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Inflation Target and its Impact on Macroeconomy in the Zero Lower Bound Environment: the case of the Czech economy

Technical Appendix

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Log-linearized model

The model is formed by 40 equations describing endogenous variables (from equation (1) to equation (40)) and by 12 equations for exogenous shocks (from equation (41) to equation (52)). An interpretation of the model variables is presented in Table 1. Interpretation of the structural parameters and the parameters related to shocks is presented in Tables 2 and 3.

Market Clearing Conditions:

$$\begin{aligned} y_{H,t} = \; & \frac{\overline{C}}{\overline{Y}_H}\frac{\gamma_c\alpha}{1+\omega}\left(c_t+(1-\gamma_c)x_t+(1-\alpha)s_t\right) \\ & +\frac{\overline{C}^*}{\overline{Y}_H}\frac{1-n}{n}\frac{\gamma_c^*(1-\alpha^*)}{1+\omega^*}\left(c_t^*+(1-\gamma_c^*)x_t^*+\alpha^*s_t\right) \\ & +\frac{\overline{I}}{\overline{Y}_H}\gamma_i\alpha\left(i_t+(1-\gamma_i)(1+\omega)x_t+(1-\alpha)s_t\right) \\ & +\frac{\overline{I}^*}{\overline{Y}_H}\frac{1-n}{n}\gamma_i^*(1-\alpha^*)\left(i_t^*+(1-\gamma_i^*)(1+\omega)x_t^*+\alpha^*s_t\right) \end{aligned} \tag{1}$$

$$\begin{aligned} y_{F,t}^* = \; & \frac{\overline{C}^*}{\overline{Y}_F^*}\frac{\gamma_c^*\alpha^*}{1+\omega^*}\left(c_t^*+(1-\gamma_c^*)x_t^*-(1-\alpha^*)s_t\right) \\ & +\frac{\overline{C}}{\overline{Y}_F^*}\frac{n}{1-n}\frac{\gamma_c(1-\alpha)}{1+\omega}\left(c_t+(1-\gamma_c)x_t-\alpha s_t\right) \\ & +\frac{\overline{I}^*}{\overline{Y}_F^*}\gamma_i^*\alpha^*\left(i_t^*+(1-\gamma_i^*)(1+\omega^*)x_t^*-(1-\alpha^*)s_t\right) \\ & +\frac{\overline{I}}{\overline{Y}_F^*}\frac{n}{1-n}\gamma_i(1-\alpha)\left(i_t+(1-\gamma_i)(1+\omega)x_t-\alpha s_t\right) \end{aligned} \tag{2}$$

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$$y_{N,t} = \frac{\overline{C}}{\overline{Y}_N}\left((1-\gamma_c)(c_t-\gamma_c x_t)+\frac{\gamma_c\omega}{1+\omega}(c_t+(1-\gamma_c)x_t)\right) + \frac{\overline{I}}{\overline{Y}_N}\left(1-\gamma_i)(i_t-\gamma_i(1+\omega)x_t\right)+\frac{\overline{G}}{\overline{Y}_N}\varepsilon_{g,t} \tag{3}$$

$$y^*_{N,t} = \frac{\overline{C}^*}{\overline{Y}^*_N}\left((1-\gamma^*_c)(c^*_t-\gamma^*_c x^*_t)+\frac{\gamma^*_c\omega^*}{1+\omega^*}(c^*_t+(1-\gamma^*_c)x^*_t)\right) + \frac{\overline{I}^*}{\overline{Y}^*_N}\left(1-\gamma^*_i)(i^*_t-\gamma^*_i(1+\omega^*)x^*_t\right)+\frac{\overline{G}^*}{\overline{Y}^*_N}\varepsilon^*_{g,t} \tag{4}$$

$$y_t = \frac{\overline{Y}_H}{\overline{Y}}y_{H,t}+\frac{\overline{Y}_N}{\overline{Y}}y_{N,t} \tag{5}$$

$$y^*_t = \frac{\overline{Y}^*_F}{\overline{Y}^*}y^*_{F,t}+\frac{\overline{Y}^*_N}{\overline{Y}^*}y^*_{N,t} \tag{6}$$

Euler Equation:

$$c_t - hc_{t-1} = E_t(c_{t+1}-hc_t)-\frac{1-h}{\sigma}E_t\left(r_t-\pi_{t+1}+\varepsilon_{d,t+1}-\varepsilon_{d,t}\right) \tag{7}$$

$$c^*_t - h^*c^*_{t-1} = E_t(c^*_{t+1}-h^*c^*_t)-\frac{1-h^*}{\sigma^*}E_t\left(r^*_t-\pi^*_{t+1}+\varepsilon^*_{d,t+1}-\varepsilon^*_{d,t}\right) \tag{8}$$

International Risk Sharing Condition:

$$q_t = \varepsilon^*_{d,t}-\varepsilon_{d,t}-\frac{\sigma^*}{1-h^*}(c^*_t-h^*c^*_{t-1})+\frac{\sigma}{1-h}(c_t-hc_{t-1}) \tag{9}$$

