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How to Define a Unit of Length

(extended version)

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Abstract

In this paper, I shall discuss the issue whether the standard meter in Paris is in fact one meter long. Whether one could meaningfully assert this proposition depends on how the unit of length meter is defined. I would like to suggest three conceivable definitions. (1) One meter long is everything that has the same length as an arbitrary chosen rod S now has. (2) According to the second definition one meter long is everything that coincides in the endpoints with the rod S when placed alongside. (3) The third definition states that one meter long is—in a literal sense—the rod S solely. Other objects are one meter long—although in a derived sense—if they coincide in the endpoints with S when placed alongside. The first definition is in essence the standpoint of Kripke, the second one can be attributed to Wittgenstein, the last definition is the proposal I would like to advocate here. In particular, I hold that the third definition can be attributed to Wittgenstein as well. A language-game of measuring presupposes a preparatory game of fixing a unit of measure. The meaning of the expression "standard meter" must thus be derived from this preparatory game. Therefore with all other objects, one can say only in a derived sense that they are one meter long or not.

Wittgenstein says that the only thing of which one can say neither that it is one meter long nor that it is not one meter long is the standard meter in Paris.[1] Kripke says in reply he must be wrong,[2] the stick which is actually the standard meter is (but might not be) one meter long as a matter of fact. Most commentators have sided with Wittgenstein.[3] Whether one could meaningfully assert that the standard meter is one meter long depends on how the unit of length the meter is defined. In this paper, I would like to suggest three conceivable definitions. Both Wittgenstein and Kripke would agree that the meter is to be defined via an ostensive definition (or an act of baptizing in a term used by Kripke). All the definitions make use of an arbitrary chosen rod, which shall play the role of the standard meter henceforward. Let us call it S. (1) The first definition is: one meter long is everything that has the same length as this rod S now (or in t_0) has. (2) According to the second definition one meter long is everything that coincides in endpoints with the rod S when placed alongside.

[1] *Philosophical Investigations*, § 50.

[2] *Naming and Necessity*, p. 54.

[3] See, e.g., Loomis 1999 or Avital 2008.

(3) The third definition states that one meter long is—in a literal sense—the rod S solely.[4] Other objects are one meter long—although in an analogical (or better synecdochical) sense—if they coincide in the endpoints with S when placed alongside. In essence, the first definition is the Kripke's standpoint, the second one can be attributed to Wittgenstein,[5] while the last definition is the proposal I would like to advocate.

Wittgenstein cannot say that S is (or is not) one meter long for a measuring instrument cannot be applied to itself. That S is one meter long is not a proposition but a rule which is actually timeless. For Kripke the statement "stick S is one meter long" is a priori but only a contingent proposition. The rod S may vary its length in time (and in counterfactual situations as well) and in this case ceases to be one meter long. The definition of mine is a reverse of that by Wittgenstein. I start by fixing the length of the standard meter and then apply this unit of measurement to all other spatial objects in a derived sense. The second definition starts by defining the length of all objects and then he cannot proceed to the standard meter, because it is a blind spot of the system.

The crucial point here is, of course, the longitudinal variability of all spatial objects. The standard meter is no exception. Note that it does not matter what is the cause of the variability (e.g. temperature, pressure, or a human action).[6] How do we detect that an object has changed its length? The answer is simple: by a measurement using the standard meter (or a measuring instrument, e.g. a ruler, which is derived from the standard meter). But how do we detect that the standard meter itself has changed its length? First, I would try to give an intuitive answer. If all particular measuring instruments were suddenly twice their length, that we would have a good reason to suppose that the standard meter has been halved. That is, I would suggest, the only possible answer, if we define the meter along Kripke's lines. This would mean, however, that the rod S has been deposed from the role of the standard meter. It is not the rod S but a set of measuring instruments (provided they are all matching) or their average or median length. Who can decide whether the rod S has been halved or the other measuring instruments have been doubled in their lengths? There is not and cannot be any evidence that could support either alternative. Kripke also admits the possibility that the standard meter could have changed its length; this possibility, however,

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[4] As far I know, this idea has been suggested only in Fogelin (2002, p. 127): "To begin with, it may not seem obvious that we cannot say of the standard meter that it is a meter long; indeed, we may be inclined to say the opposite, that it is the only thing that *really* is one meter long" (italics in the original). Fogelin, however, does not attribute this view to Wittgenstein.

[5] See Loomis 1999, p. 304.

