Parameter Drifting in an Estimated DSGE Model on the Czech Data *

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Abstract. In the paper, we investigate a possible drifting of structural parameters in an estimated small open economy DSGE model. To do this, we first estimate the model with a Bayesian method on the Czech data and discuss results. Then, we identify trajectories of structural parameters via a non-linear filtration based on the model's second-order approximation. We identify two drifting parameters, namely the import share of export and the import share of consumption whose movements are related to the significant exchange rate movements. The rest of parameters seems to be relatively stable in time.

Keywords: DSGE models, time-varying parameters, Kalman filter, Bayesian methods, Particle filter

JEL classification: D58, E32, E47, C11, C13

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1. Introduction

The stability of an economy's structural parameters in a medium term is an important assumption for many current macro models. A majority of dynamic stochastic

*Financial support of the Specific Research Fund at the Faculty of Economics and Administration and of the Research Project B2/10 at the Czech National Bank is gratefully acknowledged. The views expressed here are those of the authors, and do not necessarily reflect the position of the Czech National Bank. We are grateful to Jan Brůha, Martin Fukač and Milan Výškrabka for many helpful comments and advices. Also, we would like to thank Michal Andrle, Jaromír Beneš, David Havrlant, Tibor Hlédik, Ondra Kameník, Radek Šnobl, and Jan Vlček.

general equilibrium (DSGE) models, based on micro foundations and exogenous processes, stands on this assumption. The possible drifting of structural parameters, caused by structural changes, might thus cause a bias of many DSGE-based analyses and forecasts with a direct consequence in frequent recalibrations.

The parameter drifting in DSGE models might be influenced by the model specification. In general, parameters of DSGE models describe preferences, production structure, wage and price setting behaviour etc. However, DSGE models do not contain only equations based on agents' optimization problems. They are usually complemented with AR processes and technologies to fit a country's stylized facts.[1] These exogenous processes might possibly capture some consequences of structural changes and thus allow using a "better-specified" model for a longer time. For example, high foreign direct investment inflow and the entry of the Czech Republic into the EU have affected the volumes of trade balances. Andrle at al. [2] describe the way how to cope with these issues via openness and quality technologies in a DSGE model.

For a developing country, this issue might be more important because such an economy goes through frequent structural changes. Twenty years after the revolution, the Czech economy still remains on a converging path towards the more developed countries of the Western Europe. It has been hit by various shocks, some of them bringing structural changes. The European Union (EU) entry might be a good example. With this respect, naturally, there emerges a question about a projection of these changes and shocks into DSGE models' parameters and their drifting in time.

There are several papers that aim to identify drifting of structural parameters. Canova [7] estimates a small New-Keynesian model with parameter drifting. He finds the stability of the policy rule parameters and varying parameters of the Phillips curve and the Euler equation. Boivin [4] estimates a Taylor rule with drifting parameters. He identifies important but gradual changes in the policy rule parameters. Fernandez-Villaverde and Rubio-Ramirez [10] estimate a DSGE model using the U.S. data and allow for parameter drifting. On a basis of 184 observations, they find out the changing parameters in the Fed's behaviour and also the drifting of pricing parameters which is correlated with changes in inflation. More recently, Fernandez-Villaverde et al. [9] build a DSGE model with both stochastic volatility and parameter drifting in the Taylor rule and estimate it non-linearly using U.S. data and Bayesian methods. They find out evidence of changes in monetary policy even after controlling for stochastic volatility. Besides, there is a literature on VAR's estimation with time-varying parameters. For example, Sims and Zha [13] do not find any change in parameters either of the policy rule or of the private sector block of their model. Instead, they identify changing variances of structural disturbances.

Thus in the paper, we analyze a possible drifting of structural parameters in a relatively complex and on the Czech data estimated DSGE model.[2] To do this, we let structural parameters drift and subsequently identify their trajectories via a non-linear filtration method on the model's second-order approximation.

[1] See [15].

[2] In the paper, we do not analyze years prior to 1996 because of an incomplete data set for those years.

First we construct the model and check it properties. For our purpose, we need a sufficiently rich and general small open economy model, adapted to the Czech data.[3] The model in the paper is based on two existing models. First, we use the model of [6] designed for the Spanish economy as our backbone framework. To cope with the Czech data, we simplify the model and extend it with several features according to [2].

To check the model's performance, we run several tests. More concretely, we carry out the Bayesian estimation of time-invariant model parameters and also check the model properties.[4] These tests confirm the model usefulness for analyses based on Czech data via higher order approximations.

After the initial estimation and checks, we allow several structural parameters to drift in time. We follow directly methods proposed in [10]. First, we run the Kalman filter on the first-order approximated model. This procedure is the two-step problem where the former consists of adding AR processes into the framework whereas the latter in endogenizing the deep parameters via the AR processes. Adding new exogenous processes helps us to get the model to data. Endogenizing deep parameters then show us a time-varying structure of the model. Second, we run a Particle filter on the second-order approximated model. Nonlinear filtration is necessary for model agents to anticipate future parameter movements.

We identify two drifting parameters, namely the import share of export and the import share of consumption. We find the strongest relation between these parameters and significant exchange rate movements. To explain these findings, we employ a simple correlation analysis among these parameters and observables because the standard tools as decompositions to observables are not additive in the case of a non-linear world. For example, if final good producers anticipate considerable exchange rate depreciation, they try to substitute import intermediate goods for their domestic intermediate counterparts.

Although the Czech economy has undergone through several structural changes, our estimation does not prove another drifting of structural parameters in the model. For example, the regime switch to the inflation targeting does not influence the Taylor rule parameters. Moreover, the entry to the EU also does not strongly influence any structural parameter.

2. Model

This section provides a brief description of the model. Our objective here is to explain the motivation of the model choice, its suitability for the Czech data analysis and its basic structure. On the other hand, we do not present the full description of the model which can be found in the attached Technical Appendix. We follow

[3]We try to replicate significant Czech economy features. Some of them are modelled very simply, because we do not use the model for regular forecasting. In that case, the model would converge close to the CNB's g3 [2] model developed for these objectives.

[4]We check the model properties and its performance by the impulse response analysis, the Kalman filtration, the decomposition of endogenous variables into shocks and the decomposition of forecasts of endogenous variables from the steady-state. All of these tests are accomplished on the estimated model.

this structure for several reasons. First, the model itself is not the centre of the paper. Instead, it is a tool which we use for the estimations. Second, the model is based on two (Spanish and Czech) existing models, both sufficiently described in the literature and we not aim to replicate existing papers. And finally, it would be almost impossible to describe the model in full detail with a reasonable length with respect to other sections of the paper.

2.1. Motivation

Most monetary DSGE models are similar to each other. They consist of several sectors with few general principles of derivation. Optimization problems imply equations describing the main behaviour characteristics of economic agents. Subsequently, the model equations is extended with many features like exogenous processes or wedges to get a final model closer to country's stylized facts.[5] The number of these extensions and their variety differ with the purpose of a model, ranging from various general analyses to central banks' regular forecasts where such a core model should capture "all" the main stylized facts of an economy.

For our purposes, we need a sufficiently complex and general model, extended with several features to be closer to the Czech data. First, the model should be complex to capture national accounts, wage- and price-setting behaviour and various small open economy features. Second, we need a general model for non-linear filtrations to capture some special Czech economy characteristics. On the other hand, we still aim to use as simple model as possible to be controlled.

The model is based on two existing models. First, we use the model of [6], designed for the Spanish economy, as our backbone model. This model follows the current generation of DSGE models for the inflation targeting regime. It is sufficiently rich and general within its sectors structure and contains many well-known modelling features like real and nominal rigidities, technology growths, local currency pricing etc. Moreover, it is also described in literature in great detail.[6] To cope with the Czech data, we extend it with several features according to [2].[7] Hence, we believe that the model should provide us a sufficient rich tool for the estimations.[8]

The model has a relatively standard and general structure with optimizing agents and rational expectations. It contains a set of real (internal habit formation, capital adjustment costs etc.) and Calvo-type nominal rigidities with indexation parameters.[9] The production structure with intermediate and final goods producing firms

[5]Note that adding various features into a DSGE framework should not be ad hoc. As authors in [2] note for the case of regulated prices: "In a structural model regulated prices require structural interpretation". We believe that this holds in general.

[6]See also [10].

[7]Authors in [2] describe the new Czech National Bank's (CNB) core model and summarize the main stylized facts of the Czech economy w.r.t. their modelling principles. Also, they discuss some non-standard characteristics of the Czech economy and their corresponding ways how to structurally incorporate them into the monetary DSGE framework.

[8]We try to replicate all the Czech economy features, but some of them in a very simple way, because we do not use the model for regular forecasting. In that case, the model would converge close to the CNB's g3 model developed for those objectives.

[9]As is noted in [6], the model does not contain the Phillips curves. Instead, the derived equations

enables to capture the GDP accounts while the local currency pricing mechanism enables the incorporation of a gradual exchange rate pass-through into the model mechanism. The model is closed with a debt-elastic premium according to [12]. The overall structure of the model is described in Figure 1 and more concretely presented in the next subsections.

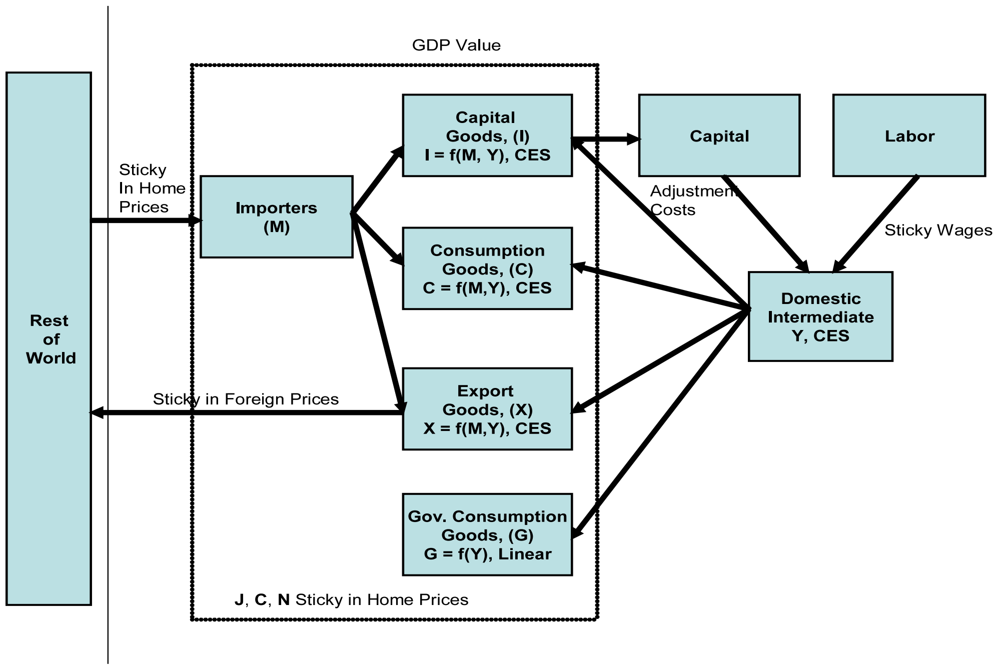

Figure 1: Structure of the Model

2.2. Households

Households consume final consumption goods, save in domestic and foreign assets, and supply differentiated labour. The individual household labour types are monopolistically competitive which provide them a degree of power for the wage setting.[10] Also, households own all firms in the model and, thus, finance them internally or receive their dividends.

Households' maximize their utility function subject to budget constraint and law of motion for the capital accumulation. The utility function is separable in consumption with internal habit formation, real money balances and the labour supply. The optimization problem has the following form[11]

from households' and firms' optimizations are left in general forms for performing higher order approximations.

[10]All differentiated commodities (labour types, intermediate goods) are assumed to be packed by a bundler and then supplied to the firms as a single composite.

[11]The first order condition with respect to real money balances is not necessary for the inflation targeting regime. Moreover, the first order condition with respect to Arrow securities is also not necessary because we assume complete markets and separable utility in labour. See [6] and (Erceg

$$
\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t^L \left[\begin{array}{c} d_t\{\log(c_{jt} - hc_{jt-1}) + \upsilon \log \frac{m_{jt}}{p_t} - \varphi_t \psi \frac{(l_{jt}^s)^{1+\vartheta}}{1+\vartheta}\} \\ -\lambda_{jt} \left\{ \begin{array}{c} (1+\tau_c)\frac{p_t^c}{p_t} c_{jt} + \frac{p_t^j}{p_t} i_{jt} + \frac{m_{jt}}{p_t} + \frac{b_{jt}}{p_t} + \frac{ex_t b_{jt}^W}{p_t} + \int q_{jt+1,t} a_{jt+1} d\omega_{j,t+1,t} \\ -(1-\tau_w) w_{jt} l_{jt}^s + (r_t(1-\tau_k) + \mu_t^{-1}\delta\tau_k) k_{jt-1} - \frac{1}{\gamma_t^L}\frac{m_{jt-1}}{p_t} - T_t - F_t \\ -R_{t-1}\frac{1}{\gamma_t^L}\frac{b_{jt-1}}{p_t} - R_{t-1}^W \frac{1}{\gamma_t^L}\frac{ex_t b_{jt-1}^W}{p_t} - \frac{1}{\gamma_t^L} a_{jt} \end{array} \right\} \\ Q_{jt}\{\gamma_t^L k_{jt} - (1-\delta) k_{jt-1} - \mu_t (1 - S\left[\gamma_t^L \frac{i_{jt}}{i_{jt-1}}\right]) i_{jt}\} \end{array} \right], \tag{1}
$$

where β is the discount parameter, γ_t^L is the growth of population, d_t is an intertemporal preference shock, c_{jt} is per capita consumption, h is the habit persistence parameter, $\frac{m_{jt}}{p_t}$ is the per capita real money balances, φ_t is the preference shock, ψ is the labour supply coefficient, ϑ is the inverse of Frisch labour supply elasticity, l_{jt}^s is the per capita hours worked, λ_{jt} is the Lagrangian multiplier associated with the budget constraint, τ_C is the tax rate of consumption, p_t^c is the price level of the consumption final good, p_t is the price level of the domestic final good, p_t^i is the price level of the investment final good, i_{jt} is per capita investment, b_{jt} is the level of outstanding debt, $ex_t b_{jt}^W$ is an amount of foreign government bonds in the domestic currency, ex_t is nominal exchange rate, a_{jt+1} of Arrow securities, τ_W is the tax rate of wage income, w_{jt} is the overall real wage index, r_t is the real rental price of capital, τ_k is the tax rate of capital income, μ_t is the investment-specific technology, δ is the depreciation rate of capital, k_{jt} is the per capita capital, T_t is the lump-sum transfer, F_t are the profits of the firms in the economy, R_t is the nominal interest rate, R_t^W is the foreign nominal interest rate, u_{jt} is the intensity of use of capital, q_t is the marginal Tobin's Q, $S\left[\gamma_t^L \frac{i_t}{i_{t-1}} z_t\right]$ is an adjustment cost function on the level of investment.