Capital Accumulation:

$$k_{t+1} = (1-\tau)k_t+\tau(i_t+\varepsilon_{i,t}) \tag{10}$$

$$k^*_{t+1} = (1-\tau^*)k^*_t+\tau^*(i^*_t+\varepsilon^*_{i,t}) \tag{11}$$

Real Costs of Capital:

$$r_{K,t} = w_t + l_t - k_t \tag{12}$$

$$r^*_{K,t} = w^*_t + l^*_t - k^*_t \tag{13}$$

Investment Demand:

$$i_t - i_{t-1} = \beta E_t(i_{t+1}-i_t)+\frac{1}{S''}(q_{T,t}+\varepsilon_{i,t})-\frac{\gamma_i(1+\omega)-\gamma_c}{S''}x_t \tag{14}$$

$$i^*_t - i^*_{t-1} = \beta^* E_t(i^*_{t+1}-i^*_t)+\frac{1}{S''^*}(q^*_{T,t}+\varepsilon^*_{i,t})-\frac{\gamma^*_i(1+\omega^*)-\gamma^*_c}{S''^*}x^*_t \tag{15}$$

Price of Installed Capital:

$$q_{T,t} = \beta(1-\tau)E_t q_{T,t+1} - (r_t - E_t\pi_{t+1}) + (1-\beta(1-\tau))E_t r_{K,t+1} \tag{16}$$

$$q^*_{T,t} = \beta^*(1-\tau^*)E_t q^*_{T,t+1} - (r^*_t - E_t\pi^*_{t+1}) + (1-\beta^*(1-\tau^*))E_t r^*_{K,t+1} \tag{17}$$

Labor Input:

$$l_t = \eta(r_{K,t} - w_t) + \frac{\overline{Y}_H}{\overline{Y}}(y_{H,t} - \varepsilon_{aH,t}) + \frac{\overline{Y}_N}{\overline{Y}}(y_{N,t} - \varepsilon_{aN,t}) \tag{18}$$

$$l^*_t = \eta^*(r^*_{K,t} - w^*_t) + \frac{\overline{Y}^*_F}{\overline{Y}^*}(y^*_{F,t} - \varepsilon^*_{aF,t}) + \frac{\overline{Y}^*_N}{\overline{Y}^*}(y^*_{N,t} - \varepsilon^*_{aN,t}) \tag{19}$$

Real Wage:

$$\begin{aligned} w_t - w_{t-1} = \; & \frac{(1-\theta_W)(1-\beta\theta_W)}{\theta_W(1+\phi_W\phi)}(mrs_t - w_t) + \beta E_t(w_{t+1} - w_t) \\ & + \beta E_t(\pi_{t+1} - \delta_W\pi_t) - (\pi_t - \delta_W\pi_{t-1}) \end{aligned} \tag{20}$$

$$\begin{aligned} w^*_t - w^*_{t-1} = \; & \frac{(1-\theta^*_W)(1-\beta^*\theta^*_W)}{\theta^*_W(1+\phi^*_W\phi^*)}(mrs^*_t - w^*_t) + \beta^* E_t(w^*_{t+1} - w^*_t) \\ & + \beta^* E_t(\pi^*_{t+1} - \delta^*_W\pi^*_t) - (\pi^*_t - \delta^*_W\pi^*_{t-1}) \end{aligned} \tag{21}$$

Marginal Rate of Substitution:

$$mrs_t = \varepsilon_{l,t} + \phi l_t - \varepsilon_{d,t} + \frac{\sigma}{1-h}(c_t - hc_{t-1}) \tag{22}$$

$$mrs^*_t = \varepsilon^*_{l,t} + \phi^* l^*_t - \varepsilon^*_{d,t} + \frac{\sigma^*}{1-h^*}(c^*_t - h^*c^*_{t-1}) \tag{23}$$

Phillips Curve for Tradable Sector:

$$\pi_{H,t} - \delta_H\pi_{H,t-1} = \beta E_t(\pi_{H,t+1} - \delta_H\pi_{H,t}) + \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}mc_{H,t} \tag{24}$$

$$\pi^*_{F,t} - \delta^*_F\pi^*_{F,t-1} = \beta^* E_t(\pi^*_{F,t+1} - \delta^*_F\pi^*_{F,t}) + \frac{(1-\theta^*_F)(1-\beta^*\theta^*_F)}{\theta^*_F}mc^*_{F,t} \tag{25}$$

Phillips Curve for Non-tradable Sector:

$$\pi_{N,t} - \delta_N\pi_{N,t-1} = \beta E_t(\pi_{N,t+1} - \delta_N\pi_{N,t}) + \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N}mc_{N,t} \tag{26}$$

$$\begin{aligned} \pi^*_{N,t} - \delta^*_N\pi^*_{N,t-1} \; & = \beta^* E_t(\pi^*_{N,t+1} - \delta^*_N\pi^*_{N,t}) + \\ & + \frac{(1-\theta^*_N)(1-\beta^*\theta^*_N)}{\theta^*_N}mc^*_{N,t} \end{aligned} \tag{27}$$

Real Marginal Costs in Tradable Sector:

$$mc_{H,t} = (1-\eta)w_t + \eta r_{K,t} - \varepsilon_{a^H,t} + (1-\alpha)s_t + (1-\gamma_c+\omega)x_t \quad (28)$$

$$mc^*_{F,t} = (1-\eta^*)w^*_t + \eta^* r^*_{K,t} - \varepsilon^*_{a^F,t} + (1-\alpha^*)s_t + (1-\gamma^*_c+\omega^*)x^*_t \quad (29)$$

