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Job Differentiation *vs*. Unemployment

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Abstract: We use a matching model in which the horizontal job differentiation results from the rationale response of firms to the state of the labor market. We show that a decrease in the labor market tightness gives firms an incentive to raise the differentiation degree of jobs. Comparative statics suggests that an increase in unemployment benefits and in the minimum wage improves productivity of skilled workers by making jobs more differentiated, and leads to a raise in unemployment rate.

Keywords: Job Differentiation, Productivity, Matching, Unemployment

Classification JEL: J64, J65

Introduction

In context of economic crisis, unemployment and the deteriorating situation of unskilled workers have become major priorities of the labor market institutions in several developed countries. In response, they try to reform their public policies such as minimum wage and income redistribution system. This paper aims to study the effects of the minimum wage and unemployment benefits in a matching model with an original presentation of interactions between unemployment and job differentiation.

In the literature, other search models with ex-ante heterogeneous workers have been proposed (Marimon and Zilibotti, 1999; Gautier and Teulings, 2004; Nickell, 2004). In these models, job differentiation is generally regarded as an exogenous parameter. In the same mind, other authors, mainly Acemoglu (1999), Gautier (1999), Albrecht and Vroman (2002), have attempted to endogenize the skill requirements. However, their models focus on vertical differentiation and consider that job productivity (when filled by a skilled worker) does not depend on the state of the labor market. In other words, job differentiation remains essentially exogenous (Mortensen and Pissarides, 1999).

The main contribution and originality of this paper focus on two points. Firstly, we argue that horizontal jobs differentiation is an endogenous variable which results from the rationale response of firms to the state of the labor market. In other words, we will show that firms will be encouraged to offer more adapted jobs to the abilities of skilled workers until unemployment (labor market state) would facilitate their recruitment. Secondly, we study this horizontal differentiation by considering that there is a skill bias in favor of skilled workers.

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We use a matching model (Pissarides, 2000) in which the horizontal job differentiation is represented as a point of a line segment. Thus, the degree of adequacy is measured by the distance between the two ends of the segment and the point. Intuitively, we assume that an increase in this degree of differentiation raises the output of well-suited workers and lowers the output of ill-suited workers. When entering the labor market, firms decide on this degree by maximizing the value of a vacancy. The hiring process between workers and firms is formalized by the usual matching function (Petrongolo and Pissarides, 2001).

We show that a decrease in the labor market tightness gives the firms an incentive to raise the differentiation degree of jobs. In this framework, we study the effects of unemployment benefits and minimum wage on productivity and unemployment. We obtain that an increase in unemployment benefits improves the productivity of skilled workers by making jobs more differentiated, and leads to a raise in unemployment rate. Comparative statics also suggests that the minimum wage has the same effects on the model variables.

The rest of paper is organized as follows: The model is presented in Section 1. Then, solving of the model and the definition of its equilibrium are discussed in Section 2. We define and study the comparative statics properties of the model in Section 3.

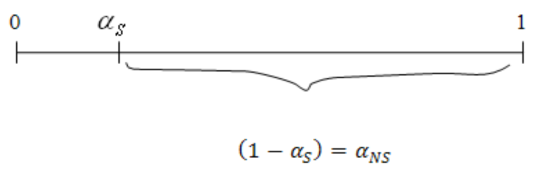
The Model

Consider an economy populated by N heterogeneous workers and by M homogeneous firms. Workers who are horizontally differentiated by their type of qualification are divided into two categories: λ_S the proportion of skilled workers, and (1-λ_S) the proportion of unskilled workers. We assume that each employee can only apply for one job per period, and that each firm offers a single job. In contrast to workers, firms in this economy are identical and their number is exogenous.

1.1 Job Differentiation

Each job is characterized by a degree of adequacy to both types of workers. This formalization is the major element of our framework. A given profile corresponds to a point of a line segment whose length is normalized to unity.

Figure 1: The degree of adequacy of a job



The distance, α_S measures the adequacy of job to the qualifications of skilled workers, while $1-\alpha_S=\alpha_{NS}$is the adequacy to unskilled workers. When α_S, the job corresponds perfectly to skilled workers and their productivity reaches the maximum, normalized at β, whereas that of unskilled is equal to its minimal value μ ($0 < \mu \leq 1$).

