8: 4$ |
| $\alpha-\mathrm{Al}_{2} \mathrm{O}_{3}$ | 4.172 | 0.83 | $6: 4$ |

$\rightarrow$ general lattice energy equation that can be applied to any crystal regardless of the crystal structure

$$
\Delta \mathrm{H}_{L}^{0}=-\frac{1.079 \cdot 10^{5} v \cdot \mathrm{Z}_{+} \mathrm{Z}_{-}}{r_{+} \cdot r_{-}}
$$

## Most important advantage of the Kapustinski equation

$\rightarrow$ it is possible to apply the equation for lattice calculations of crystals with polyatomic ions (e.g. $\mathrm{KNO}_{3}$, $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{SO}_{4} \ldots$. .
$\rightarrow$ a set of „,thermochemical radii" was derived for further calculations of lattice enthalpies

Table 1.13 Thermochemical radii of polyatomic ions*

| Ion | $p m$ | Ion | pm | Ion | $p m$ |
| :--- | :---: | :--- | :--- | :--- | :--- |
| $\mathrm{NH}_{4}^{+}$ | 151 | $\mathrm{ClO}_{4}^{-}$ | 226 | $\mathrm{MnO}_{4}^{2-}$ | 215 |
| $\mathrm{Me}_{4} \mathrm{~N}^{+}$ | 215 | $\mathrm{CN}^{-}$ | 177 | $\mathrm{O}_{2}^{2-}$ | 144 |
| $\mathrm{PH}_{4}^{+}$ | 171 | $\mathrm{CNS}^{-}$ | 199 | $\mathrm{OH}^{-}$ | 119 |
| $\mathrm{AlCl}_{4}^{-}$ | 281 | $\mathrm{CO}_{3}^{2-}$ | 164 | $\mathrm{PtF}_{6}^{2-}$ | 282 |
| $\mathrm{BF}_{4}^{-}$ | 218 | $\mathrm{IO}_{3}^{-}$ | 108 | $\mathrm{PtCl}_{6}^{2-}$ | 299 |
| $\mathrm{BH}_{4}^{-}$ | 179 | $\mathrm{~N}_{3}^{-}$ | 181 | $\mathrm{PtBr}_{6}^{2-}$ | 328 |
| $\mathrm{BrO}_{3}^{-}$ | 140 | $\mathrm{NCO}^{-}$ | 189 | $\mathrm{PtI}_{6}^{2-}$ | 328 |
| $\mathrm{CH}_{3} \mathrm{COO}^{-}$ | 148 | $\mathrm{NO}_{2}^{-}$ | 178 | $\mathrm{SO}_{4}^{2-}$ | 244 |
| $\mathrm{ClO}_{3}^{-}$ | 157 | $\mathrm{NO}_{3}^{-}$ | 165 | $\mathrm{SeO}_{4}^{2-}$ | 235 |

[^0]
## Lattice Enthalpy

Born - Lande

$$
L=N_{A} M \frac{Z_{A} Z_{B} e^{2}}{4 \pi \varepsilon_{0} d}\left(1+\frac{1}{n}\right)
$$

| El. config. | n |
| :---: | :---: |
| He | 5 |
| Ne | 7 |
| Ar | 9 |
| Kr | 10 |
| Xe | 12 |

Born - Mayer

$$
L=N_{A} M \frac{Z_{A} Z_{B} e^{2}}{4 \pi \varepsilon_{0} d}\left(1-\frac{d^{*}}{d}\right) \quad \mathrm{d}^{*}=0.345 \AA
$$

## Lattice Enthalpy

Kapustinski
$\mathrm{M} / \mathrm{v}$ je přibližně konstantní pro všechny typy struktur $\mathrm{v}=$ počet iontů ve vzorcové jednotce