Real Marginal Costs in Non-tradable Sector:

$$mc_{N,t} = (1-\eta)w_t + \eta r_{K,t} - \varepsilon_{a^N,t} - \gamma_c x_t \quad (30)$$

$$mc^*_{N,t} = (1-\eta^*)w^*_t + \eta^* r^*_{K,t} - \varepsilon^*_{a^N,t} - \gamma^*_c x^*_t \quad (31)$$

Relative Price of Non-tradable Goods:

$$x_t - x_{t-1} = \pi_{N,t} - \pi_{T,t} \quad (32)$$

$$x^*_t - x^*_{t-1} = \pi^*_{N,t} - \pi^*_{T,t} \quad (33)$$

Inflation of Tradable Goods:

$$\pi_{T,t} = \frac{1}{1+\omega}\left(\pi_{H,t} + (1-\alpha)\Delta s_t + \omega\pi_{N,t}\right) \quad (34)$$

$$\pi^*_{T,t} = \frac{1}{1+\omega^*}\left(\pi^*_{F,t} + (1-\alpha^*)\Delta s_t + \omega^*\pi^*_{N,t}\right) \quad (35)$$

Overall Inflation:

$$\pi_t = \gamma_c\pi_{T,t} + (1-\gamma_c)\pi_{N,t} \quad (36)$$

$$\pi^*_t = \gamma^*_c\pi^*_{T,t} + (1-\gamma^*_c)\pi^*_{N,t} \quad (37)$$

Real Exchange Rate:

$$q_t = (\alpha+\alpha^*-1)s_t + (1-\gamma^*_c+\omega^*)x^*_t - (1-\gamma_c+\omega)x_t \quad (38)$$

Monetary Policy Rule:

$$r_t = \rho r_{t-1} + (1-\rho)(\psi_y E_t\{y_{t+1}\} + \psi_\pi E_t\{\pi_{t+1}\}) + \varepsilon_{m,t} \quad (39)$$

$$r^*_t = \rho^* r^*_{t-1} + (1-\rho^*)(\psi^*_y E_t\{y^*_{t+1}\} + \psi^*_\pi E_t\{\pi^*_{t+1}\}) + \varepsilon^*_{m,t} \quad (40)$$

Productivity Shock in Tradable Sector:

$$\varepsilon_{a^H,t} = \rho_{a^H}\varepsilon_{a^H,t-1} + \mu_{a^H,t} \quad (41)$$

$$\varepsilon^*_{a^F,t} = \rho^*_{a^F}\varepsilon^*_{a^F,t-1} + \mu^*_{a^F,t} \quad (42)$$

Productivity Shock in Non-tradable Sector:

$$\varepsilon_{a^N,t} = \rho_{a^N}\varepsilon_{a^N,t-1} + \mu_{a^N,t} \quad (43)$$

$$\varepsilon^*_{a^N,t} = \rho^*_{a^N}\varepsilon^*_{a^N,t-1} + \mu^*_{a^N,t} \quad (44)$$

Preference Shock:

$$\varepsilon_{d,t} = \rho_d\varepsilon_{d,t-1} + \mu_{d,t} \quad (45)$$

$$\varepsilon^*_{d,t} = \rho^*_d \varepsilon^*_{d,t-1} + \mu^*_{d,t} \tag{46}$$

Labor Supply Shock:

$$\varepsilon_{l,t} = \rho_l \varepsilon_{l,t-1} + \mu_{l,t} \tag{47}$$

$$\varepsilon^*_{l,t} = \rho^*_l \varepsilon^*_{l,t-1} + \mu^*_{l,t} \tag{48}$$

Shock in Government Expenditures:

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \mu_{g,t} \tag{49}$$

$$\varepsilon^*_{g,t} = \rho^*_g \varepsilon^*_{g,t-1} + \mu^*_{g,t} \tag{50}$$

Shock in Investment Efficiency:

$$\varepsilon_{i,t} = \rho_i \varepsilon_{i,t-1} + \mu_{i,t} \tag{51}$$

$$\varepsilon^*_{i,t} = \rho^*_i \varepsilon^*_{i,t-1} + \mu^*_{i,t} \tag{52}$$

Table 1: Interpretation of Variables

variable	interpretation
c_t, c^*_t	consumption
i_t, i^*_t	investment
y_t, y^*_t	total output
$y_{H,t}, y^*_{F,t}$	output in tradable sector
$y_{N,t}, y^*_{N,t}$	output in non-tradable sector
x_t, x^*_t	internal exchange rates
s_t	terms of trade
r_t, r^*_t	nominal interest rate
q_t	real exchange rate
k_t, k^*_t	capital
$r_{K,t}, r^*_{K,t}$	payoff from renting capital
w_t, w^*_t	real wage
$q_{T,t}, q^*_{T,t}$	price of installed capital (Tobin's Q)
l_t, l^*_t	labor
mrs_t, mrs^*_t	marginal rate of substitution
π_t, π^*_t	inflation
$\pi_{T,t}, \pi^*_{T,t}$	inflation of tradable goods
$\pi_{H,t}, \pi^*_{F,t}$	inflation in tradable sector

