Laboratory federalism: Policy diffusion and yardstick competition

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Abstract

1 Introduction

The concept of ”laboratory federalism”, coined by Oates (1999), states that federations and more generally politically decentralized countries may benefit from better policies than centralized countries, thanks to a greater efficiency in identifying the best policies. Indeed, multiple small scale experimentation may foster the identification of the best policies, and reduce the aggregate risk of experimentation. The basic intuition behind this result is that the implementation of a given policy in a region generate an informational externality which allows the other regions to learn about the quality and/or adequacy of this policy. This process of learning is likely to generate better outcomes than if policy making were made by a single decision maker for all regions. In this paper I propose to explore another possible driver of policy learning in a federation. Indeed, I show that yardstick competition is an important source of policy diffusion. However, it may also have perverse effects,

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such as policy pandering or populism.

Formal models studying the link between federalism and policy innovation are presented in Rose-Ackerman (1980), Kollman et al. (2000) and Strumpf (2002), who study the experimentation of new policies in decentralized vs centralized countries. They do not consider rational model of elections. More recent work introduce some politics in the analysis, like Cai and Treisman (2009) and Kotsogiannis and Schwager (2006a). These different papers concentrate on the quantity of experimentation in a federation compared to a centralized state. The basic intuition is that there is more experimentation in a decentralized system because each lower state is doing some experimentation. But some problems may arise, the most important being the incentive to free-ride on the neighbours’ experimentation as a way to reduce the risk and costs of own experimentation. As a consequence we may observe in fact less experimentation in a federation than in a centralized state. This effect can be conter-balanced if the most innovative local politicians are promoted to the national level: this gives an inventive to experiment as a way to signal his ability to innovate.

However one may ask if the matter of the quantity of experimentation is really the whole story behind the concept of laboratory federalism. In this paper the focus is on the quality of the experimentation. Indeed, I think that one main idea behind the concept of laboratory federalism is that a decentralized system allows to learn more efficiently about the best policies, thanks to a larger sample of small scale experimentation and a larger variety of the politicians’ ability, than in centralized states. Hence in the following I do not modelize the process experimentation as the choice of a new, untried policy over a well-known policy, but as the choice of a policy in a set risky policies. Only the experience from former (domestic and in the neighbourhood) choices and expertise may guide the choice among the available policies.

Another feature of the current models of laboratory federalism is that politicians are supposed benevolents. Indeed, politicians are supposed to differ in their ability, but have the same objectives as the voters. However,
conflicts of interest between the voters and their representants are usually a reality, and may play a role in the way good policies are uncovered. I then suppose that, although all politicians learn and experiment about. This illustrates the idea that a more precise knowledge about the quality and/or the efficiency of different policies may also be used against the voters’ interests.

2 The model

I first expose the basics and framework of the model. I then solve the game, for a purpose of exposition, in the case of a single jurisdiction.

2.1 Basic setting

I first present the basic framework of the model, which I borrow to Maskin and Tirole (2004). Consider a two-periods game \( t \in \{1, 2\} \) between a representative voter and a politician. In each period, a policy \( P_t \in \{a, b\} \) has to be implemented. One of the two policies is the best and gives a higher utility to the voter. Let’s call this best policy \( x \in \{a, b\} \). \( x \) is determined by Nature and is the same over the two periods and is unobservable. At the beginning of the first period the voter has no a priori information on which policy is the best: \( \text{Prob}(x = a) = \gamma = 1/2 \).

Given a policy choice \( P \), the utility of the voter is \( u(P, x) = 1 + \varepsilon \) if \( P = x \) and \( u(P, x) = \varepsilon \) if \( P \neq x \). \( \varepsilon \) is a random shock with mean zero, pdf = \( f(.) \) and cdf = \( F(.) \).

In each period, the choice and implementation of the policy is delegated to a politician, who has a more precise information on \( x \); at the beginning of the first period the politician receives a signal \( s \) on \( x \). The signal has the following conditional distribution:

\footnote{we suppose that \( f(.) \) satisfies the MLRP}
with $q > 1/2$ being the probability for the politician to get a signal $s = a (s = b)$ if $x = a (x = b)$ (hence $s = a (s = b)$ is “good news” regarding state $a (b)$).