M nahrazeno 0.87 v , není nutno znát strukturu

$$
L=1210 v \frac{Z_{A} Z_{B}}{d}\left(1-\frac{0,345}{d}\right)
$$

Kapustinski

| structure | $\boldsymbol{M}$ | $\mathbf{C N}$ | stoichm | $\boldsymbol{M} / \boldsymbol{v}$ |
| :--- | :--- | :--- | :--- | :--- |
| CsCl | 1.763 | $(8,8)$ | AB | 0.882 |
| NaCl | 1.748 | $(6,6)$ | AB | 0.874 |
| ZnS sfalerite | 1.638 | $(4,4)$ | AB | 0.819 |
| ZnS wurtzite | 1.641 | $(4,4)$ | AB | 0.821 |
| $\mathrm{CaF}_{2}$ fluorite | 2.519 | $(8,4)$ | $\mathrm{AB}_{2}$ | 0.840 |
| $\mathrm{TiO}_{2}$ rutile | 2.408 | $(6,3)$ | $\mathrm{AB}_{2}$ | 0.803 |
| $\mathrm{CdI}_{2}$ | 2.355 | $(6,3)$ | $\mathrm{AB}_{2}$ | 0.785 |
| $\mathrm{Al}_{2} \mathrm{O}_{3}$ | 4.172 | $(6,4)$ | $\mathrm{A}_{2} \mathrm{~B}_{3}$ | 0.834 |

$\mathrm{v}=$ the number of ions in one formula unit

## Lattice Enthalpy of NaCl

Born - Lande calculation $\mathrm{L}=-765 \mathrm{~kJ} \mathrm{~mol}^{-1}$
Only ionic contribution
Experimental Born - Haber cycle $\mathrm{L}=-788 \mathrm{~kJ} \mathrm{~mol}^{-1}$
Lattice Enthalpy consists of ionic and covalent contribution

## Applications of lattice enthalpy calculations:

$\rightarrow$ thermal stabilities of ionic solids
$\rightarrow$ stabilities of oxidation states of cations
$\rightarrow$ Solubility of salts in water
$\rightarrow$ calculations of electron affinity data
$\rightarrow$ lattice enthalpies and stabilities of ,non existent" compounds

## Cation/anion Radius Ratio



$\mathbf{r}_{\text {cation }} / r_{\text {anion }}=$ ideal
Stable


Unstable

| CN | r/R |
| :---: | :---: |
| $12-$ hcp/ccp | 1.00 (substitution) |
| $8-$ cubic | $0.732-1.00$ |
| $6-$ octahedral | $0.414-0.732$ |
| $4-$ tetrahedral | $0.225-0.414$ |

Structure Types Derived from CCP $=\mathrm{FCC}$


## Pauling Rules

- Cation-Anion distance is determined by sums of ionic radii. Cation coordination environment is determined by radius ratio.
- The bond valence sum of each ion should equal oxidation state.
- Avoid shared polyhedral edges and/or faces. (particularly for cations with high oxidation state \& low coordination number)
- In a crystal containing different cations those with large valence and small coord. number tend not to share anions.
- The number of chemically different coordination environments for a given ion tends to be small.


## Structure Types Derived from CCP = FCC



## Characteristic Structures of Solids = Structure Types

Rock salt $\underline{\mathrm{NaCl}} \mathrm{LiCl}, \mathrm{KBr}, \mathrm{AgCl}, \mathrm{MgO}, \mathrm{TiO}, \mathrm{FeO}, \mathrm{SnAs}, \mathrm{UC}, \mathrm{TiN}, \ldots$
Fluorite ${\underline{\mathbf{C a F}_{2}}}_{\underline{2}} \mathbf{B a C l}_{\mathbf{2}}, \mathbf{K}_{\mathbf{2}} \mathbf{O}, \mathbf{P b O}_{\mathbf{2}} \ldots$
Lithium bismutide $\underline{\operatorname{Li}_{\underline{3}} \underline{B i}}$
Sphalerite (zinc blende) $\underline{\mathbf{Z n S}} \mathbf{C u C l}, \mathrm{HgS}, \mathrm{GaAs} . .$.
Nickel arsenide NiAs FeS, PtSn, CoS ...
Wurtzite $\underline{\mathbf{Z n S} \mathbf{Z n O}, ~ M n S, ~ S i C ~}$
Rhenium diboride $\mathrm{ReB}_{2}$