$\pi_{N,t}, \pi^*_{N,t}$	inflation of non-tradable goods
$mc_{H,t}, mc^*_{F,t}$	real marginal costs in tradable sector
$mc_{N,t}, mc^*_{N,t}$	real marginal costs in non-tradable sector
$\varepsilon_{a^H,t}, \varepsilon^*_{a^F,t}$	productivity shock in tradable sector
$\varepsilon_{a^N,t}, \varepsilon^*_{a^N,t}$	productivity shock in non-tradable sector
$\varepsilon_{d,t}, \varepsilon^*_{d,t}$	preference shock
$\varepsilon_{l,t}, \varepsilon^*_{l,t}$	labor supply shock
$\varepsilon_{g,t}, \varepsilon^*_{g,t}$	government expenditures shock
$\varepsilon_{i,t}, \varepsilon^*_{i,t}$	investment efficiency shock
$\varepsilon_{m,t}, \varepsilon^*_{m,t}$	monetary policy shock

Table 2: Interpretation of Structural Parameters

parameter	interpretation	domain
n	relative size of the domestic economy	$\langle 0,1\rangle$
$\beta,\ \beta^*$	discount factor	$\langle 0,1\rangle$
h, h^*	habit formation in consumption	$\langle 0,1\rangle$
σ, σ^*	inv. elast. of intertemporal substitution	$\langle 0,\infty)$
ϕ, ϕ^*	inv. elast. of labor supply	$\langle 0,\infty)$
$\phi_H,\ \phi_F$	elast. of subst. among tradable goods	$\langle 1,\infty)$
$\phi_N,\ \phi^*_N$	elast. of subst. among non-tradable goods	$\langle 1,\infty)$
$\phi_W,\ \phi^*_W$	elast. of subst. among labor types	$\langle 1,\infty)$
$\gamma_c,\ \gamma^*_c$	share of tradable goods in consumption	$\langle 0,1\rangle$
$\gamma_i,\ \gamma^*_i$	share of tradable goods in investment	$\langle 0,1\rangle$
$\alpha,\ \alpha^*$	share of domestic tradable goods	$\langle 0,1\rangle$
$\omega,\ \omega^*$	distribution costs	$\langle 0,\infty)$
$\tau,\ \tau^*$	capital depreciation rate	$\langle 0,1\rangle$
S'', S''^*	adjustment costs of capital	$\langle 0,\infty)$
$\eta,\ \eta^*$	elasticity of output with respect to capital	$\langle 0,1\rangle$
θ_H, θ^*_F	Calvo parameter for tradable sector	$\langle 0,1\rangle$
θ_N, θ^*_N	Calvo parameter for non-tradable sector	$\langle 0,1\rangle$
θ_W, θ^*_W	Calvo parameter for households	$\langle 0,1\rangle$
δ_H, δ^*_F	indexation in tradable sector	$\langle 0,1\rangle$
δ_N, δ^*_N	indexation in non-tradable sector	$\langle 0,1\rangle$

δ_W, δ_W^*	indexation of households	$\langle 0,1 \rangle$
ρ_i, ρ_i^*	interest rate smoothing	$\langle 0,1 \rangle$
ψ_π, ψ_π^*	elasticity of interest rate to inflation	$\langle 0,\infty)$
ψ_y, ψ_y^*	elasticity of interest rate to output	$\langle 0,\infty)$

Table 3: Interpretation of Parameters related to Shocks

parameter	interpretation	domain
ρ_{aH}, ρ^*_{aF}	persistence of productivity shocks - tradables	$\langle 0,1 \rangle$
ρ_{aN}, ρ^*_{aN}	persistence of productivity shocks - non-tradables	$\langle 0,1 \rangle$
ρ_d, ρ_d^*	persistence of preference shocks	$\langle 0,1 \rangle$
ρ_l, ρ_l^*	persistence of labor supply shocks	$\langle 0,1 \rangle$
ρ_g, ρ_g^*	persistence of shocks in government expenditures	$\langle 0,1 \rangle$
ρ_i, ρ_i^*	persistence of shocks in investment efficiency	$\langle 0,1 \rangle$
$\sigma_{aH}, \sigma^*_{aF}$	std. dev. of productivity shocks - tradables	$\langle 0,\infty)$
$\sigma_{aN}, \sigma^*_{aN}$	std. dev. of productivity shocks - non-tradables	$\langle 0,\infty)$
σ_d, σ_d^*	std. dev. of preference shocks	$\langle 0,\infty)$
σ_l, σ_l^*	std. dev. of labor supply shocks	$\langle 0,\infty)$
σ_g, σ_g^*	std. dev. of shocks in government expenditures	$\langle 0,\infty)$
σ_i, σ_i^*	std. dev. of shocks in investment efficiency	$\langle 0,\infty)$
σ_m, σ_m^*	std. dev. of monetary shocks	$\langle 0,\infty)$
$cor_{a^H, a^{F*}}$	correlation of productivity shocks - tradables	$\langle -1,1 \rangle$
$cor_{a^N, a^{N*}}$	correlation of productivity shocks - non-tradables	$\langle -1,1 \rangle$
cor_{d,d^*}	correlation of preference shocks	$\langle -1,1 \rangle$
cor_{l,l^*}	correlation of labor supply shocks	$\langle -1,1 \rangle$
cor_{g,g^*}	correlation of shocks in government expenditures	$\langle -1,1 \rangle$
cor_{i,i^*}	correlation of shocks in investment efficiency	$\langle -1,1 \rangle$
cor_{m,m^*}	correlation of shocks in investment efficiency	$\langle -1,1 \rangle$