A politician can be of two types: either congruent or dissonent\(^2\). The ex ante probability that a politician is dissonent is $\pi$. $\pi$ can also be seen as the share of dissonent politicians in the population of candidate politicians. A congruent politician share the same preferences as the voter: his prefered policy is $P = x$. A dissonent politician a opposed preferences: his prefered policy is $P \neq x$.

In addition to the implementation of his prefered policy, a politician also derives utility from the exercice of power (this can be seen as an ego-rent). The utility function of a politician is then $V = G + E$, where $G$ is the utility provided by the implementation of his prefered politicy and $E$ is the utility provided from the exercice of power. For a discount factor $\beta$, I define

$$\kappa = \frac{G}{\beta (G + R)}$$

the degree of patience of the politicians\(^3\). Hence a high $\kappa (\kappa > 1)$ means that politicians are impatient: they give more value to the current period than to the future. A lower level of $\kappa (\kappa < 1)$ means that politicians are patient. I suppose however that there is always a small share $\xi$ of totally impatient politicians.

**Timing of the game**

The sequence of events goes the following

1. Nature chooses a type of politician and the best policy $x \in \{a, b\}$

\(^2\)I borrow this nomenclature to Besley (2006)

\(^3\)\(\kappa\) is the same for both types of politicians
2. The incumbent gets a signal $s$ and chooses a policy $P$

3. The shock $\varepsilon$ is realized

4. The voter observes $u(P_1, x)$ and $P_1$

5. Given $u$, $P$ and $\pi$ the voter chooses whether to reelect the incumbent or the pick a challenger (congruent with probability $\pi$)

6. The second period’s incumbent chooses a policy $P$

7. The voter gets utility $u(P_2, x)$ and the game ends.

2.2 Policy making and elections in a single jurisdiction

In this subsection I focus on the case of a single jurisdiction. I first describe the strategies of the representative voter and the politicians and then characterize the different equilibria of the game.

The simplest way to solve this two-periods game is to reason backwards. In the last period, since there is no more electoral incentives, the incumbent simply chooses his favorite policy. The nature of $x$ is indirectly observable from the first period policy choice and outcome; then a second period politician gets utility $G + E$. The expected second period utility of the voter is then a function of the reputation of the second term incumbent. If this reputation is $\pi_2$, the voter gets the expected utility $\pi_2$ for the second period.

In the first period, a politician has to take into account the impact of his policy choice on his probability of reelection. Let’s write $\sigma_I(P, \mu_1, s) = \sigma_I(P)$ the probability of reelection of the incumbent conditionally on his policy choice $P$, the public belief $\mu_1$ about $x$ in $t = 1$ and the private signal $s$. The objective of an incumbent in the first period is then to maximize

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4 A reelected politician knows $x$; a newly appointed politician can learn once in office the policy choice and outcome of his predecessor.

5 Recall that the voter has the expected utility $u = 1$ if $P = x$, which happens with probability $\pi_2$, and zero otherwise.
$$\text{Max}_P G + R + \sigma_I (P, \mu_1, s) \beta (G + R)$$ (1)

To make an informed choice on $P$, the incumbent form a private belief about $x$ using the public belief $\mu_1$ and his private signal $s$. Since there is no a priori information about the good policy (the public belief $\mu_1 = 1/2$) the private belief depends only on the signal: $\tilde{\mu}(s)$. Since

$$\Pr (x = s \mid s) = q$$ (2)
$$\Pr (x \neq s \mid s) = 1 - q$$ (3)

$\tilde{\mu}(s = a) = q > 1/2$ and $\tilde{\mu}(s = b) = 1 - q < 1/2$.

The favorite policy of a congruent politician is then $P = s$. It is optimal for a congruent incumbent to implement is favorite policy if:

$$q \left( G + E + \sigma_I (P = s) \beta (G + E) \right) + (1 - q) \left( E + \sigma_I (P = s) \beta (G + E) \right) \geq q \left( E + \sigma_I (P \neq s) \beta (G + E) \right) + (1 - q) \left( G + R + \sigma_I (P \neq s) \beta (G + R) \right)$$ (4)

or if

$$\kappa \geq \frac{\sigma_I (P \neq s) - \sigma_I (P = s)}{2q - 1}$$ (5)