## Structure Types Derived from CCP = FCC

| Anions/cell ( $=4$ ) | Oct. (Max 4) | Tet. (Max 8) | Stoichiometry | Compound |
| :---: | :---: | :---: | :---: | :---: |
| 4 | $100 \%=4$ | 0 | $\mathrm{M}_{4} \mathrm{X}_{4}=\mathbf{M X}$ | $\begin{aligned} & \mathrm{NaCl} \\ & \text { (6:6 coord.) } \end{aligned}$ |
| 4 | 0 | 100\% = 8 | $\mathbf{M}_{8} \mathrm{X}_{4}=\mathrm{M}_{2} \mathbf{X}$ | $\mathbf{L i}_{2} \mathbf{O}$ <br> (4:8 coord.) |
| 4 | 0 | 50\% $=4$ | $\mathrm{M}_{4} \mathrm{X}_{4}=\mathbf{M X}$ | ZnS, sfalerite <br> (4:4 coord.) |
| 4 | 50\% = 2 | 0 | $\mathrm{M}_{2} \mathrm{X}_{4}=\mathrm{MX}_{2}$ | $\mathrm{CdCl}_{2}$ |
| 4 | 100\% = 4 | 100\% = 8 | $\mathrm{M}_{12} \mathrm{X}_{4}=\mathrm{M}_{3} \mathrm{X}$ | $\mathbf{L i}_{3} \mathbf{B i}$ |
| 4 spinel | 50\% $=2$ | 12.5\% = 1 | $\mathrm{M}_{3} \mathrm{X}_{4}$ | $\mathrm{MgAl}_{2} \mathrm{O}_{4}$, |

## Comparison between structures with filled octahedral and tetrahedral holes

| O/t | fcc (ccp) | hcp |
| :---: | :---: | :---: |
| all oct. | NaCl | NiAs |
| all tetr. | $\mathrm{CaF}_{2}$ | $\mathrm{ReB}_{2}$ |
| oft (all) | $\mathrm{Li}_{3} \mathrm{Bi}$ | $\left(\mathrm{Na}_{3} \mathrm{As}\right)$ (!) problem |
| $1 / 2 \underline{t}$ | sphalerite (ZnS) | wurtzite (ZnS) |
| $1 / 20$ | $\mathrm{CdCl}_{2}$ | $\mathrm{CdI}_{2}$ |

## Fluorite ( $\mathrm{CaF}_{2}$, antifluorite $\mathrm{Li}_{\mathbf{2}} \mathbf{O}$ )

Fluorite structure $=$ a face-centered cubic array (FCC) of cations $=$ cubic close packing (CCP) of cations with all tetrahedral holes filled by anions $=$ a simple cubic (SC) array of anions.

Antifluorite structure $=$ a face-centred cubic (FCC) array of anions = cubic close packing (CCP) of anions, with cations in all of the tetrahedral holes (the reverse of the fluorite structure).

Fluorite ( $\mathrm{CaF}_{\mathbf{2}}$, antifluorite $\left.\mathrm{Li}_{\mathbf{2}} \mathbf{O}\right)$


F/Li
$\mathrm{K}_{2}\left[\mathrm{PtCl}_{6}\right], \mathrm{Cs}_{2}\left[\mathrm{SiF}_{6}\right],\left[\mathrm{Fe}\left(\mathrm{NH}_{3}\right)_{6}\right]\left[\mathrm{TaF}_{6}\right]_{2}$