Results of estimation

Figure 1: Priors and posteriors

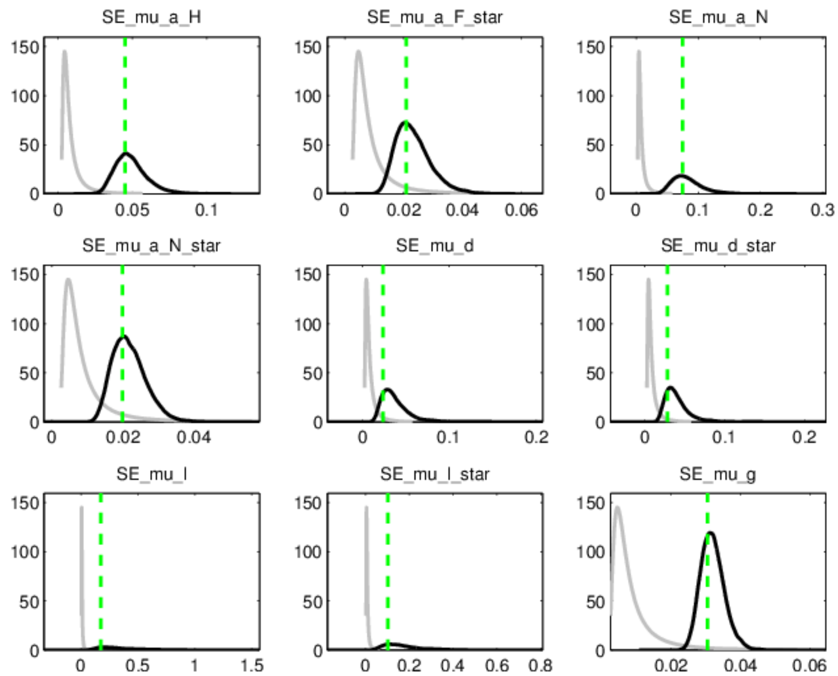


Figure 2: Priors and posteriors

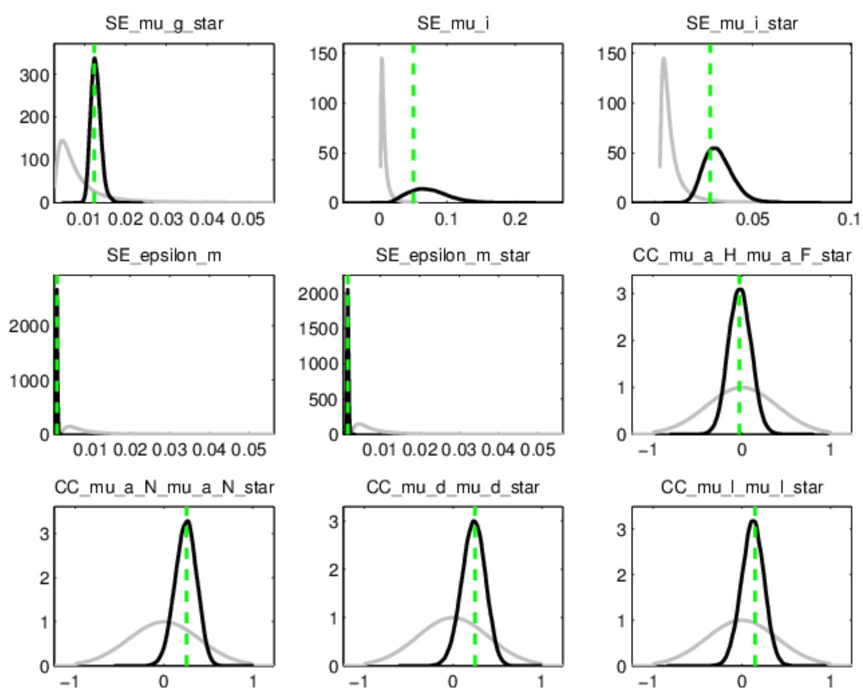


Figure 3: Priors and posteriors