Similary, for a dissonent politician, playing his favorite policy $P \neq s$ is optimal if:

$$q \left( G + E + \sigma_I (P \neq s) \beta (G + E) \right) + (1 - q) \left( E + \sigma_I (P \neq s) \beta (G + E) \right) \geq q \left( E + \sigma_I (P = s) \beta (G + E) \right) + (1 - q) \left( G + R + \sigma_I (P = s) \beta (G + R) \right)$$ (6)

or if

$$\kappa \geq \frac{\sigma_I (P = s) - \sigma_I (P \neq s)}{2q - 1}$$ (7)
The only decision the voter has to make in the game is to choose at the end of the first period whether to reelect the incumbent or to pick a new, “untried”, politician. Since, as seen above, the voter’s expected second period utility is a function of the second period incumbent’s type, the relevant decision variable for the voter is the reputation of the incumbent compared to the reputation of a challenger. The voter needs then to update the reputation of the current incumbent given the observable variables at disposal, namely the policy implemented and the utility derived from this policy.

Suppose that a congruent politician always chooses the policy the most likely to be the best by implementing $P = s$ and that the probability that a dissonent politician decides to be behave in the interest of the voter by implementing $P = s$ is $\lambda$. Being rational, the voter update the reputation of the incumbent using the Bayes rule and formulate the incumbent’s ex post reputation $\Pi$. Hence for $P = a$ and $u$, the likelihood ratio $\frac{\pi}{1-\pi}$ can be written as:

$$LR(\frac{\Pi}{1-\Pi} | P=a,u) = \frac{\pi [q\mu f(u-1) + (1-q) (1-\mu) f(u)]}{(1-\pi) ((1-\xi) \lambda [q\mu f(u-1) + (1-q) (1-\mu) f(u)] + ((1-\xi) (1-\lambda) + \xi) [q\mu f(u) + (1-q) (1-\mu) f(u-1)])}$$  \tag{8}$$

The similar argument apply when $P = b$. The condition for the re-election is obvious and is given by $\Pi \geq \pi$: it is optimal to re-elect the incumbent is at least as good as the reputation of a challenger, since the voter’s expected second period is precisely a function of this reputation. Taking into account that the public belief is uninformed: $\mu = 1/2$, the condition for the reelection can be stated as (both for $P = a$ and $P = b$):

$$LR(\frac{\Pi}{1-\Pi}) \geq \frac{\pi}{1-\pi} \tag{9}$$

Since $f(.)$ satisfies the MLRP, it is easy to see that 9 is monotonically decreasing in $u$. There is then a unique level of utility $\bar{u}$ such that the condition (9 is binding. Since the distribution function $f$ is symmetric, it is easy to see that $\bar{u} = 1/2$. Then $u \geq 1/2 \Rightarrow \Pi(u) \geq \pi$ and the incumbent is
reelected. $u < \bar{u} \Rightarrow \Pi(u) < \pi$ and the incumbent is not reelected.

Given the parameters and the strategies described above, there is two possible equilibria of the game:

Result 1

- A **pooling equilibrium** $\lambda = 1$, in which all incumbent choose the policy preferred by the voter $(P = s)$, except a small share $\xi$ of politicians who always pick their own preferred policy, exists if $\kappa \leq F(1/2) - F(-1/2)$:

- A **separating equilibrium** $(\lambda = 0)$ in which incumbent always play their preferred strategy in the first period. This equilibrium exists if $\kappa > F(1/2) - F(-1/2)$. (The demonstration for a dissonent politician is similar to the demonstration for a dissonent politician in the pooling equilibrium)

3 Policy making in a federation

The literature on yardstick competition shows that when the voters in one jurisdiction can observe the policy making in the neighbouring jurisdictions, they can partly soften the agency problem they face in their relationship with their representatives (see Besley and Case (1995). The notion of “neighbourhood” is not to understand only literally as a measure of geographical proximity. What makes yardstick competition possible is in fact: 1) the proximity in terms of political and economic institutions, and 2) a certain level of correlation in the random shocks and unobservable variables affecting the economy and the outcome of the economic policies. Hence by observing two or more signals, instead of only one, on some unobservable variable, say the performance of an incumbent politician, a voter can increases the amount and the precision of the information at his disposal. More information means in turn a less severe agency problem between the principal-voter
and the agent-politician.