## Fluorite structures $\left(\mathrm{CaF}_{2}\right.$, antifluorite $\left.\mathrm{Li}_{\mathbf{2}} \mathbf{O}\right)$



Oxides: $\mathbf{N a}_{\mathbf{2}} \mathbf{O}, \mathrm{K}_{\mathbf{2}} \mathbf{O}, \mathbf{U O}_{\mathbf{2}}$, $\mathbf{Z r O}_{\mathbf{2}}, \mathbf{T h O}_{\mathbf{2}}$
alkali metal sulfides, selenides and tellurides
$\mathrm{K}_{2}\left[\mathrm{PtCl}_{6}\right],\left(\mathrm{NH}_{4}\right)_{2}\left[\mathrm{PtCl}_{6}\right]$, $\mathrm{Cs}_{2}\left[\mathrm{SiF}_{6}\right]$, $\left[\mathrm{Fe}\left(\mathrm{NH}_{3}\right)_{6}\right]\left[\mathrm{TaF}_{6}\right]_{2}$.

$$
\begin{aligned}
& \mathrm{CaF}_{2}, \mathrm{SrF}_{2}, \mathrm{SrCl}_{2}, \mathrm{BaF}_{2}, \mathrm{BaCl}_{2}, \mathrm{CdF}_{2,}, \mathrm{HgF}_{2}, \mathrm{EuF}_{2}, \beta-\mathrm{PbF}_{2,}, \mathrm{PbO}_{2} \\
& \mathrm{Li}_{2} \mathrm{O}, \mathrm{Li}_{2} \mathrm{~S}, \mathrm{Li}_{2} \mathrm{Se}, \mathrm{Li}_{2} \mathrm{Te}, \mathrm{Na}_{2} \mathrm{O}, \mathrm{Na}_{2} \mathrm{~S}, \mathrm{Na}_{2} \mathrm{Se}, \mathrm{Na}_{2} \mathrm{Te}, \mathrm{~K}_{2} \mathrm{O}, \mathrm{~K}_{2} \mathrm{~S}
\end{aligned}
$$

## Sphalerite (zincblende, ZnS)



Cubic close packing of anions with $1 / 2$ tetrahedral holes filled by cations


## Sphalerite (zincblende, ZnS)



## Sphalerite (zincblende, ZnS)



13-15 compounds: BP, BAs, AlP, AlAs, GaAs, GaP, GaSb, AlSb, InP, InAs, InSb

12-16 compounds: BeS, BeSe, BeTe, $\beta-\mathrm{MnS}$ (red), $\beta$-MnSe, $\beta$-CdS, CdSe, CdTe, HgS, HgSe, $\mathbf{H g T e}, \mathbf{Z n S e}$, ZnTe

Halogenides: AgI, CuF, CuCl, CuBr, CuI, NH4F
Borides: PB, AsB
Carbides: $\beta$-SiC
Nitrides: BN

Diamond


## Diamond


$\mathrm{SiO}_{2}$ cristobalite
hexagonal

$\mathrm{SiO}_{2}$ tridymite ice

## Cubic Diamond



## Diamond Structure

C, $\mathrm{Si}, \mathrm{Ge}, \alpha-\mathrm{Sn}$


- Add 4 atoms to a FCC
- Tetrahedral bond arrangement
- Each atom has 4 nearest neighbors and 12 next nearest neighbors


## Elements of the 14th Group



|  | $a(\AA)$ | $d\left(\mathrm{~g} . \mathrm{cm}^{-3}\right)$ |
| :--- | :---: | :---: |
| C | 3.566 | 3.515 |
| Si | 5.431 | 2.329 |
| Ge | 5.657 | 5.323 |
| $\mathrm{a}-\mathrm{Sn}$ | 6.489 | 7.285 |

Diamond Lattice (100)


Diamond Lattice (110)


## Diamond Lattice (111)



Diamond Lattice (111) Hard Sphere Model


## Diamond Lattice (111) Hard Sphere Model



Face Centered Cubic Lattice (111) Hard Sphere Model


## Wurzite, ZnS



Hexagonal close packing of anions with $1 / 2$ tetrahedral holes filled by cations