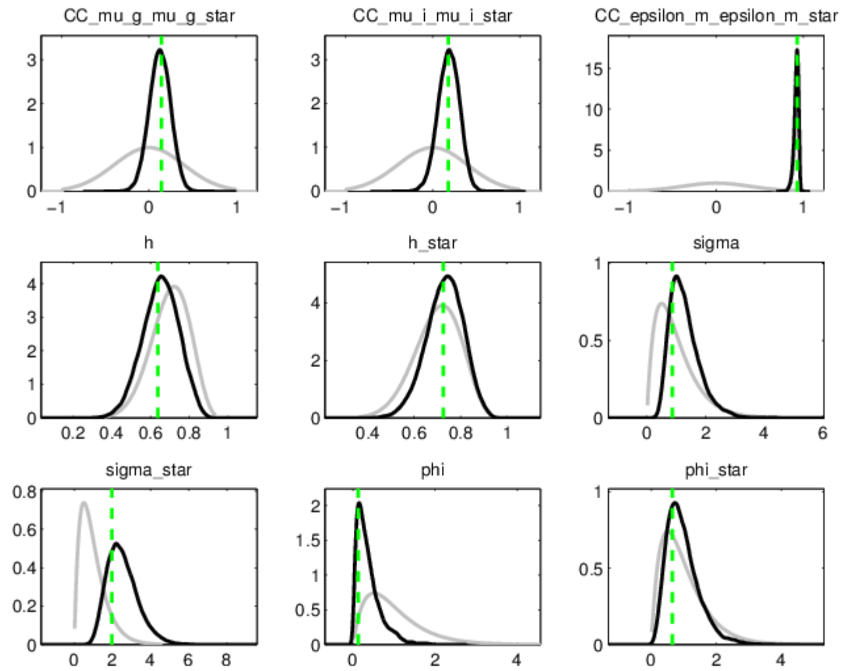


Figure 4: Priors and posteriors

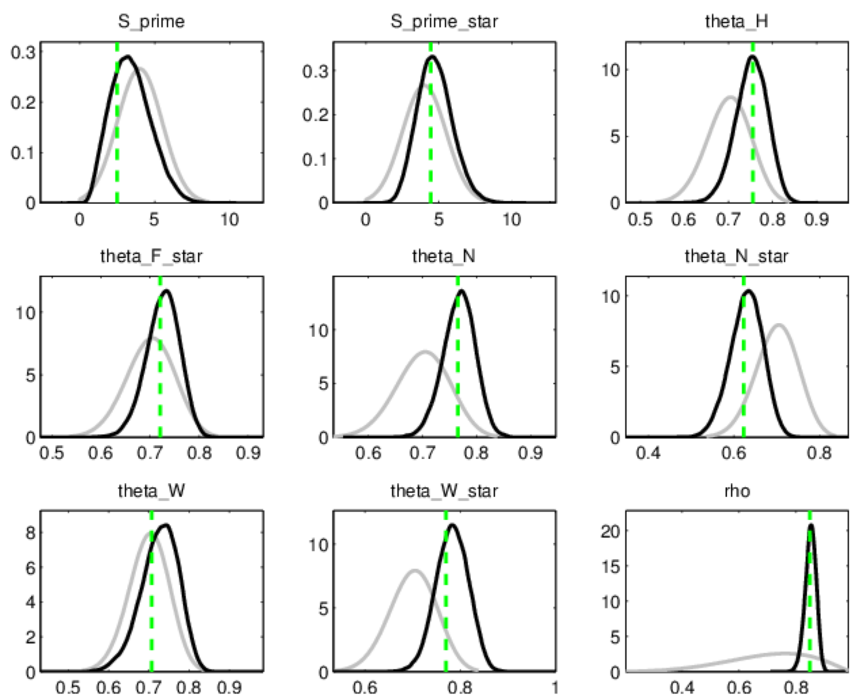


Figure 5: Priors and posteriors

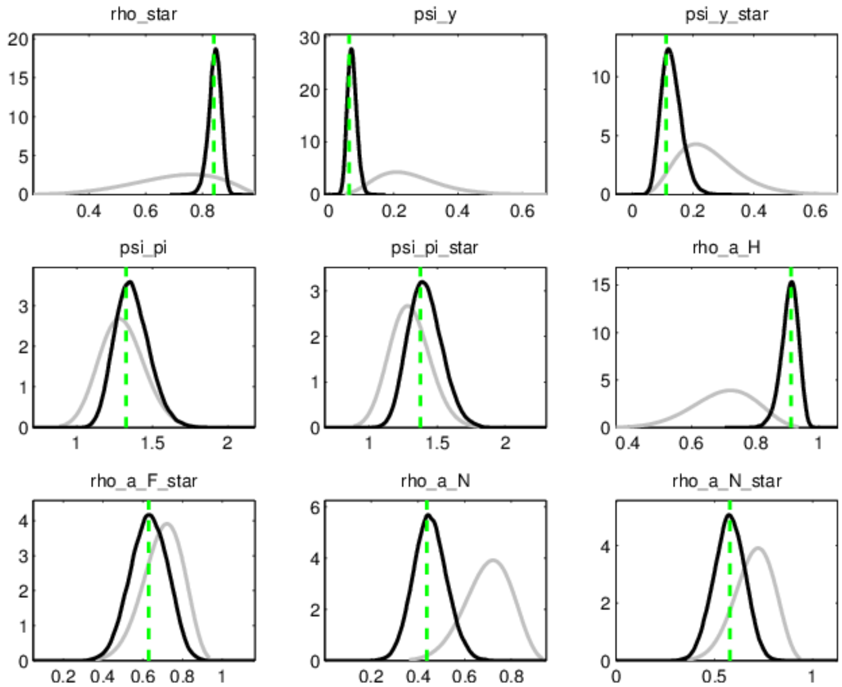


Figure 6: Priors and posteriors