To study the impact of the yardstick competition on the political economic outcomes, the literature typically modelize a game with two jurisdictions in which incumbents simultaneously make policy decisions. Representative voters can then observe policies and outcomes in both jurisdictions. Given a correlation and similarity between the two jurisdictions, voters in both regions make comparisons of policies and outcomes to infer the level of competences/effort of their own incumbent, and make a re-election decision.

In the following I propose to study, in a related framework, the role of the yardstick competition on the diffusion of policies across jurisdictions. The literature cited above focuses on the yardstick competition as an alternative or complement to the mobility of (voice vs exit, to use the conceptual framework of Hirschman (1970)) to constraint the Leviatan (Brennan and Buchanan (1978)). Decentralization introduces generates mobility/yardstick competition and provide incentives to representatives to work in voters interests.

Another role of decentralisation is to encourage experimentation and generate learning about new policies or which policies are the right policies. This is the idea of “laboratory federalism” proposed by Oates (1999) (see Rose-Ackerman (1980), Strumpf (2002), LeBorgne and Lockwood (2006), Kotsogiannis and Schwager (2006b), Kotsogiannis and Schwager (2006a)).

3.1 Policy making and learning

I now extend the framework of the preceding section to consider a federation composed of $N$ similar jurisdictions. The same sequence of event happens in both regions, but not simultaneously, unlike in the typical models presented

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6Across jurisdictions in a federation, but also across countries in a union like the EU for instance.
7si moins d’expérimentation dans les papiers cités dans le cas de la decentr. peut-être parce que décisions simultanées; mérite peut-être une comparaison: partie normative: centre vs decentre ?
8definir, elargir
in the literature on yardstick competition cited above. Indeed, I suppose that decisions (policy choice, re-election) are first made in one jurisdiction, then in the second jurisdiction, and so on. Hence voters in the jurisdiction $i$ can observe the policy choices and outcomes in other jurisdictions before making their voting decision. I consider that the order of regions in the policy making is exogeneous.

The modelize the dependence across regions I suppose that the best polices $x$ are correlated between two jurisdictions $i, j$, where $i$ states for the $i^{th}$ region to implement a policy, and $j$ states for the next region to implement a policy, according to the joint distribution in table (3.1), where $\rho > 1/2$, such that there is a positive correlation between the best policies.

The incumbent and the voter in the region $i$ make their decisions according to the sequence described in the preceding section for a single jurisdiction: the incumbent makes a policy choice $P_i$, a level of utility $u_i$ is provided to the voter, who chooses whether to reelect or not the incumbent. This sequence of events generate some information for the region $j$. This information is incorporated in the public belief.

**Evolution of the public belief**

The public belief $\mu_i^1$ incorporates all the public information about the best policy $x_i$ in state $i$ before any action is taken. In the preceding section with a single jurisdiction this prior belief was uninformed and $\mu_i^1$ was equal to $1/2$. This is not the case anymore. After the occurrence of the policy choice $P_i$ and outcome $u_i$, the public belief about $x_i$ is updated and
becomes $\mu_i^2$. This public belief $\mu_i^2$ incorporates all the public information about $x_i$ after the policy making and its outcome. Given the structure of the game, it is equivalent to formulate $\mu_i^2$ as a function of $\mu_i^1$ and the policy choice and outcome: $\mu_i^2(\mu_i^1, P_i, u_i)$, or as a function of the complete history of past policy choices and outcomes in all regions, incorporated in $I_i = \{\{P_1, u_1\}, \{P_2, u_2\}, \ldots, \{P_{i-1}, u_{i-1}\}\}$ and then $\mu_i^2(I_i, P_i, u_i)$. As we will see, this equivalence is due to the martingale property of the evolution of the public believes $\mu$ across regions.

The updating of the public belief $\mu_i$ is a function of all public knowledge and according to the Bayes rule:

$$\mu_i^2 | P = a, u = \frac{\mu_i^1 \Pr(P = x) f(u-1)}{\mu_i^1 \Pr(P = x) f(u-1) + (1 - \mu_i^1) \Pr(P \neq x) f(u)}$$

(10)

and $\mu_i^2 | P = a, u \geq \mu_i^1 \Leftrightarrow \frac{\Pr(P=x)}{\Pr(P\neq x)} \geq \frac{f(u)}{f(u-1)}$

$$\mu_i^2 | P = b, u = \frac{\mu_i^1 \Pr(P \neq x) f(u)}{\mu_i^1 \Pr(P \neq x) f(u) + (1 - \mu_i^1) \Pr(P = x) f(u-1)}$$