[6] One could raise an objection that the length of the meter cannot change, because the current definition of the meter is fixed by the speed of light. The meter is the length of the path travelled by light in vacuum during a time interval of 1⁄299 792 458 of a second. One can, however, ask—as physicists actually do—whether the speed of light is constant or changing in time, cf. the so-called *variable speed of light hypothesis*. The philosophical lesson would be the same, albeit there is no natural intuition how to detect such variability. The generalized subject of this essay is thus how to define a physical unit at all.

cannot be detected. This is due to the fact that the paradigm of one meter is not a concrete object (the rod S) but an abstraction, i.e. the length which it accidentally had in t_0.
If Wittgenstein cannot assert that the rod S is one meter long, then *a fortiori* he cannot assert that the rod S has changed its length either. Both eventualities are, however, perfectly conceivable and this makes Wittgenstein's account counterintuitive and puzzling. All change, to be sure, has to be conceived with respect to a stationary background, to a *hypokeimenon*. When we recall the dilemma exposed in the previous paragraph, by choosing S as the standard meter we had stated that all change is to be conceived relatively to S. The standard meter is thus an arbitrary chosen fixed point (or better: a fixed vector) in our (metric) system of measurement. The background (hypokeimenon) is here—and this is Wittgenstein's point—nothing metaphysical, but only a chosen role in this system.[7] That is, I think, a sufficient argument that the standard meter cannot change its length.
Gert (2002) has, however, tried to demonstrate that it was not Wittgenstein's intention to deny that one cannot say of the standard meter that it is or it isn't one meter long.[8] Wittgenstein latter writings have actually a peculiar dialogical structure which implies that not all sentences express Wittgenstein's own view. I do not want to dispute whether Gert is right or wrong. In any case, we can take her interpretation as an indication that a straightforward attribution of the definition (2) to Wittgenstein might be problematic. Wittgenstein employs the standard meter only as an analogy to the more general problem of whether primary elements can be described or named only. But such a question is a wrong one. In order to describe an object, we must be able to name it; and in order to name an object, we must assign a name to it in advance. In Wittgenstein's words: "For naming and describing do not stand on the *same* level: naming is a preparation for description. Naming is so far not a move in the language-game–any more than putting a piece in its place on the board is a move in chess."[9] So a game of chess presupposes a preparatory game of the naming of the pieces.[10] In a similar way: a language-game of measuring presupposes a preparatory game of fixing a unit of measure, its standard, and the whole method of measurement.
The definition (3) is in accord with this analysis. In a preparatory language-game, we have to choose a standard of measurement, in particular name the rod S as the standard meter. That is the first step of the definition (3): saying that the rod S solely is one meter long. After this is

[7] To recall the previous footnote, what is the background against which we could measure the variability of the speed of light? The variability of the speed of light makes sense only if the remaining fundamental physical constants (that are the electron charge and Plank's constant) are fixed. For a detailed discussion see Uzan (2003).

[8] A close reading of the § 50 of the *Philosophical Investigations* vindicating this conjecture can be found in Gierlinger (2010).

[9] *Philosophical Investigations*, § 49.

[10] Gert (2002, p. 63) argues, however, that we can do this "*even within a given language-game*". I do not find her line of reasoning convincing at this point. A naming a piece is not a move in chess, although it is a move in a language-game of ostensive teaching of chess pieces.

done, we can *play* the language-game of measurement, which takes the name of the standard meter as assigned and the length of its extension (i.e. the stick S) as invariable. This is to understand that in this language-game, the (literal) meaning of the term "standard meter" is borrowed from the preparatory game and applied in a derived sense to all other objects. Between these two language-games there is, thus, a *vertical* relation.[11]

Avital (2008) has recently argued—in favor of the definition (2)—that the endpoints of the rod S (taken as numerical coordinates in a Euclid space) exemplify an internal relation of measure. Another object R is one meter long, if its endpoints exemplify the same internal relation as the endpoints of S. We can assert "R is one meter long", because the endpoints of R may exemplify another internal relation, e.g. to be two meters distant. If we embrace the definition (3), an object R is one meter long, if it shares the same form with the standard meter which means that there is an internal relation (of sameness) between them. This interpretation is in accord with Wittgenstein's account of colors (a colored square as a paradigm of a color) or even with aspect-seeing (an exemplification of an internal relation between the concepts involved).

Literature

Avital, Doron (2008) "The Standard Metre in Paris". *Philosophical Investigations* **31**:4, 318-339.

Fogelin, Robert J. (2002) *Wittgenstein*, Second edition, New York: Routledge.

Gert, Heather J. (2002) "The Standard Meter by Any Name is Still a Meter Long". *Philosophy and Phenomenological Research* **LXV**:1, 50-68.

Gierlinger, Frederik (2010) "Darstellungsweisen. Eine Notiz zum § 50 der *Philosophischen Untersuchungen*". In: *Papers of the 33. International Wittgenstein Symposium*. Kirchberg am Wechsel: Austrian Ludwig Wittgenstein Society, 101-103.

Kripke, Saul (1980) *Naming and Necessity.* Oxford: Blackwell.

Loomis, Eric (1999) "Necessity, the a priori, and the Standard Meter". *Synthese* **121**, 291-307.

Ter Hark, Michel (1990) *Beyond the Inner and the Outer*. Dordrecht: Kluwer.

Uzan, Jean-Philippe (2003) "The Fundamental Constants and their Variation: Observational Status and Theoretical Motivations". *Rev. Mod. Phys.* **75**, 403-455. Online: http://link.aps.org/doi/10.1103/RevModPhys.75.403.

Wittgenstein, Ludwig (1998) *Philosophical Investigations*. Oxford: Blackwell.

[11] The expression "vertical relation" is due to ter Hark (1990, p. 34). The failure to consider vertical relation between language-games is called the "ground-floor fallacy", e.g. the naming and using the standard meter within the same language-game.