Households solve the optimization problems for consumption, investment, capital and its utilization, domestic and foreign bonds, real money balances and labour supply (and wage). The last first order condition (FOC) with respect to labour (and wage) implies equations for the optimal wage setting. Here the optimization problem is a two-step. First, we need to find a relation between labour and wage and, then, by substitution, we solve the corresponding part of the Lagrangian

$$
\max_{w_{jt}} \mathrm{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \gamma_\tau^L \left(-d_{t+\tau}\varphi_{t+\tau}\psi \frac{(l_{jt+\tau}^s)^{1+\vartheta}}{1+\vartheta} + \lambda_{jt+\tau} \prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} (1-\tau_w) w_{it} l_{jt+\tau}^s \right), \tag{2}
$$

$$
\text{s.t. } l_{jt+\tau}^s = \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w} w_{jt}}{\Pi_{t+s} w_{t+\tau}} \right)^{-\eta} l_{t+\tau}^d,
$$

where θ_w is the Calvo parameter for wages, Π_t is the inflation of the domestic intermediate good, χ_w is the indexation parameter for wages, η is the elasticity of substitution among different types of labour, l_t^d is the per capita labour demand.

et al., 2000).

2.3. Intermediate Goods Producing Firms

The model contains two intermediate goods producing sectors - domestic and import firms. All firms are assumed to be monopolistically competitive which provides them a degree of power for their Calvo-type price setting.

The domestic intermediate firms combine packed labour and rented capital from households. Via a Cobb-Douglas production function, they produce differentiated domestic intermediate goods. Subsequently, these goods are supplied to the final goods producing firms as their inputs.

More formally, the domestic intermediate goods producing firms solve a two-steps optimization problem. First, they are minimizing their costs with respect to the production function, given input prices

$$\min_{l_{it}^d, k_{it-1}} w_t l_{it}^d + r_t k_{it-1} \text{ s.t. } y_{it} = A_t k_{it-1}^{\alpha} (l_{it}^d)^{1-\alpha} - \phi z_t. \tag{3}$$

where y_{it} is the per capita production of the domestic final good, A_t is the neutral technology growth, α is the labour share in production of the domestic intermediate goods, ϕ is the parameter associated with the fixed cost production, z_t is the per capita long run growth.

Assuming the Calvo price-setting with the indexation parameters, the second stage consists of the profit maximization by choosing the optimal price

$$\max_{p_{it}} \mathrm{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \gamma_{\tau}^{L} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi_p} \frac{p_{it}}{p_{t+\tau}} - mc_{t+\tau} \right) y_{it+\tau}, \tag{4}$$

$$\text{s.t. } y_{it+\tau} = \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi_p} \frac{p_{it}}{p_{t+\tau}} \right)^{-\epsilon} y_{t+\tau},$$

where θ_p is the Calvo parameter for the domestic good prices, χ_p is the indexation parameter for the domestic good prices, mc_t is the real marginal cost, ε is the elasticity of substitution among different types of the domestic intermediate goods.

On the other hand, the intermediate importers costlessly differentiate the single foreign good which they import from the rest of the world. The packed intermediate imported good is then supplied to the final goods producers (except the government sector).

The optimization problem has only one step because the import intermediate firms buys only one foreign good with the straightforward specification for the marginal cost (and hence no need for optimality conditions between two inputs). The price-setting problem has the following form

$$\max_{p_{it}^M} \mathrm{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_M)^{\tau} \gamma_{\tau}^{L} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\prod_{s=1}^{\tau} (\Pi_{t+s-1}^M)^{\chi_M} \frac{p_{it}^M}{p_{t+\tau}^M} - mc_{t+\tau}^M \right) y_{it+\tau}^M, \tag{5}$$

$$\text{s.t. } y^M_{it+\tau} = \left(\prod_{s=1}^{\tau} (\Pi^M_{t+s-1})^{\chi_M} \frac{p^M_{it}}{p^M_{t+\tau}}) \right)^{-\epsilon_M} y^M_{t+\tau},$$

where p^M_{it} is the price of goods of importing firms in the domestic currency, θ_M is the Calvo for the import prices, Π^M_t is the imported good inflation, χ_M is the indexation of the imported good, $mc^M_t = \frac{ex_t p^W_t}{p^M_t}$ is the real marginal cost in the importing sector, p^W_t is the foreign price of the foreign homogenous final good in the foreign currency, $ex_t p^W_t$ is its foreign price in the domestic currency, y^M_t is the final imported good, ε_M is the elasticity of substitution among different types of imported goods.

2.4. Final Goods Producing Firms

The model contains four final goods producing sectors - consumption, investment, export and government.[12] Consumption, investment and export firms purchase both intermediate composite inputs. The monopolistic competition is only within the export sector.

Consumption, investment and export final goods producers $s = c, i, x$ maximize profits subject to their CES production functions

$$\max_{s^d_t, s^M_t} \mathrm{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \gamma^L_{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} (p^s_t s_t - p_t s^d_t - p^M_t s^M_t), \tag{6}$$

$$\text{s.t. } s_t = \left[(n^s)^{\frac{1}{\epsilon_s}} (s^d_t)^{\frac{\epsilon_s - 1}{\epsilon_s}} + (1 - n^s)^{\frac{1}{\epsilon_c}} (s^M_t (1 - \Gamma^s_t))^{\frac{\epsilon_s - 1}{\epsilon_s}} \right]^{\frac{\epsilon_s}{\epsilon_s - 1}},$$

where s^d_t are the domestic consumption, investment and export, s^M_t are the imported consumption, investment and export, p^s_t are consumption, investment, and export prices, ϵ_s are the elasticities of substitution among different types of consumption, investment and export goods, n^s are home bias in the aggregation in consumption, investment and export and Γ^s_t are adjustment costs in consumption, investment and exports sectors.

For exporting firms, there is a second stage optimization problem associated with their market power and the Calvo price-setting

$$\max_{p^x_{it}} \mathrm{E}_t \sum_{\tau=0}^{\infty} (\beta \theta_x)^{\tau} \gamma^L_{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\prod_{s=1}^{\tau} (\Pi^x_{t+s-1})^{\chi_x} \frac{p^x_{it}}{p^x_{t+\tau}} - mc^x_{t+\tau} \right) y^x_{it+\tau}, \tag{7}$$

$$\text{s.t. } y^x_{it+\tau} = \left(\prod_{s=1}^{\tau} (\Pi^x_{t+s-1})^{\chi_x} \frac{p^x_{it}}{p^x_{t+\tau}} \right)^{-\epsilon_x} y^x_{t+\tau},$$

where p^x_{it} is the price of the exported goods in the foreign currency, θ_x is the Calvo for the export prices, Π^x_t is the export prices inflation in the foreign currency, χ_x is

[12]The final government spending goods are produced from domestic intermediate goods only and thus there is no optimization exercise.

the indexation of the exported prices, $mc_t^x = \frac{p_t}{ex_t p_t^x}$ is the real marginal cost in the exporting sector, $ex_t p_t^x$ is the price of the exported goods in the domestic currency, y_t^x is the demand for the products of exporting firms, ε_x is the elasticity of substitution among different types of exported goods.

2.5. Policy Authorities

A central bank operates under the inflation targeting regime. It sets its one-period nominal interest rate through open market operations according to a Taylor-type rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left(\left(\frac{\Pi 4_{t+4}^c}{target_{t+4}}\right)^{\gamma_\Pi} \left(\frac{\gamma_t^L \frac{y_t^d}{y_{t-1}^d} z_t}{\Lambda_L \Lambda_{y^d}}\right)^{\gamma_y}\right)^{1-\gamma_R} \exp(\xi_t^m), \tag{8}$$

where R is a steady state of nominal interest rate, $target$ is the inflation target, γ_R Taylor rule parameter (rates), γ_Π Taylor rule parameter (inflation), γ_y Taylor rule parameter (output), Λ_{y^d} is the growth rate of output, Λ_L is the growth rate of population, ξ_t^m is the monetary policy shock.

Hence, it targets the four period-ahead year-on-year headline inflation $\Pi 4_{t+4}^c$. Our motivation here is to get the model closer to the official monetary policy rule of the CNB.[13]

For the fiscal policy, we assume a Ricardian setting of a fiscal policy treatment. Besides our effort to focus on the monetary policy and thus a simple fiscal policy, we are aware of possible ambiguities and uncertainties in supposed practical fiscal policy effects.[14] Thus, we assume a simple fiscal policy according to

$$g_t = \rho_g c_t + \xi_t^g, \tag{9}$$

where g_t is the per capita level of real government consumption

2.6. The Rest of the World

The rest of the world is represented by the EU and is modelled exogenously. There are many exporters in the EU and their productions enter to production function of domestic importers. Subsequently, there is a bundler. Varieties of domestic exporters are aggregated by bundler, thus exports can be also represented by one aggregate of export prices.

$$x_t = v_t^x \left[\frac{\left(\frac{ex_t p_t^x}{p_t}\right)}{\left(\frac{ex_t p_t^W}{p_t}\right)}\right]^{-\epsilon_W} y_t^W, \tag{10}$$

$$\log(R_t^W) = \rho_{R^W} \log(R_{t-1}^W) + (1 - \rho_{R_W}) \log(R^W) + \xi_t^{R^W}, \tag{11}$$

[13] The policy rule still differs from the CNB's model because the central bank in this model also targets the output with a small weight. See [2] for the description of the CNB's monetary policy rule.

[14] See [3], [5], [11].

$$\log(y_t^W) = \rho_{y^W} \log(y_{t-1}^W) + (1 - \rho_{y^W}) \log(y^W) + \xi_t^{y^W}, \tag{12}$$

$$\log(\Pi_t^W) = \rho_{\Pi^W} \log(\Pi_{t-1}^W) + (1 - \rho_{\Pi^W}) \log(\Pi^W) + \xi_t^{\pi^W}, \tag{13}$$

where v_t^x is the export prices dispersion, ϵ_W is the elasticity of substitution among different types of world trade goods, y_t^W is the world demand, Π_t^W is the foreign homogenous final good prices inflation.

3. Model Estimation

In this section, we present results of the time-invariant Bayesian estimation on quarterly Czech and Eurozone data and discuss the most interesting posterior values of model parameters. The posterior distributions are constructed with the Metropolis-Hastings algorithm[15] of the Dynare Toolbox [8]. Finally we present the most interesting results of time-varying parameter estimation.

3.1. Data

The quarterly Czech data sample covers 58 observations from 1996Q1 to 2010Q2. We use 16 time series as observables for the estimations. Seasonally adjusted national accounts data stem from the Czech Statistical Office (CZSO). Namely, we use real volumes of consumption, investment, government spending, export, import and their corresponding deflators (except the consumption and government spending deflators).[16] The headline CPI inflation also comes from the CZSO. For the wages, we seasonally adjust the time series of the average nominal wage growth in the business sector which stem from the CZSO as well. The data for the labour demand are gained from the Labour Force Sample Survey's seasonally unadjusted time series for "employed in the economy". All series are seasonally adjusted to receive its trend-cyclical component.

The exchange rate is the CZK/EUR while the domestic interest rate is the 3M PRIBOR. We use three foreign observables. The foreign interest rate is the 3M EURIBOR. The foreign inflation is the PPI of the effective Eurozone acquired from the Consensus Forecast. Finally, the foreign real economic activity is approximated by the foreign demand, acquired from the GDP of the effective Eurozone which stems also from Consensus Forecast.[17]

Because of high data uncertainty, we allow for the measurement errors in the model. Prominent examples of this uncertainty might be frequent data revisions, methodology changes, or high volatility of quarter-on-quarter dynamics of several time series, probably partly as a result of the presence of high frequency noise. Measurement errors are incorporated on levels via measurement equations where we let observables to differ from measurements.

[15] 1 million draws

[16] Instead of the consumption deflator, we use the CPI inflation. The government deflator is not necessary because of the simple fiscal policy treatment.

[17] For the definition of effective variables see Inflation Reports of the CNB.

3.2. Priors

First of all, we set steady-state growth rates parameters. The overall growth in the model is slightly above 4.5 % a year which is approximately consistent with the previous GDP growth of the Czech economy.[18] We assume that the population growth does not play any role in determining the model long-run steady-state growth, and thus, we set it to zero.

We set the steady-state nominal appreciation rate to -2.4 % a year. This value corresponds approximately to the data during the relevant period until the beginning of the crisis in 2009. Adding this year to the sample shifts the rate upwards (to the less appreciation rate) because there was a considerable depreciation. In this respect, we assume that financial crises might not affect long-run steady state of an economy. Hence, with our assumption that, ceteris paribus, the Czech economy will return to the long-term appreciation, higher value would bias the calibration. On the other hand, considering the 1998-2008 period would imply stronger appreciation.

The steady state inflation corresponds to the 2 % inflation target set in annual terms.[19] The foreign inflation steady state is calibrated according to the inflation target of the ECB which corresponds to 2 % annually as well. The foreign demand growth for the domestic export is set at a pace of 9 % a year.[20] The steady-state foreign nominal interest rate is calibrated to 4 % annually.

3.3. Posteriors

In this subsection, we present point estimates of some model parameters. Our objective here is to underline and discuss the most tangible parameters which have relatively clear counterparts in the real economy.

In general, we believe that the Bayesian estimation is an important tool for checking a model's calibration (that our calibrated priors are in line with data) and providing an appreciable informative message about an economy. On the other hand, we are aware of considerable data uncertainty (short time series, structural changes in the data, frequent revisions, gradual convergence of the Czech economy etc.) and possible model misspecification that might potentially bias the estimation.

Point estimate of habit formation parameter is relatively high with the value slightly above 0.94. The posterior thus exceeds our prior set to 0.9. We set the prior to this value for two reasons. First, the parameter corresponds to the high level of consumption smoothing in the Czech economy. Second, the decrease of consumption expenditures during the crisis was relatively low with respect to the slump of overall real economic activity and the evolution of wages. With this respect, this fact might be probably partly explained by the social security system and support from government transfers to households.

Elasticities of substitution in different sectors differ between 5.0 for the domestic goods and 9.5 for the export goods. These values correspond to the range of average markups between 25 percent and 12 percent. It might be difficult to check the

[18]The overall steady-state growth is generated via the neutral technology only.

[19]In quarterly terms relevant for the model, this target corresponds to (2/400+1=1.005).

[20]The approximation of foreign demand is four times the EU GDP growth, which is assumed to be 2.5 %. See [2].