Figure 2: The productivity of skilled worker

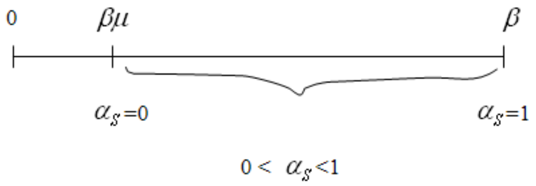
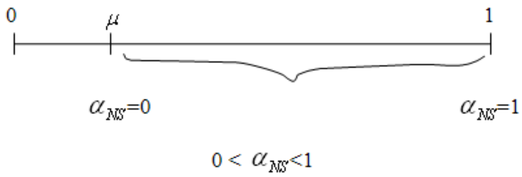


Figure 3: The productivity of unskilled worker



When α_S= 1, the productivity of unskilled is equal to unity while that of skilled is equal to its minimal value $\beta\mu$. Formally, we assume that worker productivity y_i with i={s; ns} is an increasing function f of the degree of adequacy α_i to job.

$$y_S = f(\alpha_S) \quad (1) \qquad \text{and} \qquad y_{NS} = f(\alpha_{NS}) \quad (2)$$

Considering that job is biased towards skilled workers, their productivity can be rewritten:

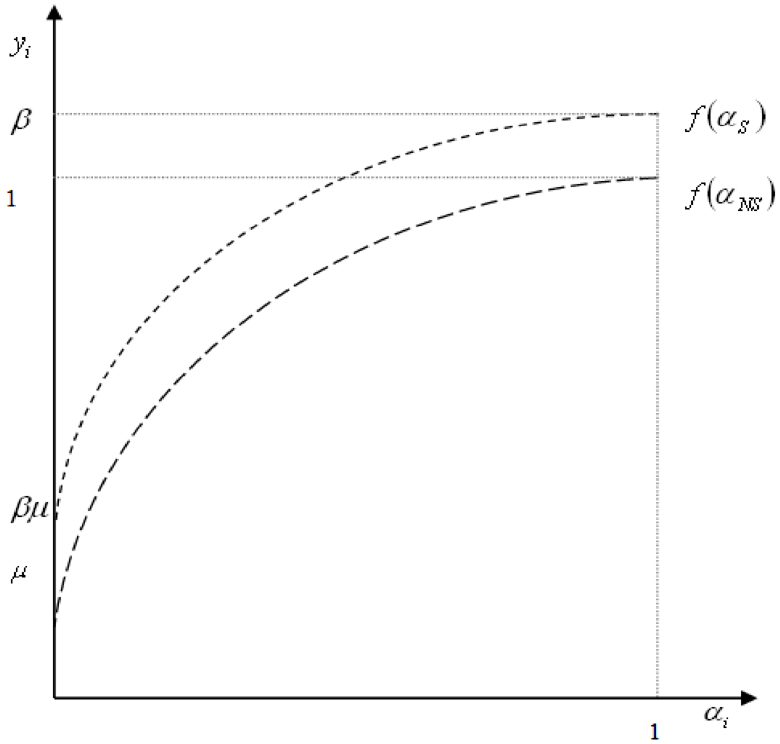
$$y_S = \beta f(\alpha_S) \quad (3)$$

with $\beta > 1$ which represents the level of qualification (skill bias) of skilled workers relative to unskilled. If job is horizontally undifferentiated, when $\alpha_{NS} = \alpha_S = 1/2$, skilled workers have higher productivity than the unskilled.

$$y_{NS} = f(\alpha_{NS}) < y_S = \beta f(\alpha_S) \quad (4)$$

This skill bias can be explained through the complementarity relation between technology and qualification. Skilled workers are more able to adapt their abilities (namely in computing) to all types of jobs. This increasing function $f(\alpha_i)$ is assumed strictly concave ($f' > 0; f'' < 0$).

Figure 4: Adequacy and productivity



Moreover, each type of job corresponds to the productivity of a skilled or unskilled worker. At this productivity y_i, the inverse function of $f(\alpha_i)$, combines a degree of adequacy to skilled and unskilled workers:

$$\alpha_S = f^{-1}\left(\frac{y_S}{\beta}\right) \quad (5) \qquad \text{and} \qquad \alpha_{NS} = f^{-1}(y_{NS}) \quad (6)$$

Given that $\alpha_{NS} + \alpha_S = 1$, we deduce the degree of adequacy of unskilled workers:

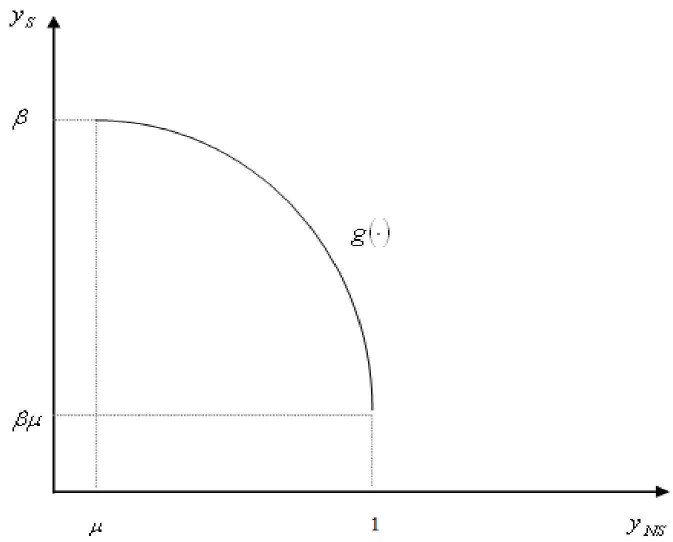
$$y_{NS} = f\left(1 - f^{-1}\left(\frac{y_S}{\beta}\right)\right) \quad (7)$$

We note $g(.)$ the function thus obtained:

$$y_{NS} = g\left(\frac{y_S}{\beta}\right) \quad (8) \qquad \text{and} \qquad y_S = \beta g(y_{NS}) \quad (9)$$

Taking into account hypotheses of the function $f(\alpha_i)$, we verify that substitution relationship (of productivities), represented by g(.), is decreasing and concave ($g' < 0; g'' < 0$).

Figure 5: Substitution relationship



Increasing y_S requires that job is better suited to the characteristics of skilled workers and less suited to those of unskilledy_{NS}whose productivity necessarily decreases. For $y_S > y_{NS}$ the slop e$g(y_S)$ has a value greater than unity and increases withy_S.

1.2 Hiring Process

There are frictions in the labor market that stops instantaneous matching of unemployed workers (skilled or unskilled) and vacant jobs. In order to give solid microeconomic foundations to meeting process between workers and firms, we use here the "urn-ball model" (Petrongolo and Pissarides, 2001). According to this model and the "job search theory" (McKenna, 1985), we assume that an unemployed worker meets one firm likely in each period. This firm is taken at random from all the firms. We also consider that workers orient correctly their job search to the extent that they meet the firms offering more or less differentiated jobs in one point which represents the whole labor market (Albrecht and al., 2003). The "firm-employee" match is done at this point which includes(λ_S N) skilled workers, $(1 - \lambda_S)$ N unskilled and M firms. Let θ, represents the labor market tightness (θ = M / N). Given that each worker sends a single application, the probability that the firm doesn't meet a skilled worker is given by:

$$\left(1 - \frac{1}{M}\right)^{N\lambda_S} \tag{10}$$

Considering that jobs (*M*) and workers (*N*) approaching to infinity, the probability to fill a job by a skilled worker is:

$$q_S = 1 - e^{-\frac{\lambda_S}{\theta}} \tag{11}$$

This probability is a decreasing function of θ. Owing to the congestion effect, a rise in the number of vacant jobs has a negative impact on the probability of filling a job. In the same way, increasing λ_S causes a rise in this probability owing to the opportunity effect. Moreover, the probability to meet an unskilled worker is given by the following equation:

$$q_{NS} = e^{-\frac{\lambda_S}{\theta}}\left(1 - e^{-\frac{(1-\lambda_S)}{\theta}}\right) \tag{12}$$

The impact of θ is identical on this probability due to the same effect. However, any increase in λ_S, causes a decrease of this probability. Finally, the probability to meet a skilled or unskilled is given by the sum of: $q = q_S + q_{NS}$

$$q = 1 - e^{-\frac{1}{\theta}} \tag{13}$$

This probability does not depend on the proportion of skilled workers but of the labor market tightness whose impact is negative (the congestion effect).