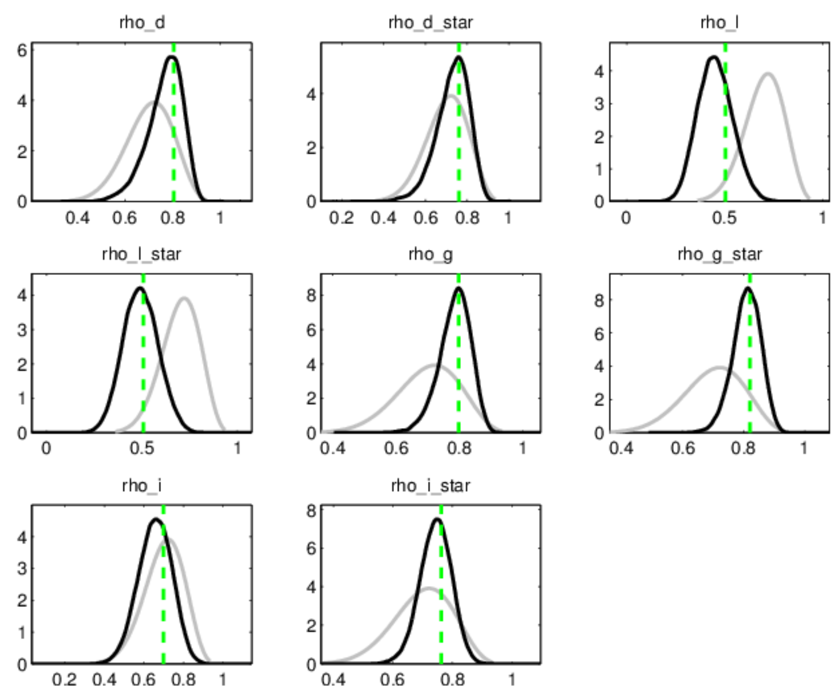


Figure 7: Multivariate diagnostics

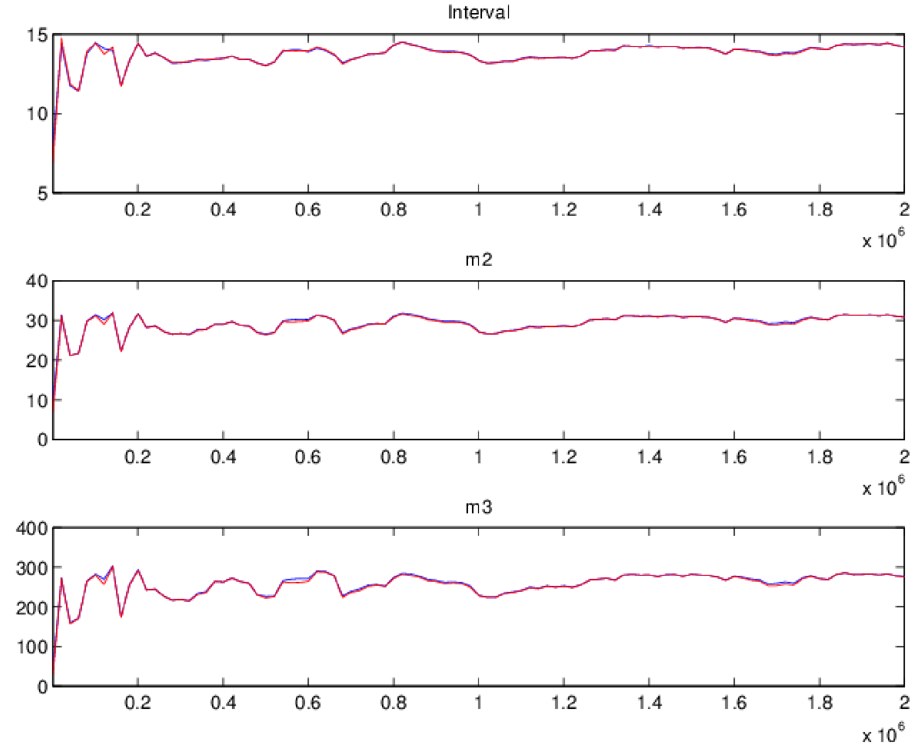


Figure 8: Smoothed innovations

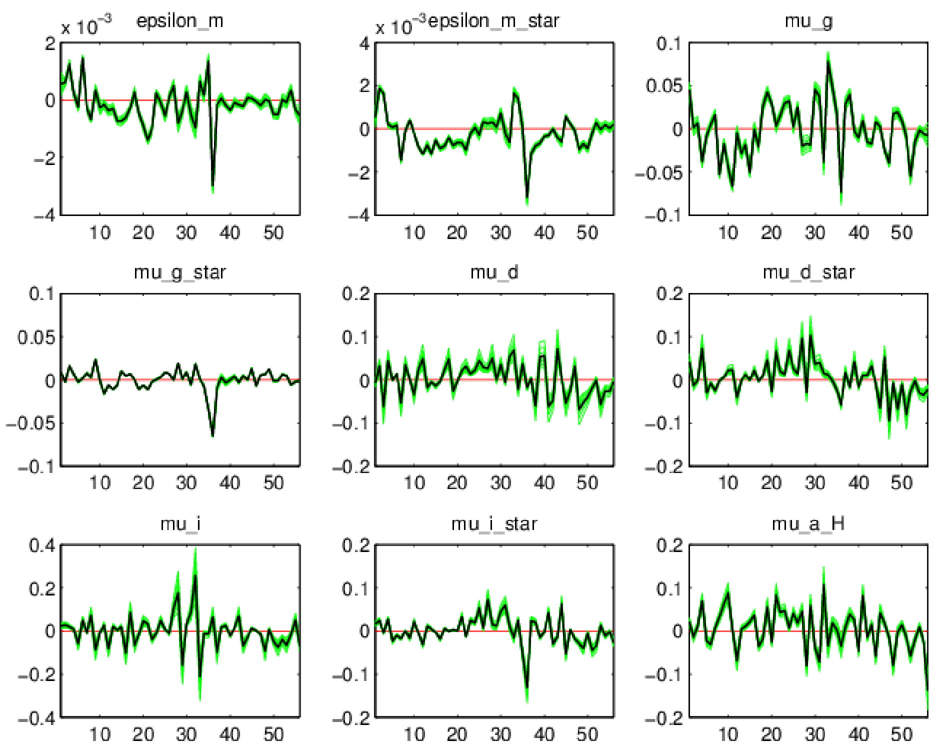
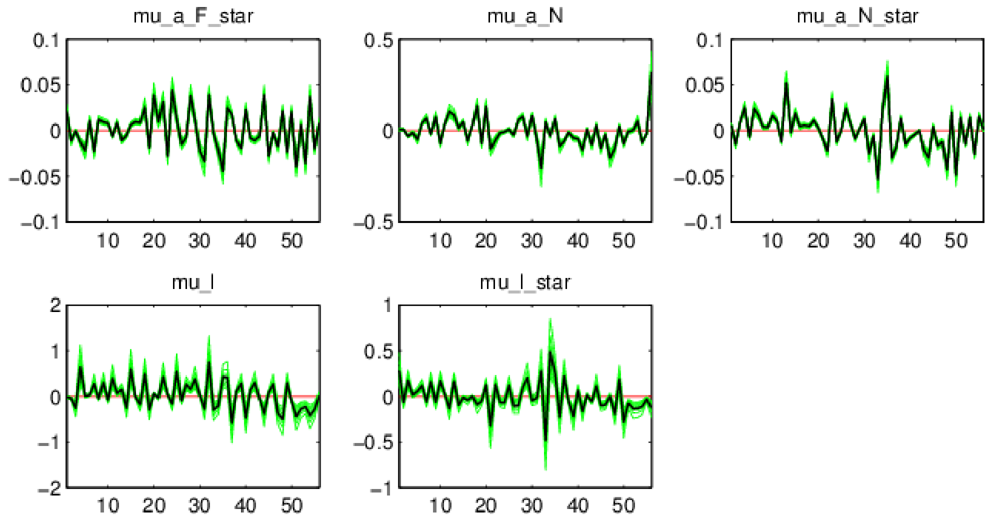