Table 1: Estimated Parameters

Parameter		**Prior Mean**	**Dist**	**Posterior Mean**	**Lower and Upper Bound of a 90 % HPD interval**
Preferences					
Habits	h	0.900	beta	0.9409	(0.9326 , 0.9513)
Labour supply coef.	ψ	8.832	gamma	8.8138	(8.7144 , 8.9094)
Frisch elasticity	ϑ	1.250	gamma	1.2556	(1.2330 , 1.2820)
Wedges					
Euler	κ^{euler}	1.0119	-	-	-
Forex	κ^{forex}	1.0034	-	-	-
Adjustment costs					
Investment	κ	20.000	gamma	20.1364	(20.0852 , 20.2006)
Capital utilization	$\gamma 2$	0.280	gamma	0.3229	(0.3054 , 0.3445)
Risk premium	Γ_{bW}	0.800	beta	0.8027	(0.7564 , 0.8553)
Elasticities of substitution					
Domestic goods	ϵ	5.000	gamma	5.0035	(4.9562 , 5.0508)
Import goods	ϵ_M	9.000	gamma	9.0690	(9.0415 , 9.1017)
Export goods	ϵ_x	9.400	gamma	9.2646	(9.1852 , 9.3299)
World goods	ϵ_W	0.860	gamma	0.6938	(0.6090 , 0.7686)
Consumption goods	ϵ_c	7.600	gamma	7.6614	(7.6166 , 7.7022)
Investment goods	ϵ_i	7.600	gamma	7.5723	(7.5363 , 7.6029)
Labour types	η	7.000	gamma	7.0235	(7.0055 , 7.0428)
Price and wage setting					
Calvo dom. prices	θ_p	0.500	norm	0.6719	(0.6197 , 0.7600)
Calvo exp. prices	θ_x	0.080	gamma	0.2386	(0.1962 , 0.2814)
Calvo imp. prices	θ_M	0.750	norm	0.7309	(0.7046 , 0.7586)
Calvo wages	θ_w	0.380	norm	0.4519	(0.4302 , 0.4744)
Index. dom. prices	χ_p	0.750	gamma	0.7111	(0.6509 , 0.7688)
Index. imp. prices	χ_M	0.500	gamma	0.4519	(0.4187 , 0.4828)
Index. exp. prices	χ_x	0.350	gamma	0.3600	(0.3157 , 0.4191)
Index. wages	χ_w	0.920	beta	0.9529	(0.9176 , 0.9859)
Monetary policy					
Taylor rule (int. rates)	γ_R	0.960	beta	0.9544	(0.9389 , 0.9716)
Taylor rule (output gap)	γ_y	0.220	gamma	0.2233	(0.2109 , 0.2363)
Taylor rule (inflation)	γ_Π	1.150	gamma	1.1458	(1.1307 , 1.1600)
Fiscal policy					
Public consumption	ρ_g	0.750	beta	0.7758	(0.7107 , 0.8488)
Home bias					
Home bias in consump.	n_c	0.280	beta	0.3453	(0.2878 , 0.4080)
Home bias in invest.	n_i	0.120	beta	0.3297	(0.2913 , 0.3608)
Home bias in export	n_x	0.350	beta	0.4178	(0.3873 , 0.4602)
Growth rates					
Invest. spec. tech.	Λ_μ	1.000	norm	1.0000	(0.9998 , 1.0002)
General tech.	Λ_A	1.009	norm	1.0090	(1.0088 , 1.0092)
Population	Λ_L	1.000	norm	1.0000	(0.9998 , 1.0001)
ER appreciation	$\dot{ex}$	0.994	norm	0.9942	(0.9940 , 0.9944)
Openness tech.	α_O	1.0035	-	-	-
Export spec. tech.	α_X	1.0058	-	-	-

resulting markups with the micro data because there are no corresponding official series for the Czech economy. The only series available are evolutions of prices in the food branch (agricultural prices, food production prices and food consumer prices) but these tables show only final prices without any detailed specifications of firms' cost. Besides price markups, the wage markup is 16 percent. This value might indicate a relatively significant bargaining power and labour market stickiness.

The posterior values of Calvo price-setting parameters are 0.67 for domestic prices, 0.24 for export prices, and 0.73 for import prices with corresponding indexation parameters 0.71, 0.36 and 0.45. These posterior values indicate relatively flexible pricing policies of domestic firms with approximate duration of three quarters. The export sector estimation might signify a higher flexibility of exporting firms with duration slightly above one quarter. On the other hand, import sector seems to be relatively sticky with an approximate duration almost a year. The indexation of wages has the posterior 0.95 implying almost the full indexation.[21]

The share of domestic consumption goods in the total consumption basket (home bias in consumption) is approximately 35 percent. The similar share is for the investment sector. The home bias for the export sector is slightly higher with the posterior 0.42.

The inflation parameter in the monetary policy rule has its posterior slightly above 1.14 whereas the output parameter is more than five times lower with the value of 0.22. The posterior of lagged interest rate parameter equals to 0.95, and thus corresponds to the standard smooth profile of interest rates.

3.4. Time-varying Parameter Estimation

As was noted, DSGE models are usually supplemented with exogenous processes to get them closer to the data. Typical examples are sector technologies which capture important features of an economy. With this respect, these processes can be understood as time-varying parameters. We decide to incorporate four exogenous processes to the model:

- First, we aim to capture some aspects of high openness of the Czech economy, especially the fact that exports are very import intensive. Thus, we assume the trade openness technology which helps to work with reexport effects in the model consistent way.

$$\log(\dot{aO_t}) = \rho_{\dot{aO}} \log(\dot{aO_{t-1}}) + (1 - \rho_{\dot{aO}}) \log(\alpha_O) + \xi_t^{aO}. \tag{14}$$

- Second, since there is only a simple relation between government spending and consumption, we assume a government specific technology.

$$\log(\dot{aG_t}) = \rho_{\dot{aG}} \log(\dot{aG_{t-1}}) + \xi_t^{aG}. \tag{15}$$

[21] These results might be influenced by presence of indexation parameters in the price- and wage-setting equations. In the model, prices (and wages) are changing due to the reoptimizing and the indexation. Thus, estimation of these two parameters together might be sometimes difficult to interpret.

- Third, since the Czech headline CPI inflation is still influenced by regulated prices, we incorporate a regulated prices technology into the model.[22] This technology is only a proxy for the regulated prices goods sector.

$$\log(\dot{aR}_t) = \rho_{\dot{aR}} \log(\dot{aR}_{t-1}) + \xi_t^{aR}, \tag{16}$$

- And fourth, we added two time-varying wedges into the first order conditions of households. Namely, we insert a wedge between long-term growth of the economy and long-term real interest rate in the Euler equation and also a wedge between domestic interest rate, foreign interest rate and the exchange rate appreciation in the UIP. All these processes are according to (Andrle et al., 2009) and have forms

$$\log(\kappa_t^{euler}) = \rho_{euler} \log(\kappa_{t-1}^{euler}) + (1 - \rho_{euler}) \log(\kappa^{euler}) + \xi_t^{euler}, \tag{17}$$

$$\log(\kappa_t^{forex}) = \rho_{forex} \log(\kappa_{t-1}^{forex}) + (1 - \rho_{forex}) \log(\kappa^{forex}) + \xi_t^{forex}. \tag{18}$$

Filtered trajectories of exogenous processes serve as a tool for comparison the model behaviour with our intuition, and thus can possibly show some problems and model misspecification. Figure 2 shows evolution of filtered exogenous processes. Filtration of the government specific technology tells that a ratio of government spending goods with respect to value added is high. A relation between the regulated prices technology and observed regulated prices would be beneficial. The slump of trade openness technology during the crisis shows that there was a huge decrease of re-exports in the Czech economy during the first quarter 2009. It is very intuitive, because not only value added is traded.

For the time-varying parameter estimation, we need to choose candidate parameters for the drifting. Our first guess comes from the Bayesian estimation. Figure 3 shows parameters whose posterior distributions are considerably bimodal. Also, we choose the parameter for the import intensity of export as a candidate because we can expect that the openness of the Czech economy was changing during the analyzed period.[23]

As the first exercise, we carry out a time-varying parameter estimation allowing the drifting of parameters when these movements are unanticipated by agents in the model. For parameters $par = \theta_p, \chi_p, n_c, n_x, \rho_g, \epsilon_W$, we set:

$$par_t = 0 par_{t-1} + par_{ss} + \xi_t^{par}.$$

Hence, it is possible to use the first order approximation of the model and Kalman filter because applying the nonlinear filtration is not necessary (See Figure 4). First, we estimate the Calvo parameter θ_p and indexation parameter χ_p of the domestic intermediate producers. The results indicate the relative stability of these two parameters where their movements are mutually compensating. Second, we focus on

[22] However, we do not incorporate a direct link to the regulated prices observable.

[23] In fact, we did not resist the temptation and tried to estimate all parameters as time-varying.

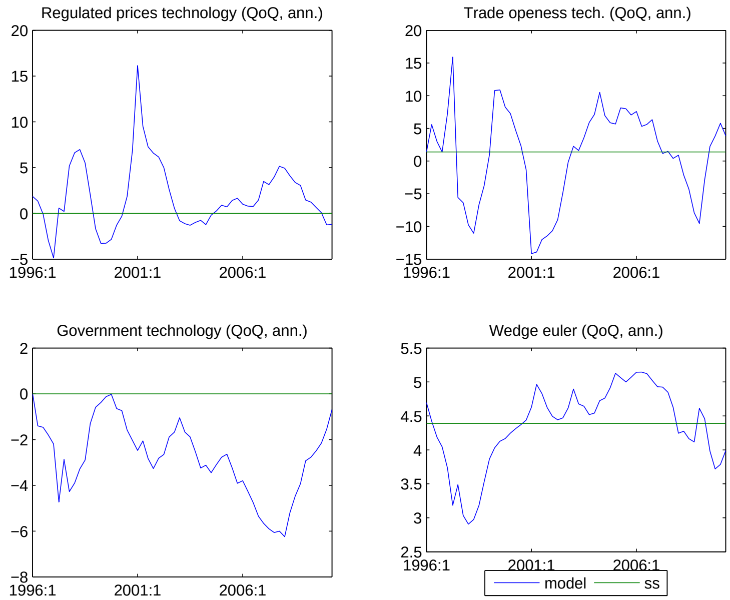

Figure 2: Filtered exogenous processes

the import shares of export and consumption (parameters n_x and n_c). The drifting of these parameters is more significant but without any trend. Another promising example is a price elasticity of exports parameter ϵ_W. Fiscal policy parameter ρ_g seems to be stable over time.

The next step is a time-varying parameter estimation allowing the drifting which is anticipated by model agents due to the higher order approximation. In such case, one needs to use a nonlinear filter because the model structure is also nonlinear. We use the Particle filter.[24] For obtaining the second order approximation, we employ the Dynare Toolbox.[25] The difference between first and second order approximations can be showed via following equations

$$y_t = y_s + Ayh_{t-1} + Bu_t$$

where ys is the steady state value of y and $yh_t = y_t - ys$.

The second order approximation is

$$y_t = ys + 0.5\Delta^2 + Ayh_{t-1} + Bu_t + 0.5C(yh_{t-1} \otimes yh_{t-1}) + 0.5D(u_t \otimes u_t) + E(yh_{t-1} \otimes u_t)$$

where ys is the steady state value of y, $yh_t = y_t - ys$, and Δ^2 is the shift effect of the variance of future shocks.

To check both models, we compare impulse responses of the first and second order approximations. The differences between the behaviour of these two approximations are relatively small when assuming one standard deviation shocks. In Figure 5, we show the comparison of five standard deviation total factor productivity shocks. The reactions are not so strong in the case of the second order approximation because risk stems into policy functions (a precautionary behaviour).

[24] See [1] and [14].

[25] See dynare manual [8]

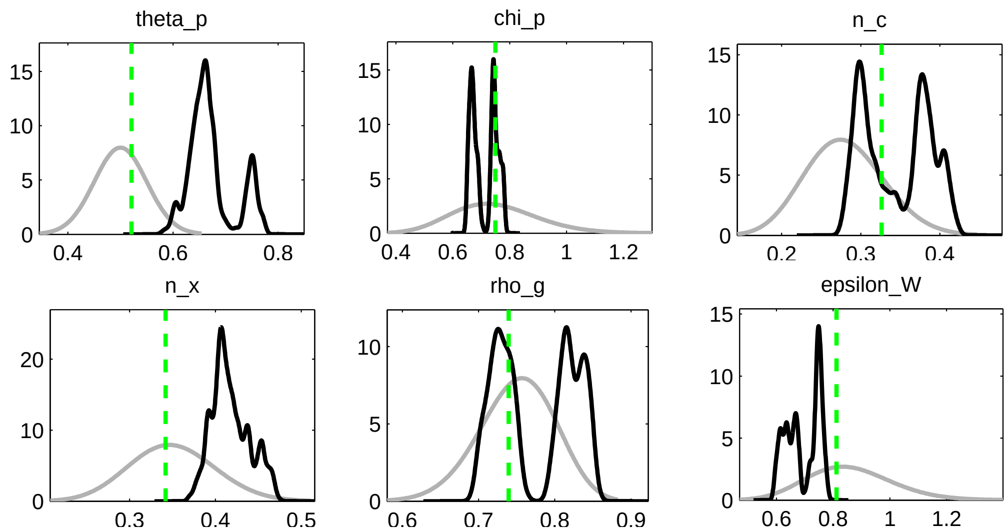

Figure 3: Bimodal posterior distributions parameters

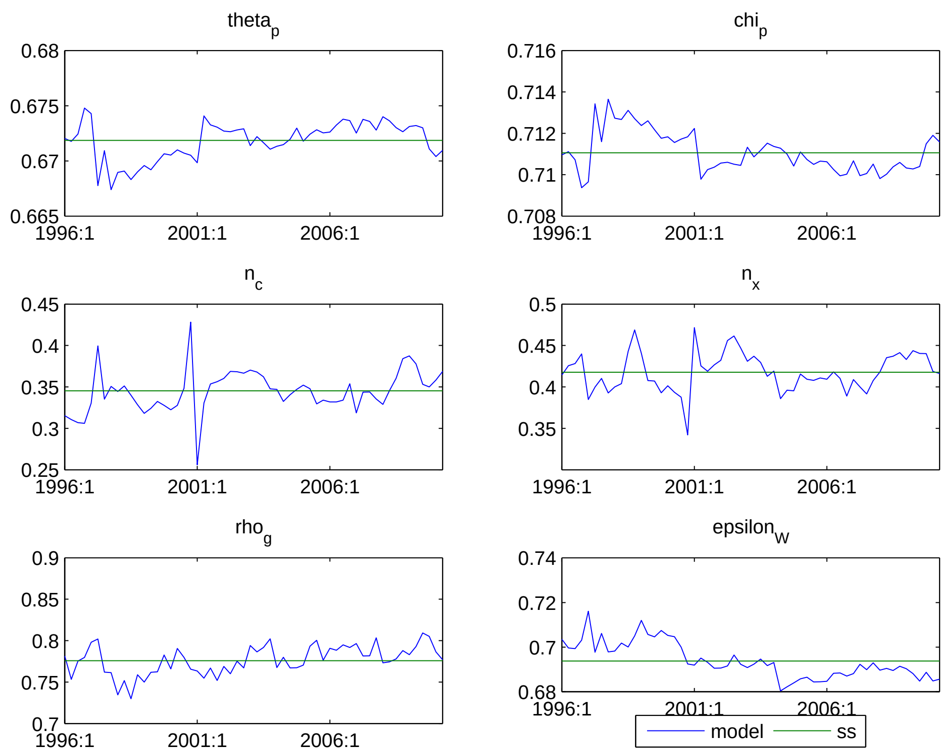

Figure 4: Time-varying parameter estimation - Kalman filter, 1st-order approx.