(11)

and $\mu_i^2 | P = b, u \geq \mu_i^1 \Leftrightarrow \frac{\Pr(P=x)}{\Pr(P\neq x)} \leq \frac{f(u)}{f(u-1)}$

The policy choice and its outcome in the region $i$ reveal some information on the best policy $x_i$ in this region. Due to the correlation between the best policies in two succeeding regions $i$ and $j$, the observable variables $P_i$ and $u_i$ also reveal some information about the best policy $x_j$ in the region $j$. This is where the interjurisdictional learning potentialy takes place. Indeed, given the updated belief in region $i$ and the correlation between the good policies across the states $i$ and $j$, the public belief about the good policy is:

$$\Pr(a_i) = \Pr(a_i, a_j) + \Pr(a_i, b_j)$$

$$\mu_j^1 = \rho \mu_i^2 + (1 - \rho) (1 - \mu_i^2)$$

(12)
Equilibrium with yardstick competition

To understand the impact of this learning/informational externality, one must examine if and under which condition the two equilibria in result (1) exist when the public belief may be informed.

Upon receiving a signal $s$, the incumbent forms his private belief $\tilde{\mu}$ about $x$:

\[
\tilde{\mu} = \Pr(x = a \mid s) = \frac{\Pr(s \mid x = a) \Pr(x = a)}{\Pr(s \mid x = a) \Pr(x = a) + \Pr(s \mid x = b) \Pr(x = b)}
\]

(13)

The favorite policy of the incumbent is then given by the relative size of the public belief about $x$ and the precision of the private signal. Hence a congruent incumbent prefers the policy annonced by his signal if

\[
q > \Pr(x \neg s)
\]

while the opposite is true for a dissonent incumbent. $\Pr(x \neg s)$ stands for the public belief about the other policy than that announced by the signal.

Since the goal of the voter is unchanged, the re-election rule keeps the same characteristics: the voter re-elect the incumbent if and only if $\Pi > \pi$. However, the public belief being now possibly informed, the utility threshold below which the incumbent is not re-elected has now to be dependant on the policy choosed. Hence I call $u_a$ and $u_b$ the thresholds for reelection for $P = a$ respectively $P = b$. The values of $u_a$ and $u_b$ are uniquely defined by the following conditions:

\[\text{Pr}(x = a \mid s) > 1/2 \Leftrightarrow q > 1 - \mu^1\]
\[
\frac{f(u_a)}{f(u_a - 1)} = \frac{\mu^1}{1 - \mu^1} \\
\frac{f(u_b)}{f(u_b - 1)} = \frac{1 - \mu^1}{\mu^1}
\]

(15)  
(16)

Given the hypothesis that the distribution function \(f\) is single peaked and symmetrical (which in turn allows us to use the MLRP), it is easy to see that both \(u_a\) and \(u_b\) are uniquely defined. Furthermore one can see that:

\[u_b = 1 - u_a\]

The next result allows us to understand the impact of the yardstick comparison on the behavior of the dissonent type:

**Result 2**

**Pooling equilibrium with multiple jurisdictions:** a dissonant incumbent pools on the voter’s favorite policy if \(\kappa \leq F(1/2) - F(-1/2)\) and:

\[\mu \in [\underline{\mu}; \bar{\mu}]\]

(17)

where \(0 < \underline{\mu} < \bar{\mu} < 1\)

- **Separating equilibrium with multiple jurisdictions:** alternatively a dissonant incumbent plays his own favorite policy if \(\kappa > F(1/2) - F(-1/2)\) or

\[\mu \notin [\underline{\mu}; \bar{\mu}]\]

(18)

Thus, when the public belief about one of the policies is strong enough (that is below \(\underline{\mu}\) or above \(\bar{\mu}\), a dissonant incumbent choose his favorite policy. The result (2) basically says that a more precise information about the best policy may reduce the disciplining incentive of election. This effect is a common result of the literature on the yardstick competition and even more generally of the principal-agent literature. This decrease in the pooling by dissonent
incumbent is of course compensated by an increase in the quality of the selection process: by choosing his favorite policy a dissonent incumbent is likely to reveal his type and to loose the re-election, especially if the public belief about one particular policy is strong.