1.3 Utilities, Profits

In accordance with traditional matching models, wages given to workers result from a bargaining process according to their bargaining power. This wage is noted w_i with $i=\{s;\ ns\}$. In this model the utility of employees is represented only by their wages and that of the unemployed corresponds to their unemployment benefits denoted b. For firms, entering the labor market and creating a vacancy impose a cost noted c. We consider the profit P_T of a firm whose job is filled by a skilled or unskilled $T=\{s;\ ns\}$. We obtain:

$$P_T = y_T - w_T \tag{14}$$

Sharing the total surplus associated with a job is done accordingly to Nash's generalized rule, depending on the bargaining power of employees and firms. We denote σ ($0<\sigma<1$) the bargaining power of workers. We obtain:

$$w_T - b = \sigma(y_T - b) \tag{15}$$

For σ =1employees capture all the surplus created by the current job; $y_T = w_T$. The firm's profit can then be rewritten:

$$P_T = (1-\sigma)(y_T - b) \tag{16}$$

The value of a vacant job is denoted by F given by:

$$F = -c + q_S P_S + q_{NS} P_{NS} \tag{17}$$

This value decreases with the cost c and increases with the probability of meeting an unemployed. However, under the assumption of labor market free-entry, the value of a vacancy is equal to zero (F=0) and we have:

$$c = q_S P_S + q_{NS} P_{NS} \tag{18}$$

2. Solving the model

Solving the model consists of establishing interactions at the stationary equilibrium, between labor market tightness and Job differentiation.

2.1 Optimal job differentiation

When entering the labor market, the firm decides on the degree of differentiation. Formally, productivity of a skilled worker y_S is obtained by maximizing the value of the vacancy under the constraint imposed by the substitution relationship:

$$\mathrm{M}ax \;\; F = -c + q_S P_S + q_{NS} P_{NS} \quad \text{s.c} \quad y_{NS} = g\left(\frac{y_S}{\beta}\right)$$

The first order condition satisfies (appendix A):

$$q_S + q_{NS} . g'\left(\frac{y_S}{\beta}\right)\frac{1}{\beta} = 0 \tag{19}$$

Taking into account the concavity of $g(.)$, we easily obtain the following result (appendix A):

Proposition 1: *In the optimum of profits, productivity* (y_S) *is an increasing function of the ratio*(q_S/q_{NS}).

This result is very intuitive and is interpreted in the following way. An increase in the probability to meet a skilled worker (increase in(q_S/q_{NS})) has the effect of facilitating its recruitment. Therefore, firms adapt their job-technology to the labor market state by creating jobs more differentiated in favor of skilled workers. This improvement of the matching quality leads to an increase in productivity. The concavity of $g(.)$ ensures that the first order condition is satisfied (Amine andLages,2010, 2011).

2.2 Labor market tightness and unemployment

In this section, we establish the interactions between the labor market tightness (unemployment) and productivity. The ratio(q_S/q)can be written as follows:

$$\left(\frac{q_S}{q}\right) = \frac{1 - e^{-\frac{\lambda_S}{\theta}}}{1 - e^{-\frac{1}{\theta}}} \tag{20}$$

The derivative of this ratio with respect to θ depends on the sign of the following expression:

$$\frac{1-e^{-\frac{\lambda_S}{\theta}}}{\frac{\lambda_S}{\theta}e^{-\frac{\lambda_S}{\theta}}}-\frac{1-e^{-\frac{1}{\theta}}}{\frac{1}{\theta}e^{-\frac{1}{\theta}}} \qquad (21)$$

To obtain the sign of this expression, we consider the following function: $\varepsilon(x)$is strictly increasing for $x > 0.$ (appendix B)

$$\varepsilon(x)=\frac{1-e^{-x}}{xe^{-x}} \qquad (22)$$

Knowing that$\lambda_S < 1$, we deduce that the expression (21) is negative. Therefore, the derivative of the ratio (q_S/q) with respect to θ is negative. Given the *proposition1*, we can then state the following result:

Proposition 2: *In a matching model with heterogeneous workers, an increase in the labor market tightness reduces the job differentiation.*

The result is interpreted with great simplicity when the ratio (y_S/θ)isgreater than unity. Indeed, any increase in the labor market tightness (i.e. decrease in unemployment) produces a sufficient decrease in the probability to meet a skilled worker relative to unskilled. Considering the *proposition 1*, firms react by reducing the differentiation of jobs, thus deteriorating the productivity. Therefore, *proposition 2* implies a positive relationship between unemployment and job differentiation.