Another important difference between these two approximations is a shift of the steady state. Table 2 presents the shift effects when all time-varying parameters are incorporated. The neutral technological shock variance plays the most important role in explaning the shift.

As the second exercise, we focus on application of the standard particle filter (see [1]). Preliminary results of nonlinear filter on the second-order approximated model can be seen in the Technical Appendix. The trajectories on Figure 6 are computed

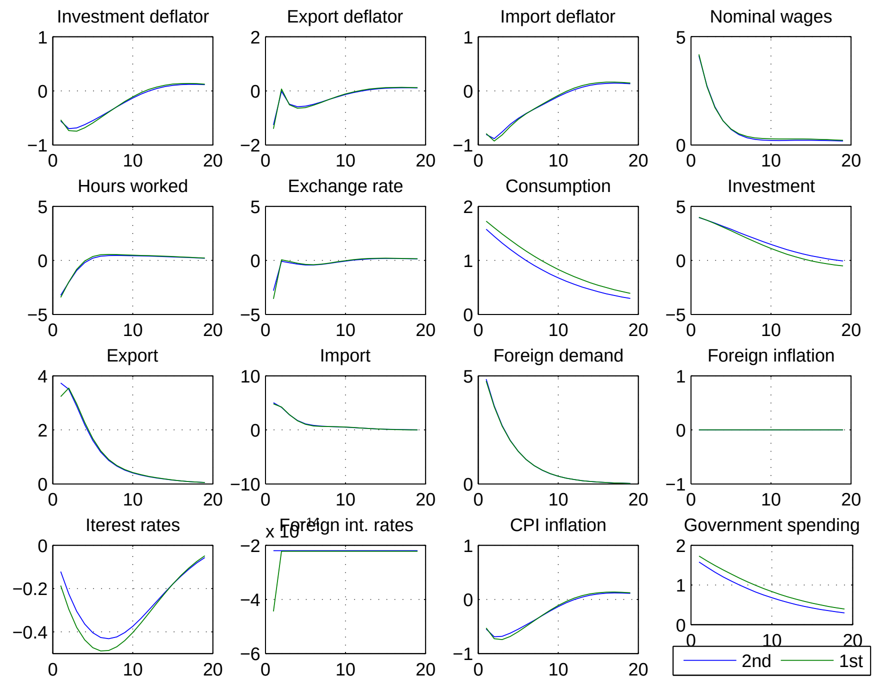

Figure 5: Technology shock impulse responses (in deviation from the steady state in p.p.)

Table 2: Δ^2 - shift effect (all estimated time-varying parameters)

Variable	**Shift (%)**
Investment deflator	0.10
Export deflator	0.98
Import deflator	0.21
Nominal wages	-0.05
Hours worked	-0.17
Exchange rate	-1.11
Consumption growth	-0.31
Investment growth	-0.44
Export growth	-1.47
Import growth	-1.64
Foreign demand growth	0.00
Foreign inflation growth	0.00
Interest rate	-0.00
Foreign interest rate	-0.00
CPI inflation	0.09
Government spending growth	-0.31

averages among 50 rerunned non-linear filtrations. The nonlinear estimation confirms significant movements in imports intensity parameters. Especially during 2002 and 2003 we identify a big increase of such parameters which indicate an increase of domestic component in producing consumption and export. Other parameters seem to be stable over time.

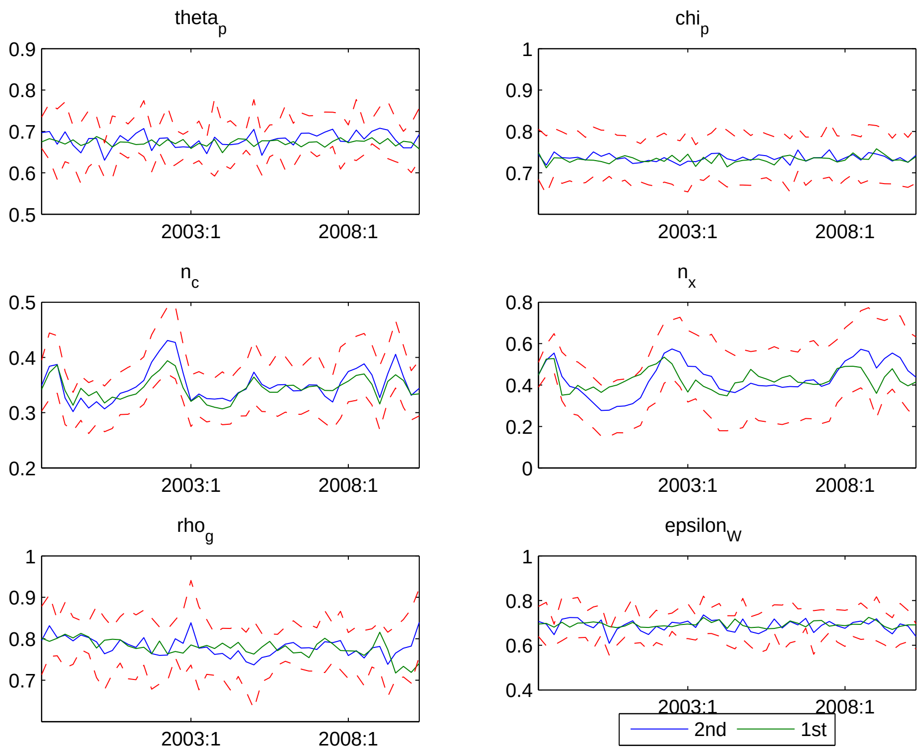

Figure 6: Nonlinear parameter filtration - Particle filter

To complete the analysis, we need a tool for finding out, which observables are responsible for the parameter drifting. Traditional tools like endogenous variables decompositions into observables are not additive because of a nonlinear world. Thus, we employ a simple correlation analysis which shows lead and lag correlations between the drifting and observed time series. From the Figure 7, we can see a strong correlation between parameters and exchange rate movements.

We find out the negative correlation between current exchange rate and current import intensity parameters. The higher domestic component of consumption and export implies lower import and thus positive net foreign assets and appreciated exchange rate. This finding is also in line with negative correlation between import and intensity parameters (mainly in the case of the second order approximation). Moreover, we find out a positive correlation between future exchange rate movements and intensity parameters. The depreciation anticipation, in this case, is a strong incentive for consumption and export goods producers to increase the domestic component. Such finding could not be revealed in the case of the first order approximation when agents do not anticipate parameter drifting.

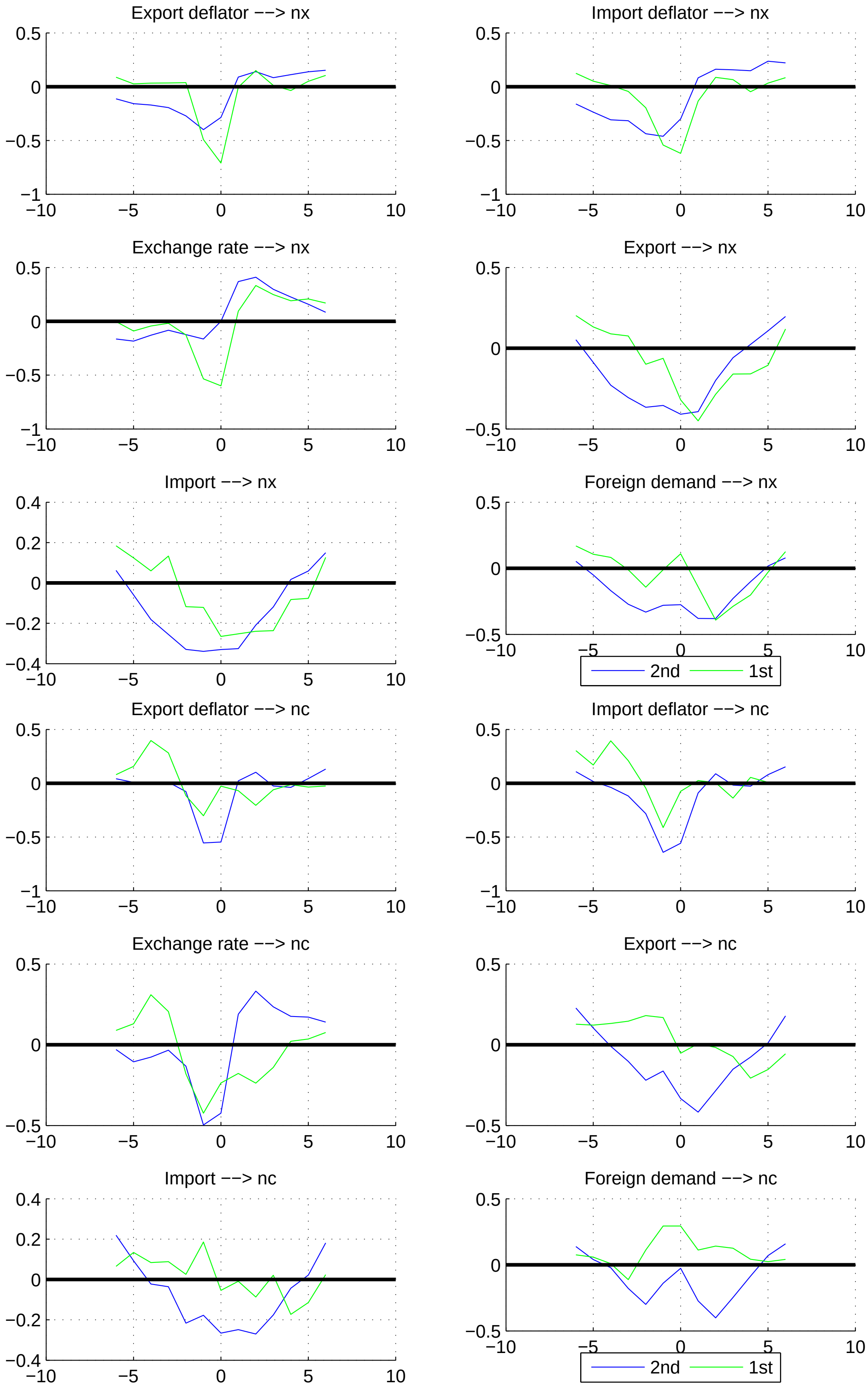

Figure 7: Time-varying parameter in time t and observables in time $t+j$ correlations

4. Conclusion

In the paper, we analyze a possible drifting of structural parameters in a relatively complex and on the Czech data estimated DSGE model. The model is based on two existing models. First, we use the model designed for the Spanish economy as our backbone framework. Second, we extend the original framework by implementing several important mechanisms tailor-made for the Czech economy. Our motivation is to combine a standard approach of building DSGE models with some original ideas to obtain this type of the model in order to study essential behavioural mechanisms. To verify the model properties, we estimate the model using Bayesian technique on the quarterly Czech and Eurozone data and discuss the results.

After the initial estimation and checks, we allow several parameters to drift in time. First, we impose some time-varying parameters through exogenous processes as openness technology, regulated prices technology or government specific technology into the model. Then we run the Kalman filter on the first-order approximated model with deep parameter drifting. The nonlinear filtration of the second-order approximated model is understood as the final step, when agents are aware of time-varying nature of the world.

We identify two drifting parameters, namely import share of export and import share of consumption. We find out that the strongest relation is between these parameters and significant exchange rate movements. We employ a simple correlation analysis among such parameters and observables to explain these findings, because standard tools as decomposition to observables are not additive in the case of non-linear world. If final producers anticipate significant exchange rate depreciation, they try to substitute import goods for domestic goods.

Although our economy has undergone many changes during the previous fifteen years, our estimation does not confirm that structural parameters have changed in the model. For example, the switch to the inflation targeting does not influence Taylor rule parameters as well as the Czech Republic entry to the EU did not influence any structural parameter. Incorporating exogenous processes like the trade openness technology, regulated prices technology or government specific technology significantly increases model ability to replicate data and thus there is no need to add time-varying parameters. On the other hand, interpretation of exogenous processes filtration might not have a direct structural linkage.

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5. Technical Appendix

We recapitulate original model equations in the first section. Note that their ordering and description is the same as in the original article [6]. This is our intention, because then we can clearly specify our modifications of the original equations and also some added equations. Further we present a derivation of a steady state for the modified model. Plots of priors and posteriors are given in the fourth section.

5.1. Original Model Equations and Variables

The full set of equilibrium conditions (and their derivation) can be found in [6]. We recapitulate them here. Note that all variables are stationarized, so we do not introduce a special notation to emphasize it. Steady states are denoted as variables without time index. We also omit an expectation term.