3.2 Pandering

In the preceding result I made the assumption that congruent incumbents would play the voter’s favorite policy, since such a policy also is their favorite, and since they would maximize their likelihood of reelection by maximizing the voter’s expected utility. However, as we shall see, some particular situations may arise in which a congruent incumbent chooses a policy against the voter’s preferences.

If the private belief $\tilde{\mu}$ of a congruent incumbent goes the same way as the public belief about which policy is to be implemented, then of course this policy will be choosen. It could be however that the incumbent’s private belief about the best policy diverges from the public belief, and designates another policy as the most likely to be the best. Two cases may arise:

- Suppose that $\mu > 1/2$ and that the incumbent receive a signal $s = b$; if $\mu < q$, then $\tilde{\mu} < 1/2$.
- Suppose that $\mu < 1/2$ and that the incumbent receive a signal $s = a$; if $\mu > 1 - q$, then $\tilde{\mu} > 1/2$.

Then, the situation $\tilde{\mu} < 1/2 < \mu$ may arise if the incumbent gets a signal $s = b$ and $\mu < q$; the situation $\tilde{\mu} > 1/2 > \mu$ may arise if the incumbent gets a signal $s = a$ and $\mu > 1 - q$. In such cases the question for the congruent incumbent is therefore whether to choose the policy the most likely to be the best as pointed by his private belief $\tilde{\mu}$, or whether to implement the policy the most popular, as pointed by the public belief $\mu$. Indeed, implementing a policy the most likely to be the best but going against the public belief may be costly in terms of re-election likelihood. Implementing the best policy
(that is following the private belief $\bar{\mu}$) is rational for a congruent politician as long as:

$$\kappa > \frac{\bar{\mu} [F(u_a - 1) - F(1-u_a)] + (1 - \bar{\mu}) [F(u_a) - F(-u_a)]}{2\bar{\mu} - 1}$$

(19)

Let's take the case where $\mu > 1/2 > \bar{\mu}$. The condition (19) can be written as:

$$\kappa > \frac{(1-q)\mu [F(u_a - 1) - F(1-u_a)] + q (1-\mu) [F(u_a) - F(-u_a)]}{\mu - q}$$

(20)

When the public belief $\mu$ is close to 1/2, the condition (20) is respected. However, as shown in the appendix, there is a threshold $\bar{\mu}$ such that when $q > \mu > \bar{\mu}$ then the RHS of equation (20) is positive. The same reasoning applies to the case where $\mu < 1/2 < \bar{\mu}$: there is a threshold $\bar{\mu}$ such that when $\bar{\mu} > \mu > 1 - q$ the RHS of condition (19) is positive. As stated in the following proposition, there is a parameters set for which the condition (19) is not satisfied. In such cases, the behaviour of the congruent incumbent is guided by the public belief.

**Proposition 1 Pandering equilibrium**

When the variance of the distribution function $F(.)$ is high enough relative to the level of patience $\kappa$, a congruent incumbent may pandering to voter by choosing the popular policy with no consideration for his private information about the best policy. This happens whenever $\mu \in ]q; \mu_p[ \text{ or } \mu \in ]1-q; 1-\mu_p[$, where $\mu_p$ is a threshold for pandering, whose existence is defined in the appendix.

The consequence of Proposition (1) is that, in some circumstances, when the voter has a strong belief about a given policy, even a congruent politician may have the incentive to follow the public belief and to neglect his own private information. Of course this outcome is costly in terms of utility. But another important consequence arise if we consider again the mechanism
of interjuridictional learning. Indeed, by pandering to the public belief, a politician slows the process of learning, since he do not use his private signal. As long as the shock $\varepsilon$ is not too high, the public belief in the next jurisdiction is unlikely to be largely updated and the wrong policy is more likely to be implemented.
References


Proof

All proof will be provided upon request.