## Wurzite, ZnS



ZnO, ZnS, ZnSe, ZnTe, BeO, CdS, CdSe, MnS, AgI, AIN

## Semiconductors of 13-15 and 12-16 type

## Structure of III-V and II-VI Compound Semiconductors



Zinc blende


Wurtzite


## Rock Salt, $\mathbf{N a C l}$



Cubic close packing of anions with all octahedral holes filled by cations

Rock Salt, $\mathbf{N a C l}$

$\mathrm{Na}^{+}$

## Rock Salt, NaCl


(a)

(b)

Anion and cation sublattices

## Rock Salt Crystal Structure



## Rock salt structures ( $\mathbf{N a C l}$ )



Hydrides: LiH, NaH, KH

Borides: ZrB, HfB
Carbides: TiC, ZrC, VC, UC
Nitrides: $\mathbf{S c N}, \mathbf{T i N}, \mathbf{U N}, \mathbf{C r N}, \mathbf{V N}, \mathbf{Z r N}$
Oxides: $\mathrm{MgO}, \mathrm{CaO}, \mathrm{SrO}, \mathrm{BaO}, \mathrm{TiO}, \mathrm{VO}, \mathrm{MnO}, \mathrm{FeO}$, CoO, NiO
Chalcogenides: MgS, CaS, SrS, BaS, $\alpha-M n S, ~ M g S e, ~$ CaSe, SrSe, BaSe, CaTe

Halides: LiF, LiCl, LiBr, LiI, NaF, NaBr, NaI, KF, $\mathrm{KCl}, \mathrm{KBr}, \mathrm{KI}, \mathrm{RbF}, \mathrm{RbCl}, \mathrm{RbBr}, \mathrm{AgCl}, \mathrm{AgF}, \mathrm{AgBr}$

Intermetallics: SnAs
Other
$\mathrm{FeS}_{2}$ (pyrite), $\mathrm{CaC}_{2}$

## NiAs - type



Hexagonal close packing of anions with all octahedral holes filled by cations

NiS, NiAs, NiSb, NiSe, NiSn, NiTe, FeS, FeSe, FeTe, FeSb, PtSn, CoS, CoSe, $\mathrm{CoTe}, \mathrm{CoSb}, \mathrm{CrSe}, \mathrm{CrTe}, \mathrm{CoSb}$,

PtB (anti-NiAs structure)

## ReB $_{2}$ - type



Hexagonal close packing of anions with all tetrahedral holes filled by cations

## $\mathrm{Li}_{3} \mathbf{B i}$ - type (anti $\mathrm{BiF}_{3}$ )


$\left[\mathrm{Cr}\left(\mathrm{NH}_{3}\right)_{6}\right] \mathrm{Cl}_{3}, \mathrm{~K}_{3}\left[\mathrm{Fe}(\mathrm{CN})_{6}\right]$
bcc

## $\mathrm{Li}_{3} \mathrm{Bi}$ - type (anti $\mathrm{BiF}_{3}$ )



$$
\begin{aligned}
& \mathrm{Fe}_{3} \mathrm{Al} \\
& {\left[\mathrm{Cr}\left(\mathrm{NH}_{3}\right)_{6}\right] \mathrm{Cl}_{3}} \\
& \mathrm{~K}_{3}\left[\mathrm{Fe}(\mathrm{CN})_{6}\right]
\end{aligned}
$$

Cubic close packing of anions with all tetrahedral and octahedral holes filled by cations


CsCl
Primitive cubic packing of anions with all cubic holes filled by cations


## CsCl


$\mathrm{CsBr}, \mathrm{CsI}, \mathrm{CsCN}, \mathrm{NH}_{4} \mathrm{Cl}, \mathrm{NH}_{4} \mathrm{Br}, \mathrm{TlCl}, \mathrm{TlBr}, \mathrm{TlI}, \mathrm{CuZn}, \mathrm{CuPd}, \mathrm{LiHg}$