(1) The FOC of households with respect to consumption, bonds, foreign bonds, capital utility, capital stock, investment, real money balances, wages and per capita hours worked where d_t is an intertemporal preference shock, c_t is a per capita consumption, h is the habit persistence parameter, β is the discount parameter, z_t is the per capita long run growth, γ_t^L is the growth of population, λ_t is the Lagrangian multiplier associated with the budget constraint, τ_C is the tax rate of consumption, p_t^c is the price level of the consumption final good, p_t is the price level of the domestic final good

$$d_t \frac{1}{c_t - \frac{h}{z_t} c_{t-1}} - h\beta\gamma_{t+1}^L d_{t+1} \frac{1}{c_{t+1} z_{t+1} - hc_t} = \lambda_t (1+\tau_C) \left(\frac{p_t^c}{p_t}\right) \tag{19}$$

where R_t is the nominal interest rate, Π_t is the inflation of the domestic intermediate good

$$\lambda_t = \beta \frac{\lambda_{t+1}}{z_{t+1}} \frac{R_t}{\Pi_{t+1}} \tag{20}$$

where R_t^W is the foreign nominal interest rate, $ex_t b_t^W$ is an amount of foreign government bonds in the domestic currency, ex_t is the nominal exchange rate

$$\lambda_t = \beta \frac{\lambda_{t+1}}{z_{t+1}} \frac{R_t^W \Gamma(ex_t b_t^W, \xi_t^{b^W})}{\Pi_{t+1}} \frac{ex_{t+1}}{ex_t} \tag{21}$$

where $\Gamma(ex_t b_t^W, \xi_t^{b^W})$ is the premium associated with buying foreign bonds, Γ^{b^W} and $\xi_t^{b^W}$ are the parameter and shock associated with the premium

$$\Gamma(ex_t b_t^W, \xi_t^{b^W}) = e^{-\Gamma^{b^W}(ex_t b_t^W - ex b^W) + \xi_t^{b^W}}$$

r_t is the real rental price of capital, u_t is the intensity of use of capital, τ_k is the tax rate of capital income, μ_t is the investment-specific technology

$$r_t = \frac{\mu_t^{-1} \Phi'[u_t]}{1-\tau_k} \tag{22}$$

where $\mu_t^{-1}\Phi[u_t]$ is the physical cost of use of capital in resource terms

$$\Phi[u_t] = \phi_1(u_t - 1) + \frac{\phi_2}{2}(u_t - 1)^2$$

where q_t is the marginal Tobin's Q, δ is the depreciation rate of capital

$$q_t \gamma_{t+1}^L = \beta \gamma_{t+1}^L \frac{\lambda_{t+1}}{\lambda_t z_{t+1} \mu_{t+1}} \left[(1-\delta) q_{t+1} + r_{t+1} u_{t+1}(1-\tau_k) + \delta\tau_k - \Phi[u_{t+1}]\right] \tag{23}$$

where i_t is the per capita investments

$$\left(\frac{p_t^i}{p_t}\right) = q_t\left(1 - S\left[\gamma_t^L \frac{i_t}{i_{t-1}} z_t\right] - S'\left[\gamma_t^L \frac{i_t}{i_{t-1}} z_t\right]\gamma_t^L \frac{i_t}{i_{t-1}} z_t\right) + \beta q_{t+1}\frac{\lambda_{t+1}}{\lambda_t z_{t+1}} S'\left[\gamma_{t+1}^L \frac{i_{t+1}}{i_t} z_{t+1}\right]\left(\gamma_{t+1}^L \frac{i_{t+1}}{i_t} z_{t+1}\right)^2 \tag{24}$$

where $S\left[\gamma_t^L \frac{i_t}{i_{t-1}} z_t\right]$ is an adjustment cost function on the level of investment, Λ_i is the growth rate of investment

$$S\left[\gamma_t^L \frac{i_t}{i_{t-1}} z_t\right] = \frac{\kappa}{2}\left(\gamma_t^L \frac{i_t}{i_{t-1}} z_t - \Lambda_i\right)^2$$

where $\frac{m_t}{p_t}$ is the per capita real money balances

$$\frac{m_t}{p_t} = d_t v\left(\beta \frac{\lambda_{t+1}}{z_{t+1}} \frac{R_t - 1}{\Pi_{t+1}}\right)^{-1} \tag{25}$$

where η is the elasticity of substitution among different types of labour, τ_W is the tax rate of wage income, w_t^* is the optimal real wage in terms of the domestic final good, w_t is the overall real wage index, l_t^d is the per capita labour demand, θ_w is the Calvo parameter for wages, χ_w is the indexation parameter for wages

$$f_t = \frac{\eta - 1}{\eta}(1 - \tau_W)(w_t^*)^{1-\eta}\lambda_t w_t^\eta l_t^d + \beta\theta_w\gamma_{t+1}^L\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{1-\eta}\left(\frac{w_{t+1}^* z_{t+1}}{w_t^*}\right)^{\eta-1} f_{t+1} \tag{26}$$

where ψ is the labour supply coefficient, φ_t is the preference shock, $\Pi_t^{*w} = \frac{w_t^*}{w_t}$ is the optimal wage inflation, ϑ is the inverse of Frisch labour supply elasticity.

$$f_t = \psi d_t \varphi_t \left(\Pi_t^{*w}\right)^{-\eta(1+\vartheta)}\left(l_t^d\right)^{1+\vartheta} + \beta\theta_w\gamma_{t+1}^L\left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}}\right)^{-\eta(1+\vartheta)}\left(\frac{w_{t+1}^* z_{t+1}}{w_t^*}\right)^{\eta(1+\vartheta)} f_{t+1} \tag{27}$$

(2) The intermediate domestic firms can change prices where mc_t is a real marginal cost, y_t^d is the per capita aggregate demand of the domestic final good, θ_p is the Calvo parameter for the domestic good prices, χ is the indexation parameter for the domestic good prices, ε is the elasticity of substitution among different types of the domestic intermediate goods

$$g_t^1 = \lambda_t mc_t y_t^d + \beta\theta_p\gamma_{t+1}^L\left(\frac{\Pi_t^\chi}{\Pi_{t+1}}\right)^{-\varepsilon} g_{t+1}^1 \tag{28}$$

where Π_t^* is the optimal domestic intermediate goods prices inflation

$$g_t^2 = \lambda_t \Pi_t^* y_t^d + \beta\theta_p\gamma_{t+1}^L\left(\frac{\Pi_t^\chi}{\Pi_{t+1}}\right)^{1-\varepsilon}\left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{t+1}^2 \tag{29}$$

$$\varepsilon g_t^1 = (\varepsilon - 1) g_t^2 \tag{30}$$

where FOC of firms with respect to labour and capital inputs with k_t is a per capita capital, α is the labour share in production of the domestic intermediate goods

$$\frac{u_t k_{t-1}}{l_t^d} = \frac{\alpha}{1-\alpha} z_t \mu_t \frac{w_t}{r_t} \tag{31}$$

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^\alpha w_t^{1-\alpha} r_t^\alpha \tag{32}$$

(3) FOC of importing firms with respect to price where p_t^W is the foreign price of the foreign homogenous final good in the foreign currency, $ex_t p_t^W$ is its foreign price in the domestic currency, p_t^M is the price of goods of importing firms in the domestic currency, y_t^M is the final imported good, Π_t^M is the imported good inflation, θ_M is the Calvo for the import prices, χ_M is the indexation of the imported good, ε_M is the elasticity of substitution among different types of imported goods

$$g_t^{M_1} = \lambda_t \left(\frac{\frac{ex_t p_t^W}{p_t}}{\frac{p_t^M}{p_t}} \right) y_t^M + \beta\theta_M \gamma_{t+1}^L \left(\frac{(\Pi_t^M)^{\chi_M}}{\Pi_{t+1}^M} \right)^{-\varepsilon_M} g_{t+1}^{M_1} \tag{33}$$

where p_t^x is the price of the exported goods in the foreign currency, $ex_t p_t^x$ is the price of the exported goods in the domestic currency, y_t^x is the demand for the products of exporting firms, θ_x is the Calvo for the export prices, Π_t^W is the foreign homogenous final good prices inflation, Π_t^x is the export prices inflation in the foreign currency, χ_x is the indexation of the export prices, ε_x is the elasticity of substitution among different types of exported goods

$$g_t^{x_1} = \lambda_t \left(\frac{1}{\frac{ex_t p_t^x}{p_t}} \right) y_t^x + \beta\theta_x \gamma_{t+1}^L \left(\frac{(\Pi_t^W)^{\chi_x}}{\Pi_{t+1}^x} \right)^{-\varepsilon_x} g_{t+1}^{x_1} \tag{34}$$

where Π_t^{M*} is the optimal import prices inflation

$$g_t^{M_2} = \lambda_t \Pi_t^{M*} y_t^M + \beta\theta_M \gamma_{t+1}^L \left(\frac{(\Pi_t^M)^{\chi_M}}{\Pi_{t+1}^M} \right)^{1-\varepsilon_M} \left(\frac{\Pi_t^{M*}}{\Pi_{t+1}^{M*}} \right) g_{t+1}^{M_2} \tag{35}$$

where Π_t^{x*} is the optimal export prices inflation in the foreign currency

$$g_t^{x_2} = \lambda_t \Pi_t^{x*} y_t^x + \beta\theta_x \gamma_{t+1}^L \left(\frac{(\Pi_t^W)^{\chi_x}}{\Pi_{t+1}^x} \right)^{1-\varepsilon_x} \left(\frac{\Pi_t^{x*}}{\Pi_{t+1}^{x*}} \right) g_{t+1}^{x_2} \tag{36}$$

$$\varepsilon_M g_t^{M_1} = (\varepsilon_M - 1) g_t^{M_2} \tag{37}$$

$$\varepsilon_x g_t^{x_1} = (\varepsilon_x - 1) g_t^{x_2} \tag{38}$$

(4) Wages and prices evolve according to

$$1 = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t} \right)^{1-\varepsilon} + (1-\theta_p)(\Pi_t^*)^{1-\varepsilon} \tag{39}$$

$$1 = \theta_w \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t} \right)^{1-\eta} \left(\frac{w_{t-1}}{w_t} \frac{1}{z_t} \right)^{1-\eta} + (1-\theta_w) \left(\frac{w_t^*}{w_t} \right)^{1-\eta} \tag{40}$$

$$1 = \theta_M \left(\frac{(\Pi_{t-1}^M)^{\chi_M}}{\Pi_t^M} \right)^{1-\varepsilon_M} + (1-\theta_M)(\Pi_t^{M*})^{1-\varepsilon_M} \tag{41}$$

$$1 = \theta_x \left(\frac{(\Pi_{t-1}^W)^{\chi_x}}{\Pi_t^x} \right)^{1-\varepsilon_x} + (1-\theta_x)(\Pi_t^{x*})^{1-\varepsilon_x} \tag{42}$$

(5) Monetary and fiscal policy - Taylor rule, government's budget constraint and fiscal rule where R is a steady state of nominal interest rate, Π is the inflation target, γ_R Taylor rule parameter (rates), γ_Π Taylor rule parameter (inflation), γ_y Taylor rule parameter (output), Λ_{y^d} is the growth rate of output, Λ_L is the growth rate of population, ξ_t^m is the monetary policy shock,

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left(\left(\frac{\Pi_t}{\Pi} \right)^{\gamma_\Pi} \left(\frac{\gamma_t^L \frac{y_t^d}{y_{t-1}^d} z_t}{e^{\Lambda_L + \Lambda_{y^d}}} \right)^{\gamma_y} \right)^{1-\gamma_R} \exp(\xi_t^m) \tag{43}$$

where b_t is the level of outstanding debt w.r.t. nominal output, g_t is the per capita level of real government consumption,

$$b_t = \frac{g_t}{y_t^d} + \frac{T_t}{y_t^d} + \frac{m_{t-1}}{p_{t-1}} \frac{1}{\gamma_t^L y_t^d z_t \Pi_t} + R_{t-1} b_{t-1} \frac{y_{t-1}^d}{\gamma_t^L y_t^d z_t \Pi_t} - (r_t u_t - \delta)\tau_K \frac{k_{t-1}}{y_t^d z_t \mu_t} - \tau_W \frac{w_t l_t^d}{y_t^d} - \tau_C \frac{p_t^c}{p_t} \frac{c_t}{y_t^d} - \frac{m_t}{p_t} \frac{1}{y_t^d} \tag{44}$$

where T_t are the per capita lump-sum taxes,

$$\frac{T_t}{y_t^d} = T_0 - T_1(b_t - b) \tag{45}$$

(6) Net foreign assets evolve where y_t^W is the world demand, M_t is the per capita real imports, Δex_t is the nominal exchange rate depreciation

$$\begin{aligned} ex_t b_t^W = {} & R_{t-1}^W \Gamma(\Delta ex_t ex_{t-1} b_{t-1}^W, \xi_{t-1}^{b^W}) \Delta ex_t \frac{y_{t-1}^d}{\gamma_t^L y_t^d z_t \Pi_t} ex_{t-1} b_{t-1}^W + \\ & + \left(\frac{ex_t p_t^W}{p_t}\right)^{\epsilon_W} \left(\frac{ex_t p_t^x}{p_t}\right)^{1-\epsilon_W} \left(\frac{y_t^W}{y_t^d}\right) - \left(\frac{ex_t p_t^W}{p_t}\right) \left(\frac{M_t}{y_t^d}\right) \end{aligned} \tag{46}$$

(7) Aggregate imports and exports evolve where v_t^M is the import prices dispersion, v_t^x is the export prices dispersion, x_t is the real per capita exports, ϵ_c is the elasticity of substitution among different types of consumption goods, ϵ_i is the elasticity of substitution among different types of investment goods, ϵ_W is the elasticity of substitution among different types of world trade goods, n^c is a home bias in the aggregation in consumption, n^i is a home bias in the aggregation in investment

$$M_t = v_t^M \left[\Omega_{t+1}^c (1-n^c) \left[\frac{\frac{p_t^M}{p_t}}{\frac{p_t^c}{p_t}} \right]^{-\epsilon_c} c_t + \Omega_{t+1}^i (1-n^i) \left[\frac{\frac{p_t^M}{p_t}}{\frac{p_t^i}{p_t}} \right]^{-\epsilon_i} i_t \right] \tag{47}$$

$$x_t = v_t^x \left[\frac{\left(\frac{ex_t p_t^x}{p_t}\right)}{\left(\frac{ex_t p_t^W}{p_t}\right)} \right]^{-\epsilon_W} y_t^W \tag{48}$$

where for $s = c, i$ Π_t^c is the consumption good inflation, Π_t^i is the investment good inflation, c_t^M is the imported consumption, i_t^M is the imported investment

$$\Omega_{t+1}^s = \frac{\left[1 - \beta(1-n^s)^{\frac{1}{\epsilon_s}} \frac{\gamma_{t+1}^L \lambda_{t+1}}{\lambda_t z_{t+1}} \left[\frac{\left(\frac{p_t^s}{p_t}\right)}{\left(\frac{p_t^M}{p_t}\right)} \right] \Pi_{t+1}^s \left(\frac{s_{t+1}}{s_t^M (1-\Gamma_{t+1}^s)} \right)^{\frac{1}{\epsilon_s}} \Gamma_{t+1}^{s\prime} \frac{(\Delta s_{t+1}^M)^2}{\Delta s_{t+1}} \right]^{-\epsilon_s}}{(1-\Gamma_t^s) \left[1 - \Gamma_t^s - \Gamma_t^{s\prime} \left(\frac{\Delta s_t^M}{\Delta s_t} \right) \right]^{-\epsilon_s}} \tag{49}$$

where Γ_t^s are adjustment costs

$$\Gamma_t^s = \frac{\Gamma^s}{2} \left(\frac{\left(\frac{s_t^M}{s_t}\right)}{\left(\frac{s_{t-1}^M}{s_{t-1}}\right)} - 1 \right)^2$$

$$v_t^M = \theta_M \left(\frac{(\Pi_{t-1}^M)^{\chi_M}}{\Pi_t^M} \right)^{-\varepsilon_M} v_{t-1}^M + (1-\theta_M)(\Pi_t^{M*})^{-\varepsilon_M} \tag{50}$$

$$v_t^x = \theta_x \left(\frac{(\Pi_{t-1}^W)^{\chi_x}}{\Pi_t^x} \right)^{-\varepsilon_x} v_{t-1}^x + (1-\theta_x)(\Pi_t^{x*})^{-\varepsilon_x} \tag{51}$$