2.3 Equilibrium

The equilibrium of the labor market can be defined as a couple$(y_S;\theta)$ which satisfies the following expressions:

$$\begin{cases} -c+(1-\sigma)\left(q_S(y_S-b)+q_{NS}(y_{NS}-b)\right)=0 & (23) \\ q_S+q_{NS}.g'\left(\dfrac{y_S}{\beta}\right)\dfrac{1}{\beta}=0 & (19) \end{cases}$$

This equilibrium is obtained from equations (16), (18) and (19). Once the labor market tightness is determined, we derive the equilibrium unemployment noted *U*:

$$U=(\lambda_S N+(1-\lambda_S)N)-(q_S+q_{NS})M \qquad (24)$$

The equilibrium unemployment rate, denoted *u*, is then given by:

$$u=1-\theta q \qquad (25)$$

3. Comparative Statics

In this section, we study the effects of the unemployment benefits and the minimum wage on variables of the model and particularly on job differentiation.

3.1 Unemployment benefits effects

In order to deduce the unemployment benefits effect, we totally differentiate the equilibrium equation (23) according to this parameter. We obtain:

$$(1-\sigma)\left(\frac{\partial q_S}{\partial \theta}(y_S-b)+\frac{\partial q_{NS}}{\partial \theta}\left(g\left(\frac{y_S}{\beta}\right)-b\right)\right)d\theta-(1-\sigma)q\,db=0 \tag{26}$$

Given that $\frac{\partial q_S}{\partial \theta}<0$,the expression $\left(\frac{\partial q_S}{\partial \theta}(y_S-b)+\frac{\partial q_{NS}}{\partial \theta}\left(g\left(\frac{y_S}{\beta}\right)-b\right)\right)<0$ is negative. We obtain the effect of b on θ:

$$\frac{d\theta}{db}=\frac{q}{\left(\frac{\partial q_S}{\partial \theta}(y_S-b)+\frac{\partial q_{NS}}{\partial \theta}\left(g\left(\frac{y_S}{\beta}\right)-b\right)\right)}<0 \tag{27}$$

Using the *propositions 1 and 2*, we establish the effect on productivity y_S while the impact on unskilled productivity is given by the substitution relationship (equation (8)). Concerning the effect on wages is given by differentiation of the equation (15):

$$dw_S=\sigma dy_S+(1-\sigma)db \quad >0 \tag{28}$$

$$dw_{NS}=\sigma dy_{NS}+(1-\sigma)db \quad =? \tag{29}$$

Given that $\frac{\partial q}{\partial \theta}<0$,the impact on the unemployment rate is deduced as follows:

$$du=d\theta q-\theta\frac{\partial q}{\partial \theta}dq \quad >0 \tag{30}$$

The increase in unemployment benefits reduces the expected profits of firms and leads to the decrease in tightness θ by reducing the creation of vacancies. Consequently, unemployment rate rises as well as the probability to meet a skilled worker. Therefore and according to the *proposition 1 and 2*, firms react face to this labor market state by adapting their technology and by creating more differentiated jobs in favor to skilled workers. The matching quality is thus improved and the productivity of skilled workers rises, while that of unskilled is reduced. Moreover, the positive impact on the wages of skilled workers is explained partly by the increased productivity, and secondly by the increase in utility of the unemployed *b*. On the contrary, the effect on the wages of unskilled remains undetermined because we have two opposite effects.

Table 1: Unemployment benefits effects

	θ	y_S	y_{NS}	q	u	w_S	w_{NS}
b	-	+	-	+	+	+	?

3.2 Minimum wage

Keeping the same analytical framework, we study here the effects of minimum wage on variables of the model. We note *m* the minimum wage paid to unskilled workers. Indeed, introducing this public policy does not imply any changes in the functions of productivity of the two workers, or in the meeting process presented in the previous sections. Nevertheless, the profit of a job filled by an unskilled worker can be rewritten as follows:

$$P_{NS} = y_{NS} - m \tag{31}$$

While the profit of a job filled by a skilled always depends on the bargaining power of workers and on their unemployment benefits:

$$P_S = (1-\sigma)(y_S - b) \tag{32}$$

As previously, the optimal differentiation of jobs is obtained by maximizing the value of the vacancy under the constraint of the substitution relationship. Consequently, the two main model results are verified with the introduction of the minimum wage. At the equilibrium, the couple$(y_S;\theta)$verifiesthe following two equations:

$$\begin{cases} -c + (1-\sigma)\left(q_S\left(y_S - b\right) + q_{NS}\left(y_{NS} - m\right)\right) = 0 & (33) \\ (1-\sigma)q_S + q_{NS}.g'\left(\dfrac{y_S}{\beta}\right)\dfrac{1}{\beta} = 0 & (34) \end{cases}$$