## $\mathrm{ReO}_{3}$



NaCl structure with $3 / 4$ of cations removed and $1 / 4$ of anions removed

$$
\mathbf{U O}_{3}, \mathrm{MoF}_{3}, \mathrm{NbF}_{3}, \mathrm{TaF}_{3}, \mathrm{Cu}_{3} \mathbf{N}
$$

## Perovskite, $\mathrm{CaTiO}_{3}$

Two equvivalent views of the unit cell of perovskite


O

Cubic "close packing" of Ca and O with $1 / 4$ octahedral holes filled by Ti cations

## Perovskite structure $\mathbf{C a T i O}_{3}$


$\mathrm{TiO}_{6}-$ octahedra
$\mathrm{CaO}_{12}$ - cuboctahedra
$\left(\mathrm{Ca}^{2+}\right.$ and $\mathrm{O}^{2-}$ form a cubic close packing)
preferred structure of piezoelectric, ferroelectric and superconducting materials

## Perovskite, $\mathrm{CaTiO}_{3}$

Cubic "close packing" of A and X with $1 / 4$ octahedral holes filled by B cations


Similarity to CsCl

## Perovskite, $\mathrm{CaTiO}_{3}$

## Perovskite Crystal Structure



A
$\mathrm{KNbO}_{3}, \mathrm{KTaO}_{3}, \mathrm{KIO}_{3}$, $\mathrm{NaNbO}_{3}, \mathrm{NaWO}_{3}, \mathrm{LaCoO}_{3}$, $\mathrm{LaCrO}_{3}, \mathrm{LaFeO}_{3}, \mathrm{LaGaO}_{3}$, $\mathrm{LaVO}_{3}, \mathrm{SrTiO}_{3}, \mathrm{SrZrO}_{3}$, $\mathrm{SrFeO}_{3}$.

## Rutile, $\mathbf{T i O}_{2}$



CN - stoichiometry Rule $\mathrm{A}_{\mathrm{x}} \mathrm{B}_{\mathrm{y}}$
$\mathrm{CN}(\mathrm{A}) / \mathrm{CN}(\mathrm{B})=\mathrm{y} / \mathrm{x}$
Distorted hexagonal close packing of anions with $1 / 2$ octahedral holes filled by cations (giving a tetragonal lattice)

## Rutile, $\mathbf{T i O}_{2}$

## Rutile Crystal Structure



$$
\begin{gathered}
\mathrm{GeO}_{2}, \mathrm{CrO}_{2}, \mathrm{IrO}_{2}, \mathrm{MoO}_{2}, \mathrm{NbO}_{2}, \beta-\mathrm{MnO}_{2}, \mathrm{OsO}_{2}, \mathrm{VO}_{2} \\
(>340 \mathrm{~K}), \mathrm{RuO}_{2}, \mathrm{CoF}_{2}, \mathrm{FeF}_{2}, \mathrm{MgF}_{2}, \mathrm{MnF}_{2}
\end{gathered}
$$

## The rutile structure: $\mathbf{T i O}_{\mathbf{2}}$


$\mathrm{TiO}_{6}$ - octahedra
$\mathrm{OTi}_{3}$ - trigonal planar
(alternative to $\mathrm{CaF}_{2}$ for highly charged smaller cations)

## The spinel structure: $\mathbf{M g A l}_{2} \mathbf{O}_{4}$


fcc array of $\mathrm{O}^{2-}$ ions, $\mathrm{A}^{2+}$ occupies $1 / 8$ of the tetrahedral and $\mathrm{B}^{3+} 1 / 2$ of the octahedral holes
$\rightarrow$ normal spinel:
$\mathrm{AB}_{2} \mathrm{O}_{4}$
$\rightarrow \underline{\text { inverse spinel: }}$
$\mathrm{B}[\mathrm{AB}] \mathrm{O}_{4}\left(\mathrm{Fe}_{3} \mathrm{O}_{4}\right)$ :
$\mathrm{Fe}^{3+}\left[\mathrm{Fe}^{2+} \mathrm{Fe}^{3+}\right] \mathrm{O}_{4}$
$\rightarrow$ basis structure for several magnetic materials