The production of importing and exporting firms

$$y_t^M = c_t^M + i_t^M \tag{52}$$

$$y_t^x = \left[\frac{\left(\frac{ex_t p_t^x}{p_t}\right)}{\left(\frac{ex_t p_t^W}{p_t}\right)} \right]^{-\epsilon_W} y_t^W \tag{53}$$

Demands for consumption, investments imports where c_t^d is the domestic consumption, i_t^d is the domestic investment

$$\frac{c_t^M}{c_t^d} = \frac{\Omega_{t+1}^c(1-n^c)}{n^c}\left(\frac{p_t^M}{p_t}\right)^{-\epsilon_c} \tag{54}$$

$$\frac{i_t^M}{i_t^d} = \frac{\Omega_{t+1}^i(1-n^i)}{n^i}\left(\frac{p_t^M}{p_t}\right)^{-\epsilon_i} \tag{55}$$

(8) Market clearing condition - aggregate demand and supply where ϕ is the parameter associated with the fixed cost production, v_t^p is the dispersion of the domestic intermediate goods prices, A_t is the neutral technology growth

$$y_t^d = n^c\left(\frac{p_t^c}{p_t}\right)^{\epsilon_c} c_t + n^i\left(\frac{p_t^i}{p_t}\right)^{\epsilon_i} i_t + g_t + \frac{1}{z_t\mu_t}\Phi[u_t]k_{t-1} + x_t \tag{56}$$

$$y_t^d = \frac{A_t\frac{1}{z_t}(u_t k_{t-1})^{\alpha}(l_t^d)^{1-\alpha} - \phi}{v_t^p} \tag{57}$$

where market clearing condition - labour market where v_t^w is the dispersion of wages, l_t is the per capita hours worked

$$l_t = v_t^w l_t^d \tag{58}$$

$$v_t^p = \theta_p\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon} v_{t-1}^p + (1-\theta_p)(\Pi_t^*)^{-\varepsilon} \tag{59}$$

$$v_t^w = \theta_w\left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t}\right)^{-\eta}\left(\frac{w_{t-1}}{w_t}\right)^{-\eta}\left(\frac{1}{z_t}\right)^{-\eta} v_{t-1}^w + (1-\theta_w)\left(\frac{w_t^*}{w_t}\right)^{-\eta} \tag{60}$$

and capital accumulation

$$\gamma_t^L k_t z_t \mu_t = (1-\delta)k_{t-1} + z_t\mu_t\left(1 - S\left[\gamma_t^L\frac{i_t}{i_{t-1}}z_t\right]\right) i_t \tag{61}$$

Aggregate consumption and investment evolves

$$c_t = \left[(n^c)^{\frac{1}{\epsilon_c}}(c_t^d)^{\frac{\epsilon_c-1}{\epsilon_c}} + (1-n^c)^{\frac{1}{\epsilon_c}}(c_t^M(1-\Gamma_t^c))^{\frac{\epsilon_c-1}{\epsilon_c}}\right]^{\frac{\epsilon_c}{\epsilon_c-1}} \tag{62}$$

$$i_t = \left[(n^i)^{\frac{1}{\epsilon_i}}(i_t^d)^{\frac{\epsilon_i-1}{\epsilon_i}} + (1-n^i)^{\frac{1}{\epsilon_i}}(i_t^M(1-\Gamma_t^i))^{\frac{\epsilon_i-1}{\epsilon_i}}\right]^{\frac{\epsilon_i}{\epsilon_i-1}} \tag{63}$$

(9) Relative consumption and investment prices evolve

$$\frac{p_t^c}{p_t} = \left[n^c + \Omega_t^c(1-n^c)\left(\frac{p_t^M}{p_t}\right)^{1-\epsilon_c}\right]^{\frac{1}{1-\epsilon_c}} \tag{64}$$

$$\frac{p_t^i}{p_t} = \left[n^i + \Omega_t^i(1-n^i)\left(\frac{p_t^M}{p_t}\right)^{1-\epsilon_i}\right]^{\frac{1}{1-\epsilon_i}} \tag{65}$$

(10) Identities for inflations rates

$$\Pi_t^c = \frac{\left(\frac{p_t^c}{p_t}\right)}{\left(\frac{p_{t-1}^c}{p_{t-1}}\right)}\Pi_t \tag{66}$$

$$\Pi_t^i = \frac{\left(\frac{p_t^i}{p_t}\right)}{\left(\frac{p_{t-1}^i}{p_{t-1}}\right)}\Pi_t \tag{67}$$

$$\Pi_t^M = \frac{\left(\frac{p_t^M}{p_t}\right)}{\left(\frac{p_{t-1}^M}{p_{t-1}}\right)} \Pi_t \tag{68}$$

$$\Pi_t^x = \frac{\left(\frac{ex_t p_t^x}{p_t}\right)}{\left(\frac{ex_{t-1} p_{t-1}^x}{p_{t-1}}\right)} \frac{\Pi_t}{\Delta ex_t} \tag{69}$$

$$\Pi_t^W = \frac{\left(\frac{ex_t p_t^W}{p_t}\right)}{\left(\frac{ex_{t-1} p_{t-1}^W}{p_{t-1}}\right)} \frac{\Pi_t}{\Delta ex_t} \tag{70}$$

(10) Relation among technologies and AR processes where Λ_μ is the growth rate of investment-specific technology, Λ_A is the growth rate of neutral technology

$$z_t = A_t^{\frac{1}{1-\alpha}} \mu_t^{\frac{\alpha}{1-\alpha}} \tag{71}$$

$$\mu_t - e^{\Lambda_\mu + \xi_t^\mu} = 0 \tag{72}$$

$$\log d_t - \rho_d \log d_{t-1} - \xi_t^d = 0 \tag{73}$$

$$\log \varphi_t - \rho_\varphi \log \varphi_{t-1} - \xi_t^\varphi = 0 \tag{74}$$

$$A_t - e^{\Lambda_A + \xi_t^A} = 0 \tag{75}$$

$$\gamma_t^L - e^{\Lambda_L + \xi_t^L} = 0 \tag{76}$$

and exogenous processes

$$\log(R_t^W) = \rho_{R^W} \log(R_{t-1}^W) + (1 - \rho_{R_W}) \log(R^W) + \xi_t^{R^W} \tag{77}$$

$$\log(y_t^W) = \rho_{y^W} \log(y_{t-1}^W) + (1 - \rho_{y^W}) \log(y^W) + \xi_t^{y^W} \tag{78}$$

$$\log(\Pi_t^W) = \rho_{\Pi^W} \log(\Pi_{t-1}^W) + (1 - \rho_{\Pi^W}) \log(\Pi^W) + \xi_t^{\pi^W} \tag{79}$$

$$\log(g_t) = \rho_g \log(g_{t-1}) + (1 - \rho_g) \log(g) + \xi_t^g \tag{80}$$

5.2. Modified and Added Model Equations

(1) FOC of households with respect to consumption is modified by adding $\dot{aR_t}$ regulated prices proxy

$$d_t \frac{1}{c_t - \frac{h}{z_t} c_{t-1} \dot{aR_t}} - h\beta\gamma_{t+1}^L d_{t+1} \frac{1}{c_{t+1} z_{t+1} - h c_t \dot{aR}_{t+1}} = \lambda_t (1+\tau_C) \left(\frac{p_t^c}{p_t} \right) \tag{81}$$

FOC of households with respect to domestic bonds incorporates κ_t^{euler} time-varying parameter

$$\lambda_t = \beta \frac{\lambda_{t+1}}{z_{t+1}} \frac{R_t}{\Pi_{t+1}} \kappa_t^{euler} \tag{82}$$

FOC of households with respect to foreign bonds incorporates $prem_t$ premium, κ_t^{forex} and κ_t^{euler} time-varying parameters and uip shock ξ_t^{uip}

$$\lambda_t = \beta \frac{\lambda_{t+1}}{z_{t+1}} \frac{R_t^W prem_t e^{\xi_t^{uip}} \kappa_t^{forex} \kappa_t^{euler}}{\Pi_{t+1}} \frac{ex_{t+1}}{ex_t} \tag{83}$$

where $prem_t$ has its own equation similar to the original

$$\log(prem_t) = \rho^{prem} \log(prem_{t-1}) + \epsilon_t^{prem} - \rho^{exb^w} ex_t b_t^W \tag{84}$$

We incorporate the identity for optimal wage inflation

$$\Pi_t^{*w} = \frac{w_t^*}{w_t} \tag{85}$$

(2) FOC of firms is not changed

(3) FOC of exporting firms has the $\tilde{\Pi}_t^x$ export prices inflation in the foreign currency, q_t^x is the cost of exporting firm, p_t^x is the price of the exporting firm,

$$g_t^{x_1} = \lambda_t \left(\frac{\frac{q_t^x}{p_t}}{\frac{p_t^x}{p_t}} \right) x_t^d + \beta\theta_x \gamma_{t+1}^L \left(\frac{(\Pi_t^W)^{\chi_x}}{\tilde{\Pi}_{t+1}^x} \right)^{-\varepsilon_x} g_{t+1}^{x_1} \tag{86}$$

where $\tilde{\Pi}_t^{x*}$ is the optimal (star) export prices inflation in the foreign currency (tilde)

$$g_t^{x_2} = \lambda_t \tilde{\Pi}_t^{x*} x_t^d + \beta\theta_x \gamma_{t+1}^L \left(\frac{(\Pi_t^W)^{\chi_x}}{\tilde{\Pi}_{t+1}^x} \right)^{1-\varepsilon_x} \left(\frac{\tilde{\Pi}_t^{x*}}{\tilde{\Pi}_{t+1}^{x*}} \right) g_{t+1}^{x_2} \tag{87}$$

(4) Export prices dispersion is modified

$$1 = \theta_x \left(\frac{(\Pi_{t-1}^W)^{\chi_x}}{\tilde{\Pi}_t^x} \right)^{1-\varepsilon_x} + (1-\theta_x)(\tilde{\Pi}_t^{x*})^{1-\varepsilon_x} \tag{88}$$

(5) Taylor rule and fiscal rule are modified,

we adjust Taylor rule to capture inflation target in the Czech Republic and remove the bug[26] $e^{\Lambda_L + \Lambda_{y^d}}$ to $\Lambda_L + \Lambda_{y^d}$ where $\Pi4^c$ is year-on-year CPI inflation and *target* is the year-on-year inflation target

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left(\left(\frac{\Pi4_{t+4}^c}{target_{t+4}} \right)^{\gamma_\Pi} \left(\frac{\gamma_t^L \frac{y_t^d}{y_{t-1}^d} z_t}{\Lambda_L \Lambda_{y^d}} \right)^{\gamma_y} \right)^{1-\gamma_R} \exp(\xi_t^m) \tag{89}$$

and fiscal rule instead of (44) and (45)

$$g_t = \rho_g c_t + \xi_t^g \tag{90}$$

[26] It depends whether parameters Λ are in logs or not. We express them in logs.

(6) Net foreign assets evolution looses premium

$$ex_t b_t^W = R_{t-1}^W \Delta ex_t \frac{y_{t-1}^d}{\gamma_t^L y_t^d z_t \Pi_t} ex_{t-1} b_{t-1}^W + \tag{91}$$
$$+ \left(\frac{ex_t p_t^W}{p_t} \right)^{\epsilon_W} \left(\frac{q_t^x}{p_t} \right)^{1-\epsilon_W} \left(\frac{y_t^W}{y_t^d} \right) - \left(\frac{ex_t p_t^W}{p_t} \right) \left(\frac{M_t}{y_t^d} \right)$$

(7) Aggregate imports must contain a component for exports

$$M_t = v_t^M \left[\Omega_{t+1}^c (1-n^c) \left[\frac{\frac{p_t^M}{p_t}}{\frac{p_t^c}{p_t}} \right]^{-\epsilon_c} c_t + \Omega_{t+1}^i (1-n^i) \left[\frac{\frac{p_t^M}{p_t}}{\frac{p_t^i}{p_t}} \right]^{-\epsilon_i} i_t + \Omega_{t+1}^x (1-n^x) \left[\frac{\frac{p_t^M}{p_t}}{\frac{ex_t p_t^x}{p_t}} \right]^{-\epsilon_x} x_t \right] \tag{92}$$

where we omit adjustment costs

$$\Gamma_t^s = 0 \qquad \Gamma_t^{s'} = 0 \tag{93}$$

$$v_t^x = \theta_x \left(\frac{(\Pi_{t-1}^W)^{\chi_x}}{\tilde{\Pi}_t^x} \right)^{-\varepsilon_x} v_{t-1}^x + (1-\theta_x)(\tilde{\Pi}_t^{x*})^{-\varepsilon_x} \tag{94}$$

Demand for imports must contain a component of exports x_t^M

$$y_t^M = c_t^M + i_t^M + x_t^M \tag{95}$$

where demand for imported exports where x_t^d are the domestic exports

$$\frac{x_t^M}{x_t^d} = \frac{\Omega_{t+1}^x (1-n^x)}{n^x} \left(\frac{p_t^M}{p_t} \right)^{-\epsilon_x} \tag{96}$$

(8) Market clearing condition has aggregate exports x_t

$$y_t^d = n^c \left(\frac{p_t^c}{p_t} \right)^{\epsilon_c} c_t + n^i \left(\frac{p_t^i}{p_t} \right)^{\epsilon_i} i_t + g_t + \frac{1}{z_t \mu_t} \Phi[u_t] k_{t-1} + n^x \left(\frac{q_t^x}{p_t} \right)^{\epsilon_x} x_t \tag{97}$$

$$x_t = \left[(n^x)^{\frac{1}{\epsilon_x}} (x_t^d)^{\frac{\epsilon_x - 1}{\epsilon_x}} + (1-n^x)^{\frac{1}{\epsilon_x}} (x_t^M (1 - \Gamma_t^x))^{\frac{\epsilon_x - 1}{\epsilon_x}} \right]^{\frac{\epsilon_x}{\epsilon_x - 1}} \tag{98}$$

(9) Relative export costs evolve

$$\frac{q_t^x}{p_t} = \left[n^x + \Omega_t^x (1-n^x) \left(\frac{p_t^M}{p_t} \right)^{1-\epsilon_x} \right]^{\frac{1}{1-\epsilon_x}} \tag{99}$$