To deduce the impacts of a minimum wage on the equilibrium variables, we

$$(1-\sigma)\left(\frac{\partial q_S}{\partial \theta}(y_S - b) + \frac{\partial q_{NS}}{\partial \theta}(y_{NS} - m)\right)d\theta - (1-\sigma)q_{NS}dm = 0 \tag{35}$$

Given that, $\dfrac{\partial q_S}{\partial \theta} < 0, m > 0, \sigma > 0$, the expression $\left(\frac{\partial q_S}{\partial \theta}(y_S - b) + \frac{\partial q_{NS}}{\partial \theta}(y_{NS} - m)\right) < 0$ is negative. We obtain the effect of *m* on θ:

$$\frac{d\theta}{dm} = \frac{q_{NS}}{\left(\frac{\partial q_S}{\partial \theta}(y_S - b) + \frac{\partial q_{NS}}{\partial \theta}(y_{NS} - m)\right)} < 0 \qquad (36)$$

Using the *propositions 1 and 2*, we establish the effect on productivity y_S while the impact on unskilled productivity is given by the substitution relationship (equation (8)). Concerning the effect on skilled wages is given by differentiation of the equation (15):

$$dw_S = \sigma dy_S \quad > 0 \qquad (37)$$

The impact on the unemployment rate is deduced in the same way as unemployment benefits. We can then state the following result:

Table 2: Minimum wage effects

	θ	y_S	y_{NS}	q	u	w_S
m	-	+	-	+	+	+

Proposition 3: *In a matching model with horizontal jobs differentiation, enhancing the minimum wage leads to improve productivity of skilled workers but to increase the unemployment rate.*

The increase in the minimum wage paid to unskilled workers reduces incentives to create jobs since the firms share in the profit decreases. Therefore, the labor market tightness is reduced and the probability to meet a skilled worker increases. To cope with this increase in the minimum wage, firms become selective by creating highly differentiated jobs in favor of skilled workers. The matching quality is therefore improved thus increasing the productivity and wages of skilled.

Conclusion

Using an original formalization of job differentiation, the model results focus on three key points. The first result concerns the relationship between differentiation and unemployment rate. We have showed that firms react face to unemployment by adapting the characteristics of their job in favor to skilled workers. The second result concerns the unemployment benefits which accentuate the job differentiation and increase unemployment rate. The last result shows a negative relationship between minimum wage and the labor market tightness while providing more differentiated jobs at the expense of unskilled workers.

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Appendix

The first order condition

$$\mathrm{M}ax\ \ F = -c + q_S P_S + q_{NS} P_{NS} \qquad \text{s.c} \quad y_{NS} = g\left(\frac{y_S}{\beta}\right)$$

Using equation (16), F can be rewritten as follows:

$$F = -c + (1-\sigma)\left(q_S\left(y_S - b\right) + q_{NS}\left(g\left(\frac{y_S}{\beta}\right) - b\right)\right)$$

The first order condition satisfies:

$$\frac{\partial F}{\partial y_S} = (1-\sigma)\left(q_S + q_{NS}\, g'\left(\frac{y_S}{\beta}\right)\frac{1}{\beta}\right) = 0 \quad \Leftrightarrow \quad q_S + q_{NS}\, g'\left(\frac{y_S}{\beta}\right)\frac{1}{\beta} = 0$$

By differentiating this equilibrium expression,

$$q_S + q_{NS}\, g'\left(\frac{y_S}{\beta}\right)\frac{1}{\beta} = 0 \quad \Leftrightarrow \quad \frac{q_S}{q_{NS}} = -g'\left(\frac{y_S}{\beta}\right)\frac{1}{\beta}$$

We obtain:

$$d\left(\frac{q_S}{q_{NS}}\right) = -\frac{1}{\beta}\, g''\left(\frac{y_S}{\beta}\right) dy_S$$

Taking into account the concavity of $g(.)$, we deduce easily the *proposition 1*.

Study of the function $\varepsilon(x) = \dfrac{1-e^{-x}}{xe^{-x}}$

Its derivative is given by: $\varepsilon'(x) = \dfrac{e^{-x}(e^{-x} - 1 + x)}{(xe^{-x})^2}$

This derivative has the same sign as the expression $(e^{-x} - 1 + x)$. The latter term is null in zero and its derivative $(1 - e^{-x})$ is strictly positive for $x > 0$. The function $\varepsilon(x)$ is thus increasing for $x > 0$.