## Spinel

$\mathrm{AB}_{2} \mathrm{X}_{4}$ Spinel normal: Cubic close packing of anions with $1 / 2$ octahedral holes filled by B cations and $1 / 8$ tetrahedral holes by A cations
$\mathrm{MgAl}_{2} \mathrm{O}_{4}, \mathrm{CoAl}_{2} \mathrm{O}_{4}, \mathrm{MgTi}_{2} \mathrm{O}_{4}, \mathrm{Fe}_{2} \mathrm{GeO}_{4}, \mathrm{NiAl}_{2} \mathrm{O}_{4}, \mathrm{MnCr}_{2} \mathrm{O}_{4}$
$\mathrm{AB}_{2} \mathrm{X}_{4}$ Spinel inverse: As for spinel but A cations and 1/2 of B cations interchanged
$\mathrm{MgFe}_{2} \mathrm{O}_{4}, \mathrm{NiFe}_{2} \mathrm{O}_{4}, \mathrm{MgIn}_{2} \mathrm{O}_{4}, \mathrm{MgIn}_{2} \mathrm{~S}_{4}, \mathrm{Mg}_{2} \mathrm{TiO}_{4}, \mathrm{Zn}_{2} \mathrm{TiO}_{4}, \mathrm{Zn}_{2} \mathrm{SnO}_{4}$, $\mathrm{FeCo}_{2} \mathrm{O}_{4}$.

## Layered Structures

$\mathrm{CdI}_{2}$ Hexagonal close packing of anions with $\mathbf{1 / 2}$ octahedral holes filled by cations
$\mathrm{CoI}_{2}, \mathrm{FeI}_{2}, \mathrm{MgI}_{2}, \mathrm{MnI}_{2}, \mathrm{PbI}_{2}, \mathrm{ThI}_{2}, \mathrm{TiI}_{2}, \mathrm{TmI}_{2}, \mathrm{VI}_{2}, \mathrm{YbI}_{2}, \mathrm{ZnI}_{2}, \mathrm{VBr}_{2}$, $\mathrm{TiBr}_{2}, \mathrm{MnBr}_{2}, \mathrm{FeBr}_{2}, \mathrm{CoBr}_{2}, \mathrm{TiCl}_{2}, \mathrm{TiS}_{2} ., \mathrm{TaS}_{2}$.
$\mathrm{CdCl}_{2}$ Cubic close packing of anions with $\mathbf{1 / 2}$ octahedral holes filled by cations
$\mathbf{C d C l}_{2}, \mathrm{CdBr}_{2}, \mathrm{CoCl}_{2}, \mathrm{FeCl}_{2}, \mathrm{MgCl}_{2}, \mathrm{MnCl}_{2}, \mathbf{N i C l}_{2}, \mathbf{N i I}_{2}, \mathrm{ZnBr}_{2}, \mathrm{ZnI}_{2}$, $\mathrm{Cs}_{2} \mathrm{O}^{*}$ (anti-CdCl ${ }_{2}$ structure)

## $\mathrm{CdI}_{2}$ Hexagonal close packing



## $\mathrm{CdCl}_{2}$ Cubic close packing



## $\mathrm{CdCl}_{2}$ Cubic close packing



Fázové přeměny za zvýšeného tlaku


Zvýšení koordinačního čísla<br>Důsledky zvýšení tlaku Zvýšení hustoty<br>Prodloužení vazebných délek Přechod ke kovovým modifikacím

## X-ray structure analysis with single crystals



## Principle of a four circle X-ray diffractometer for single crystal structure analysis






[^0]:    *J.E. Huheey (1983) Inorganic Chemistry, 3rd edn, Harper and Row, London, based on data from H.D.B. Jenkins and K.P. Thakur (1979) J. Chem. Ed., 56, 576.