(10) Identities for inflations rates

$$\Pi_t^w = \frac{w_t}{w_{t-1}} z_t \Pi_t \tag{100}$$

$$\Pi_t^c = \frac{\left(\frac{p_t^c}{p_t} \right)}{\left(\frac{p_{t-1}^c}{p_{t-1}} \right)} \Pi_t a\dot{R}_t \tag{101}$$

$$\Pi_t^i = \frac{\left(\frac{p_t^i}{p_t} \right)}{\left(\frac{p_{t-1}^i}{p_{t-1}} \right)} \Pi_t \tag{102}$$

$$\Pi_t^M a\dot{X}_t = \frac{\left(\frac{p_t^M}{p_t} \right)}{\left(\frac{p_{t-1}^M}{p_{t-1}} \right)} \Pi_t \tag{103}$$

$$\Pi_t^x \dot{aX}_t = \frac{\left(\frac{p_t^x}{p_t}\right)}{\left(\frac{p_{t-1}^x}{p_{t-1}}\right)} \Pi_t \quad (104)$$

$$\Pi_t^x = \Delta ex_t \tilde{\Pi}_t^x \quad (105)$$

$$\Pi_t^{W*} = \Delta ex_t \Pi_t^W; \quad (106)$$

$$\Pi_t^{W*} \dot{aX}_t = \frac{\frac{ex_t p_t^W}{p_t}}{\frac{ex_{t-1} p_{t-1}^W}{p_{t-1}}} \Pi_t \quad (107)$$

$$\dot{y_t^d} = \frac{y_t^d}{y_{t-1}^d} z_t \gamma_L \quad (108)$$

$$\dot{c}_t \dot{aR}_t = \frac{c_t}{c_{t-1}} z_t \gamma_L \quad (109)$$

$$\dot{i}_t = \frac{i_t}{i_{t-1}} z_t \gamma_L; \quad (110)$$

$$\dot{x}_t = \frac{x_t}{x_{t-1}} z_t \gamma_L \dot{aO}_t \dot{aX}_t \quad (111)$$

$$\dot{m}_t = \frac{M_t}{M_{t-1}} z_t \gamma_L \dot{aO}_t \dot{aX}_t \quad (112)$$

$$\dot{g}_t = \frac{g_t}{g_{t-1}} z_t \gamma_L \dot{aG}_t \quad (113)$$

$$\dot{y^W}_t = \frac{y_t^W}{y_{t-1}^W} z_t \gamma_L \dot{aO}_t \dot{aX}_t \quad (114)$$

$$\Pi 4_t^c = \Pi_t^c \Pi_{t-1}^c \Pi_{t-2}^c \Pi_{t-3}^c \quad (115)$$

(11) Relation among technologies and AR processes

$$\log(\mu_t) = \rho_\mu \log(\mu_{t-1}) + (1 - \rho_\mu) \log(\Lambda^\mu) + \xi_t^\mu \quad (116)$$

$$\log(A_t) = \rho_A \log(A_{t-1}) + (1 - \rho_A) \log(\Lambda^A) + \xi_t^A \quad (117)$$

and time-varying parameters

$$\log(\dot{aR}_t) = \rho_{\dot{aR}} \log(\dot{aR}_{t-1}) + \xi_t^{aR} \quad (118)$$

$$\log(\dot{aO}_t) = \rho_{\dot{aO}} \log(\dot{aO}_{t-1}) + (1 - \rho_{\dot{aO}}) \log(\alpha_O) + \xi_t^{aO} \quad (119)$$

$$\log(\dot{aX}_t) = \rho_{\dot{aX}} \log(\dot{aX}_{t-1}) + (1 - \rho_{\dot{aX}}) \log(\alpha_X) + \xi_t^{aX} \quad (120)$$

$$\log(\dot{aG}_t) = \rho_{\dot{aG}} \log(\dot{aG}_{t-1}) + \xi_t^{aG} \quad (121)$$

$$\log(\kappa_t^{euler}) = \rho_{euler} \log(\kappa_{t-1}^{euler}) + (1 - \rho_{euler}) \log(\kappa^{euler}) + \xi_t^{euler} \quad (122)$$

$$\log(\kappa_t^{forex}) = \rho_{forex} \log(\kappa_{t-1}^{forex}) + (1 - \rho_{forex}) \log(\kappa^{forex}) + \xi_t^{forex} \quad (123)$$

$$\log(target_t) = \rho_{target}\log(target_{t-1}) + (1-\rho_{target})\log(\Pi^4) + \xi_t^{target} \tag{124}$$

and time-varying deep parameters

$$\theta_{p,t} = \rho_{\theta_p}\theta_{p,t-1} + (1-\rho_{\theta_p})\theta_{p,t} + \xi_t^{\theta_p} \tag{125}$$

$$\chi_t = \rho_\chi \chi_{t-1} + (1-\rho_\chi)\chi + \xi_t^\chi \tag{126}$$

(12) Connection to observables are

$$100\log(\Pi_t^i) = mes_t^{PI} - mes_{t-1}^{PI} \qquad obs_t^{PI} = mes_t^{PI} + \omega_t^{PI} \tag{127}$$

$$100\log(\Pi_t^x) = mes_t^{PX} - mes_{t-1}^{PX} \qquad obs_t^{PX} = mes_t^{PX} + \omega_t^{PX} \tag{128}$$

$$100\log(\Pi_t^M) = mes_t^{PM} - mes_{t-1}^{PM} \qquad obs_t^{PM} = mes_t^{PM} + \omega_t^{PM} \tag{129}$$

$$100\log(\Pi_t^w) = mes_t^{W} - mes_{t-1}^{W} \qquad obs_t^{W} = mes_t^{W} + \omega_t^{W} \tag{130}$$

$$100\log\left(\frac{l_t^d}{l_{t-1}^d}\right) = mes_t^{L} - mes_{t-1}^{L} + 100\log(\gamma_t^L) \qquad obs_t^{L} = mes_t^{L} + \omega_t^{L} \tag{131}$$

$$100\log(\Delta ex_t) = mes_t^{EX} - mes_{t-1}^{EX} \qquad obs_t^{EX} = mes_t^{EX} + \omega_t^{EX} \tag{132}$$

$$100\log(\dot{c}_t) = mes_t^{C} - mes_{t-1}^{C} \qquad obs_t^{C} = mes_t^{C} + \omega_t^{C} \tag{133}$$

$$100\log(\dot{i}_t) = mes_t^{I} - mes_{t-1}^{I} \qquad obs_t^{I} = mes_t^{I} + \omega_t^{I} \tag{134}$$

$$100\log\left(\frac{x}{x_{t-1}}\right) = mes_t^{X} - mes_{t-1}^{X} - 100\log(z_t a\dot{X}_t a\dot{O}_t) \qquad obs_t^{X} = mes_t^{X} + \omega_t^{X} \tag{135}$$

$$100\log\left(\frac{M_t}{M_{t-1}}\right) = mes_t^{M} - mes_{t-1}^{M} - 100\log(z_t a\dot{X}_t a\dot{O}_t) \qquad obs_t^{M} = mes_t^{M} + \omega_t^{M} \tag{136}$$

$$100\log(y\dot{^W}_t) = mes_t^{YW} - mes_{t-1}^{YW} \qquad obs_t^{YW} = mes_t^{YW} + \omega_t^{YW} \tag{137}$$

$$100\log(\pi_t^W) = mes_t^{PIW} - mes_{t-1}^{PIW} \qquad obs_t^{PIW} = mes_t^{PIW} + \omega_t^{PIW} \tag{138}$$

$$400(R_t - 1) = mes_t^{R} \qquad obs_t^{R} = mes_t^{R} + \omega_t^{R} \tag{139}$$

$$400(R_t^W - 1) = mes_t^{RW} \qquad obs_t^{RW} = mes_t^{RW} + \omega_t^{RW} \tag{140}$$

$$100\log(\Pi_t^c) = mes_t^{CPI} - mes_{t-1}^{CPI} \qquad obs_t^{CPI} = mes_t^{CPI} + \omega_t^{CPI} \tag{141}$$

$$100\log(\dot{g}_t) = mes_t^{G} - mes_{t-1}^{G} \qquad obs_t^{G} = mes_t^{G} + \omega_t^{G} \tag{142}$$

5.3. Steady State

Now we are interested in finding a steady state of the model. The equilibrium is given by all model equations when we remove time index. Obtaining some steady states is more straightforward, because it is delivered directly from an individual equation. Thus steady states of technologies are immediately given as $z = \Lambda_z$, $A = \Lambda_A$, $\mu = \Lambda_\mu$ and $\gamma^L = \Lambda_L$. Further we assume that $u = d = \varphi = 1$.

A level of domestic prices is numeraire, so $p = 1$ and the law of one price must hold $\frac{exp^W}{p} = 1$. We assume a nominal exchange rate appreciation for the Czech economy at pace of -2.37% annually, which implies $\Delta ex = \frac{-2.37}{400} + 1$. Export specific technology is defined in the steady state as $\dot{aX} = \frac{1}{\Delta ex}$ Inflation growths are derived from the steady state inflation which is inflation target. Inflation target $target_t$ is defined in annual timing, 2% annually. Due to the fact that our model works with quarterly data, the steady state of the domestic price inflation is $\Pi = \frac{target}{400} + 1$. Then $\Pi^c = \Pi \dot{aR}$ and $\Pi^i = \Pi\mu$ (eqs (101) and (102)). $\Pi 4^c = (\Pi^c)^4 = target$ (eq. (115)). The steady state of regulated prices technology is set as $\dot{aR} = 1$. We put wedges $\kappa^{euler} = \frac{1}{\beta \frac{R}{z\Pi}}$, $\kappa^{forex} = \frac{R}{R^W \Delta ex}$ computed from equations (82) and (83).

Foreign inflation steady state Π^W is the inflation target of the ECB, 2% annually. Together with the assumption of nominal exchange rate appreciation it delivers the steady state of the foreign inflation expressed in domestic currency, thus $\Pi^{W*} = \Delta ex \Pi^W$ (eq. 106). Inflation of exports and imports prices expressed in domestic currency must be relevant to domestic inflation and nominal exchange rate appreciation, from equations (104) and (103), we get $\Pi^x = \Pi \Delta ex$ and $\Pi^M = \Pi \Delta ex$. Inflation of domestic exports prices in foreign currency must be the same as the foreign inflation in foreign currency, $\tilde{\Pi}^x = \frac{\Pi^x}{\Delta ex}$ (equation (105)).

The steady state growths of consumption, investments, domestic product follow the overall economy growth Λ_z modified by exogenous processes, so $\dot{c} = \frac{z\gamma^L}{aR} = \Lambda_c$ (eq. (109)), $\dot{i} = z\gamma^L = \Lambda_i$ (eq. (110)), $\dot{y^d} = z\gamma^L = \Lambda_{y^d}$ (eq. (108)). Growht of nominal wages is given by real wage growth and inflation target $\Pi^w = z\Pi$ (eq. (70)). Growth of real government spending we get from eq (113) $\dot{g} = z\gamma^L \dot{aG}$. Government specific technology growth is $\dot{aG} = 1$. Trade openness technology growth is defined as $\dot{aO} = \frac{1.5}{400} + 1$ in the steady state. Exports and imports are given by overall economy growth and specific technologies in these sectors, so $\dot{x} = z\gamma^L \dot{aX}\dot{aO}$, $\dot{M} = z\gamma^L \dot{aX}\dot{aO}$ (eqs. (112) and (111)). Foreign demand growth for the domestic exports is given ad hoc at pace of 9% a year, which implies $\dot{y^W} = z\gamma^L \dot{aX}\dot{aO}$ (eq. (114)).

Adjustment costs are zero in the steady state, so $\Omega^c = 1$, $\Omega^i = 1$, $\Omega^x = 1$, $\gamma^c = 0$, $\gamma^i = 0$, $\gamma^x = 0$, $\gamma^{c,der} = 0$, $\gamma^{i,der} = 0$, $\gamma^{x,der} = 0$.

Also steady states of domestic and foreign nominal interest rates are given, domestic interest rate is 3% annually, it implies $R = \frac{3}{400} + 1$, and foreign interest rate is 4% annually, it implies $R^W = \frac{4}{400} + 1$.

Steady states of technologies and AR processes can be easily seen from equations (72) – (126), for example for (72), it is Λ^μ or if the term is missing, it is 1.

Finally our observations *obs* are in levels (100 log), thus observables (127) – (142) usually start from the level of the first observation. Measurement variables *mes* then ensure a proper connection to model variables.

Above steady states are given adhoc and can be obtained from the data, we label them big numbers. Computing other steady states can be more difficult. Begin with the easiest.