Figure 9: Smoothed innovations



Sensitivity analysis

This section compares results of simulations for different setting of the parameters. First, the model was estimated on shorter period (2000:Q2 – 2008:Q1) which excludes financial crises. Figure 10 shows the differences of median between ZLB and noZLB distributions for three cases. "Benchmark" corresponds to the model estimated for the whole period, "short period, bench rir" corresponds to the model estimated for the shorter period, but the equilibrium real interest rate for simulation remained on benchmark value (0.24 %) and "short period, actual rir" corresponds to the model estimated for the shorter period, but with equilibrium real interest rate calculated on shorter period (0.37 %). We can see that the distortions are generally lower for the model estimated on shorter period, especially for target inflation lower than two percent. The difference can be up to one percentage point compared to the benchmark. Only the inflation rate is slightly higher and thus increases the welfare cost.

Second sensitivity analysis check deals with different setting of structural parameters. For the simulation at occurrence of ZLB, all the model parameters are kept at their estimated values (posterior mean) except of the particular parameter that is set to different value. Results of this exercise are shown in Figure 11. With higher price and wage rigidities (Calvo parameter were set $\theta_H = 0.8$ or $\theta_W = 0.8$, respectively)[2] the distortions are lower. On the other hand, with higher investment adjustment cost ($S' = 8$ instead of $S' = 3.4$), the distortions are higher for output and consumption, but lower for investment. Inflation is more negative in ZLB case, but given the scale, the quantitative difference is negligible.

To sum it up, the main results are quite robust for inflation target between 2 and 4 percent. For lower values of the target (especially close to zero), the distortions vary according to the parameters setting. The difference can be up to one percentage point. All in all the distortions remain substantial and the main message of the paper is still valid.

[2] Benchmark values were $\theta_H = 0.75$ and $\theta_W = 0.73$

Figure 10: Sensitivity analysis: shorter estimation period

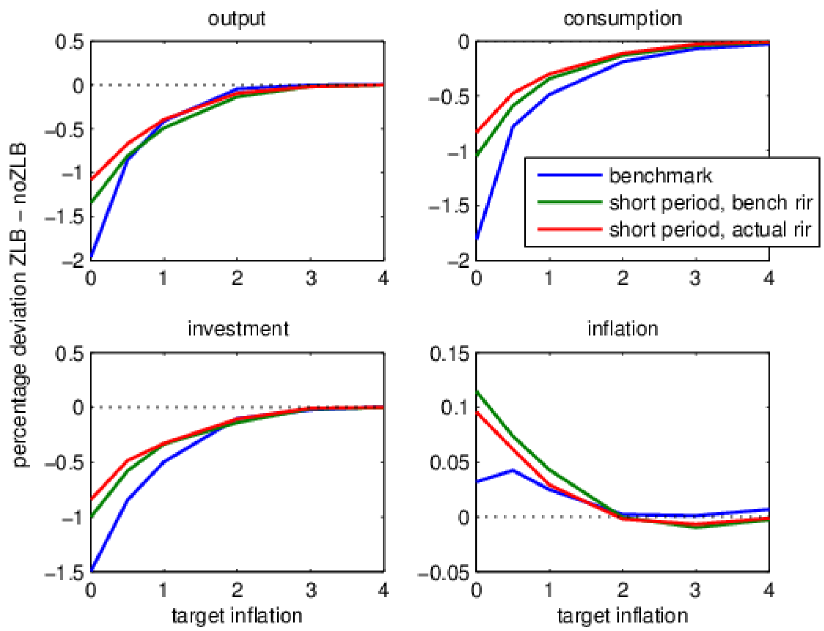


Figure 11: Sensitivity analysis: different values of parameters