Given \( \mu_2 \) and \( s = b \), the dissonent type chooses \( P = b \) if \( \mu_2 < q \), that is if \( \mu^p < 1/2 \), and if

\[
(1 - \mu^p) [\kappa + \Pr (\varepsilon \geq -\bar{u})] + \mu^p \Pr (\varepsilon \geq 1 - \bar{u}) \geq \mu^p [\kappa + \Pr (\varepsilon \geq \bar{u} - 1)] + (1 - \mu^p) \Pr (\varepsilon \geq \bar{u})
\]

\[
(1 - 2\mu^p) \kappa + (1 - \mu^p) [\Pr (\varepsilon \geq -\bar{u}) - \Pr (\varepsilon \geq \bar{u})] + \mu^p [\Pr (\varepsilon \geq 1 - \bar{u}) - \Pr (\varepsilon \geq \bar{u} - 1)] \geq 0
\]

\[
(1 - 2\mu^p) \kappa + (1 - \mu^p) (2\Pr (\varepsilon < \bar{u}) - 1) + \mu^p (2\Pr (\varepsilon < \bar{u} - 1) - 1) \geq 0
\]

\[
\Rightarrow \kappa \geq 1 - \frac{2((1 - \mu^p) \Pr (\varepsilon < \bar{u}) + \mu^p \Pr (\varepsilon < \bar{u} - 1))}{(1 - 2\mu^p)}
\]

Trouble when: (example for \( s = b \)) \( \mu^p < 1/2 < \mu < q \): in that case, the dissonent type would prefer \( P = b \). He plays \( P = b \), thus maximizing voter’s welfare, if

- \( \mu^p < 1/2 \rightarrow \mu < q \), and

- \[
(1 - 2\mu^p) \kappa \geq 1 - (1 - \mu^p) 2\Pr (\varepsilon < \bar{u}) - \mu^p 2\Pr (\varepsilon < \bar{u} - 1) \quad \text{(cond1)}
\]

The LHS of cond1 is monotonically decreasing in \( \mu \): \( \frac{\partial \kappa}{\partial \mu} < 0 \), whereas the RHS is monotonically increasing: \( \frac{\partial \kappa}{\partial \mu} (F(\bar{u}) - F(\bar{u} - 1)) - 2\frac{\partial \kappa}{\partial \mu} ((1 - \mu^p) f (\bar{u}) + \mu^p f (\bar{u} - 1)) > 0 \)

When \( \mu \rightarrow q \): \( (1 - 2\mu^p) \kappa \rightarrow 0 \), and \( 1 - (1 - \mu^p) 2\Pr (\varepsilon < \bar{u}) - \mu^p 2\Pr (\varepsilon < \bar{u} - 1) \rightarrow 1 - F(u(q)) - F(u(q) - 1) \), which is positive for \( q > 1/2 \).

Then

- For \( \mu = q \) cond1 is not fullfilled.

- cond1 is monotonic in \( \mu \)
• There is then a unique $\tilde{\mu}$ such that cond1 is binding

• There is an interval $[\tilde{\mu}, q]$ such that cond1 is not fulfilled

**Proof pooling equ:** Given $\text{Prob}(x = a) = 1/2$ and $q$, $\text{Prob}(x = i \mid s = i) = q$, for $i \in \{a, b\}$. Then given the reelection rule $\bar{u}$, it is optimal for a dissonent politician to choose policy $P = s$ in the first period only if

$$q [R + \text{Prob}(1 + \varepsilon \geq 1/2) \beta (G + R)] + (1 - q) [G + R + \text{Prob}(\varepsilon \geq 1/2) \beta (G + R)] \geq q [G + R + \text{Prob}(\varepsilon \geq 1/2) \beta (G + R)] + (1 - q) [R + \text{Prob}(1 + \varepsilon \geq 1/2) \beta (G + R)]$$

(21)

Since $q > 1/2$ and given the cdf $F(.)$ the condition can be written as

$$(1 - F(-1/2)) \beta (G + R) \geq G + (1 - F(1/2)) \beta (G + R)$$

(22)

or, given the definition of $\kappa$

$$F(1/2) - F(-1/2) \geq \kappa$$

(23)

**Proof sep equ**

It is easy to see that it is always optimal for a congruent incumbent to implement his (and voter’s) preferred strategy. Indeed, choosing $P \neq s$ would make sense for a congruent politician only if

$$q [G + R + (1 - F(\bar{u} - 1)) \beta (G + R)] + (1 - q) [G + R + (1 - F(\bar{u})) \beta (G + R)] < q [R + (1 - F(\bar{u})) \beta (G + R)] + (1 - q) [G + (1 - F(\bar{u} - 1)) \beta (G + R)]$$

which would be the case if $\kappa < F(\bar{u} - 1) - F(\bar{u}) < 0$. Indeed this case can be excluded from the definition of $\kappa$. 

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