Optimal domestic intermediate goods inflation and optimal wages are derived from equations (39), (40) and from parameters.

$$\Pi^{*}=\left(\frac{1-\theta_p\Pi^{-(1-\epsilon)(1-\chi)}}{1-\theta_p}\right)^{\frac{1}{1-\epsilon}}$$

and denoting $\Pi^{w*}=\frac{w^*}{w}$

$$\Pi^{w*}=\left(\frac{1-\theta_w\Pi^{-(1-\eta)(1-\chi_w)}z^{-(1-\eta)}}{1-\theta_w}\right)^{\frac{1}{1-\eta}}.$$

Marginal costs are from equations (28), (29) and Π^*,Π, so

$$mc=\Pi^{*}\frac{\epsilon-1}{\epsilon}\frac{1-\beta\gamma^L\theta_p\Pi^{\epsilon(1-\chi)}}{1-\beta\gamma^L\theta_p\Pi^{-(1-\epsilon)(1-\chi)}}.$$

When we know Π^*,Π^{w*} and Π, we can compute from (59), (60)

$$v^p=\frac{(1-\theta_p)(\Pi^*)^{-\epsilon}}{1-\theta_p\Pi^{(1-\chi)\epsilon}},\qquad v^w=\frac{(1-\theta_w)(\Pi^{w*})^{-\eta}}{1-\theta_w\Pi^{(1-\chi_w)\eta}z^{\eta}}.$$

Again if we know Π^M and parameters, we obtain from (41), (50)

$$\Pi^{M*}=\left(\frac{1-\theta_M(\Pi^M)^{-(1-\epsilon_M)(1-\chi_M)}}{1-\theta_M}\right)^{\frac{1}{1-\epsilon_M}},\qquad v^M=\frac{(1-\theta_M)(\Pi^{M*})^{-\epsilon_M}}{1-\theta_M(\Pi^M)^{(1-\chi_M)\epsilon_M}}$$

and with $\tilde{\Pi}^x,\Pi^W$, (88), (94)

$$\tilde{\Pi}^{x*}=\left(\frac{1-\theta_X\left(\frac{(\Pi^W)^{\chi_x}}{\tilde{\Pi}^x}\right)^{1-\epsilon_x}}{1-\theta_X}\right)^{\frac{1}{1-\epsilon_x}},\qquad v^x=\frac{(1-\theta_X)(\tilde{\Pi}^{x*})^{-\epsilon_x}}{1-\theta_X(\tilde{\Pi}^x)^{(1-\chi_x)\epsilon_x}}.$$

Now we can set steady state levels of prices. Start with $\frac{p^M}{p}$. From Π^{M*},Π^M and equations (33) and (35)

$$\frac{p^M}{p}=\frac{\epsilon_M}{\epsilon_M-1}\frac{\frac{exp^W}{p}}{\Pi^{M*}}\frac{1-\beta\theta_M\gamma^L\left(\frac{(\Pi^M)^{\chi_M}}{\Pi^M}\right)^{1-\epsilon_M}}{1-\beta\theta_M\gamma^L\left(\frac{(\Pi^M)^{\chi_M}}{\Pi^M}\right)^{-\epsilon_M}},$$

then from eqs (65) and (99)

$$\frac{p^i}{p}=\left(n^i+\Omega^i(1-n^i)\left(\frac{p^M}{p}\right)^{1-\epsilon_i}\right)^{\frac{1}{1-\epsilon_i}},\qquad \frac{q^x}{p}=\left(n^x+\Omega^x(1-n^x)\left(\frac{p^M}{p}\right)^{1-\epsilon_x}\right)^{\frac{1}{1-\epsilon_x}}.$$

From $\tilde{\Pi}^x, \tilde{\Pi}^{x*}, \Pi^W$ and equations (86) and (87), we get

$$\frac{p^x}{p} = \frac{\epsilon_x}{\epsilon_x - 1} \frac{\frac{q^x}{p}}{\tilde{\Pi}^{x*}} \frac{1 - \beta\theta_x\gamma^L \left(\frac{(\Pi^W)^{\chi_x}}{\tilde{\Pi}^x}\right)^{1-\epsilon_x}}{1 - \beta\theta_x\gamma^L \left(\frac{(\Pi^W)^{\chi_x}}{\tilde{\Pi}^x}\right)^{-\epsilon_x}}. \tag{143}$$

$\frac{p^i}{p}$, adjustment costs and eq. (24) deliver

$$q = \frac{\frac{p^i}{p}}{1 - \frac{\kappa}{2}(\gamma^L z - \Lambda_i)^2 - \kappa(\gamma^L z - \Lambda_i)(\gamma^L z) + \beta\frac{1}{z}\kappa(\gamma^L z - \Lambda_i)(\gamma^L z)^2},$$

and q with eq. (23) delivers

$$r = \frac{\frac{q\,z\,\mu}{\beta} - (1-\delta)q - \delta\tau_K}{(1-\tau_K)u}.$$

From mc and r and eqs. (32), (85)

$$w = (1-\alpha)\left(mc\left(\frac{\alpha}{r}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}, \quad w^* = \Pi^{w*}w.$$

$\frac{p^c}{p}$ is from $\frac{p^M}{p}$ and eq. (64)

$$\frac{p^c}{p} = \left(n^c + \Omega_C(1-n^c)\left(\frac{p^M}{p}\right)^{1-\epsilon_c}\right)^{\frac{1}{1-\epsilon_c}}.$$

Setting steady states of above variables is really straightforward. Much more interesting is searching for equilibrium values of real variables $k, y^d, i, x, c, g, M, \lambda$ and l^d. Solving of 9 non-linear equations (31),(57),(61),($x\frac{p^x}{p} = v^x\frac{exp^W}{P}M$ nominal exports equal nominal imports, see details in [6]),(97),(90),(92),(81) and recursive (26) and (27) is necessary.

$$k = \frac{\alpha}{1-\alpha}\frac{w}{r}z\mu l^d, \qquad y^d = \frac{\frac{A}{z}k^{\alpha}(l^d)^{1-\alpha}}{v^p}, \qquad i = \frac{\gamma^L - (1-\delta)}{z\mu}k, \qquad x = v^x\frac{\frac{exp^W}{p}}{\frac{p^x}{p}}M$$

$$c = \frac{1}{n^c}\left(\frac{p^c}{p}\right)^{-\epsilon_c}\left[y^d - n^i\left(\frac{p^i}{p}\right)^{\epsilon_i}i - n^x\left(\frac{q^x}{p}\right)^{\epsilon_x}v^x\frac{\frac{exp^W}{p}}{\frac{p^x}{p}}M - g\right], \qquad g = \rho_g c$$

$$M = v^M\left[\Omega^c(1-n^c)\left(\frac{\frac{p^M}{p}}{\frac{p^c}{p}}\right)^{-\epsilon_c}c + \Omega^i(1-n^i)\left(\frac{\frac{p^M}{p}}{\frac{p^i}{p}}\right)^{-\epsilon_i}i + \Omega^x(1-n^x)\left(\frac{\frac{p^M}{p}}{\frac{q^x}{p}}\right)^{-\epsilon_x}x\right]$$

$$\lambda = \frac{z - h\beta\gamma^L}{(1+\tau_C)(z-h)(\frac{p^c}{p})}\frac{1}{c}, \qquad (l^d)^{\vartheta} = \frac{\left[1 - \beta\theta_w z^{\eta(1+\vartheta)}\Pi^{\eta(1-\chi_w)(1+\vartheta)}\gamma^L\right]\frac{\eta-1}{\eta}(1-\tau_W)w^*}{\left[1 - \beta\theta_w z^{\eta-1}\Pi^{-(1-\eta)(1-\chi_w)}\gamma^L\right]\psi((\Pi^{w*}))^{-\eta\vartheta}}\lambda$$

To do this we set some auxiliary parameters.

$$a_1 = \frac{\alpha}{1-\alpha}\frac{w}{r}z\mu, \quad a_2 = \frac{\frac{A}{z}}{v^p}, \quad a_3 = \frac{\gamma^L - (1-\delta)}{z\mu}, \quad a_4 = \frac{1}{n^c}\left(\frac{p^c}{p}\right)^{-\epsilon_c}, \quad a_5 = a_4 n^i \left(\frac{p^i}{p}\right)^{\epsilon_i}$$

$$a_6 = a_4 v^x \frac{\frac{exp^W}{p}}{\frac{p^x}{p}}, \quad a_7 = n^x \left(\frac{q^x}{p}\right)^{\epsilon_x}, \quad a_8 = v^M \Omega_X (1-n^x)\left(\frac{\frac{p^M}{p}}{\frac{q^x}{p}}\right)^{-\epsilon_x}, \quad a_9 = a_4\rho_g,$$

$$a_{10} = v^M \Omega^c (1-n^c)\left(\frac{\frac{p^M}{p}}{\frac{p^c}{p}}\right)^{-\epsilon_c}, \quad a_{11} = v^M \Omega^i (1-n^i)\left(\frac{\frac{p^M}{p}}{\frac{p^i}{p}}\right)^{-\epsilon_i}, \quad a_{12} = \frac{z - h\beta\gamma^L}{(1+\tau_C)(z-h)\left(\frac{p^c}{p}\right)},$$

$$a_{13} = \frac{\left[1 - \beta\theta_w z^{\eta(1+\vartheta)}\Pi^{\eta(1-\chi_w)(1+\vartheta)}\gamma^L\right]\frac{\eta-1}{\eta}(1-\tau_W)w^*}{\left[1-\beta\theta_w z^{\eta-1}\Pi^{-(1-\eta)(1-\chi_w)}\gamma^L\right]\psi(\Pi^{w*})^{-\eta\vartheta}}, \quad a_{14} = \vartheta, \quad a_{15} = \alpha.$$

Substituting these parameters into above equations, we get

$$k = a_1 l^d, \quad y^d = a_2 k^{a_{15}}(l^d)^{1-a_{15}}, \quad i = a_3 k, \quad x = \frac{a_6}{a_4}M, \quad c = a_4 y^d - a_5 i - a_6 a_7 M - a_4 g,$$

$$g = \rho_g c, \quad M = a_{10}c + a_{11}i + a_8 x, \quad \lambda = a_{12}\frac{1}{c}, \quad (l^d)^{a_{14}} = a_{13}\lambda.$$

After some easy algebra

$$M = a_{10}c + a_{11}i + a_8\frac{a_6}{a_4}M \Rightarrow M = \frac{a_{10}c + a_{11}i}{1 - a_8\frac{a_6}{a_4}} \text{ where } a_{16} = 1 - a_8\frac{a_6}{a_4}$$

$$c = a_4 y^d - a_5 i - a_6 a_7 M - a_9 c \Rightarrow c = \frac{a_{16}a_4 y^d - (a_5 a_{16} - a_6 a_7 a_{11})i}{a_{16} + a_6 a_7 a_{10} + a_9 a_{16}}$$

$$= \frac{a_{16}a_4 a_2 a_1^{a_{15}} - (a_5 a_{16} - a_6 a_7 a_{11})a_1 a_3}{a_{16} + a_6 a_7 a_{10} + a_9 a_{16}} l^d$$

$$(l^d)^{a_{14}} = a_{13}\frac{a_{12}}{c} \Rightarrow l^d = \left(\frac{a_{17}}{a_{18}}\right)^{\frac{1}{1+a_{14}}}$$

where $a_{17} = a_{12}a_{13}(a_{16} + a_6 a_7 a_{10} + a_9 a_{16})$, $a_{18} = a_{16}a_4 a_2 a_1^{a_{15}} - (a_5 a_{16} - a_6 a_7 a_{11})a_1 a_3$

When we know l^d, we can simply derive steady states of $k, i, y^d, c, M, \lambda, x$ and g.

$$k = a_1 l^d, \quad i = a_3 k, \quad y^d = a_2 k^\alpha (l^d)^{1-\alpha}, \quad c = \frac{a_{16}a_4 y^d - (a_5 a_{16} - a_6 a_7 a_{11})i}{a_{16} + a_6 a_7 a_{10} + a_9 a_{16}},$$

$$M = \frac{a_{10}c + a_{11}i}{1 - a_8\frac{a_6}{a_4}}, \quad \lambda = \frac{a_{12}}{c}, \quad x = \frac{a_6}{a_4}M, \quad g = \rho_g c.$$

From equations (58), (97), (54), (55), (96), (95) and from the above, we get

$$l = v_w l^d, \quad c^d = n^c\left(\frac{p^c}{p}\right)^{\epsilon_c} c, \quad i^d = n^i\left(\frac{p^i}{p}\right)^{\epsilon_i} i, \quad x^d = n^x\left(\frac{q^x}{p}\right)^{\epsilon_x} x,$$

$$c^M = c^d \Omega^c \frac{1-n^c}{n^c}\left(\frac{p^M}{p}\right)^{-\epsilon_c}, \quad i^M = i^d \Omega^i \frac{1-n^i}{n^i}\left(\frac{p^M}{p}\right)^{-\epsilon_i}, \quad x^M = x^d \Omega^x \frac{1-n^x}{n^x}\left(\frac{p^M}{p}\right)^{-\epsilon_x},$$

$$y^M = c^M + i^M + x^M.$$

From equations (33), (37), (34), (38), (26), (28), (30) and from the above, we get

$$
\begin{aligned}
g^{M_1} &= \frac{\lambda \frac{\frac{exp^W}{p}}{\frac{p^M}{p}} y^M}{1 - \beta\theta_M \gamma^L \left(\frac{(\Pi^M)^{\chi_M}}{\Pi^M}\right)^{-\epsilon_M}}, \qquad g^{M_2} = \frac{\epsilon_M}{\epsilon_M - 1} g^{M_1}, \\
g^{x_1} &= \frac{\lambda \frac{\frac{q^x}{p}}{\frac{p^x}{p}} x^d}{1 - \beta\theta_X \gamma^L \left(\frac{(\Pi^W)^{\chi_x}}{\tilde{\Pi}^x}\right)^{-\epsilon_x}}, \qquad g^{x_2} = \frac{\epsilon_x}{\epsilon_x - 1} g^{X_1}, \\
f &= \frac{d\varphi\psi(\Pi^{w*})^{-\eta(1+\vartheta)}(l^d)^{1+\vartheta}}{1 - \beta\theta_w \gamma^L z^{\eta(1+\vartheta)} \Pi^{\eta(1-\chi_w)(1+\vartheta)}} \\
g^1 &= \frac{\lambda\, mc\, y^d}{1 - \beta\theta_p \gamma_L \Pi^{(\chi-1)(-\epsilon)}}, \qquad g^2 = \frac{\epsilon}{\epsilon - 1} g^1.
\end{aligned}
$$

Net foreign assets evolution is derived from eq. (91)

$$
exb^W = \frac{(\frac{exp^W}{p})^{\epsilon_W} (\frac{p^x}{p})^{1-\epsilon_W} (\frac{y^W}{y^d}) - \frac{exp^W}{p} \frac{M}{y^d}}{1 - R^W \dot{ex} \frac{y^d}{\gamma_L z \Pi y^d}}.
$$

From eq. (84) and from the fact that $exb^W = 0$, we get

$$
prem = 1
$$

and from eq. (48)

$$
y^W = \frac{x}{v_x \left(\frac{\frac{p^x}{p}}{\frac{exp^W}{p}}\right)^{-\epsilon_W}}
$$

To recapitulate, first we define steady states for z, A, μ, γ_L, u, d, φ, $\frac{exp^W}{p}$, Δex, $\dot{aX}$ $target$, Π, Π^c, $\dot{aR}$, κ^{euler}, κ^{forex}, Π^i, $\Pi 4^c$, Π^W, Π^{W*}, Π^x, Π^M, $\tilde{\Pi}^x$, $\dot{c}$, $\dot{i}$, $\dot{y}^d$, Π^w, $\dot{g}$, $\dot{aG}$, $\dot{aO}$, $\dot{x}$, $\dot{M}$, $\dot{y}^W$, γ^c, γ^i, γ^x, $\gamma^{c,der}$, $\gamma^{i,der}$, $\gamma^{x,der}$, R, R^W and for deep parameters θ_p and χ and observables (127) – (142).

From simple subsequent computation we derive steady states for Π^*, Π^{w*}, mc, v^p, v^w, Π^{M*}, v^M, $\tilde{\Pi}^{x*}$, v^x, $\frac{p^M}{p}$, $\frac{p^i}{p}$, $\frac{q^x}{p}$, $\frac{p^x}{p}$, q, r, w, w^*, $\frac{p^c}{p}$.

Solving 9 non-linear equations delivers steady states for k, y^d, i, x, c, g, M, λ, l^d.

Then we can compute the rest of steady states as c^d, i^d, x^d, c^M, i^M, x^M, y^M, l, g^{M_1}, g^{M_2}, g^{x_1}, g^{x_2}, f, g^1, g^2, exb^W, $prem$, y^W. We thus have **120 steady state** values from **120 equations**.

5.4. Posterior distributions

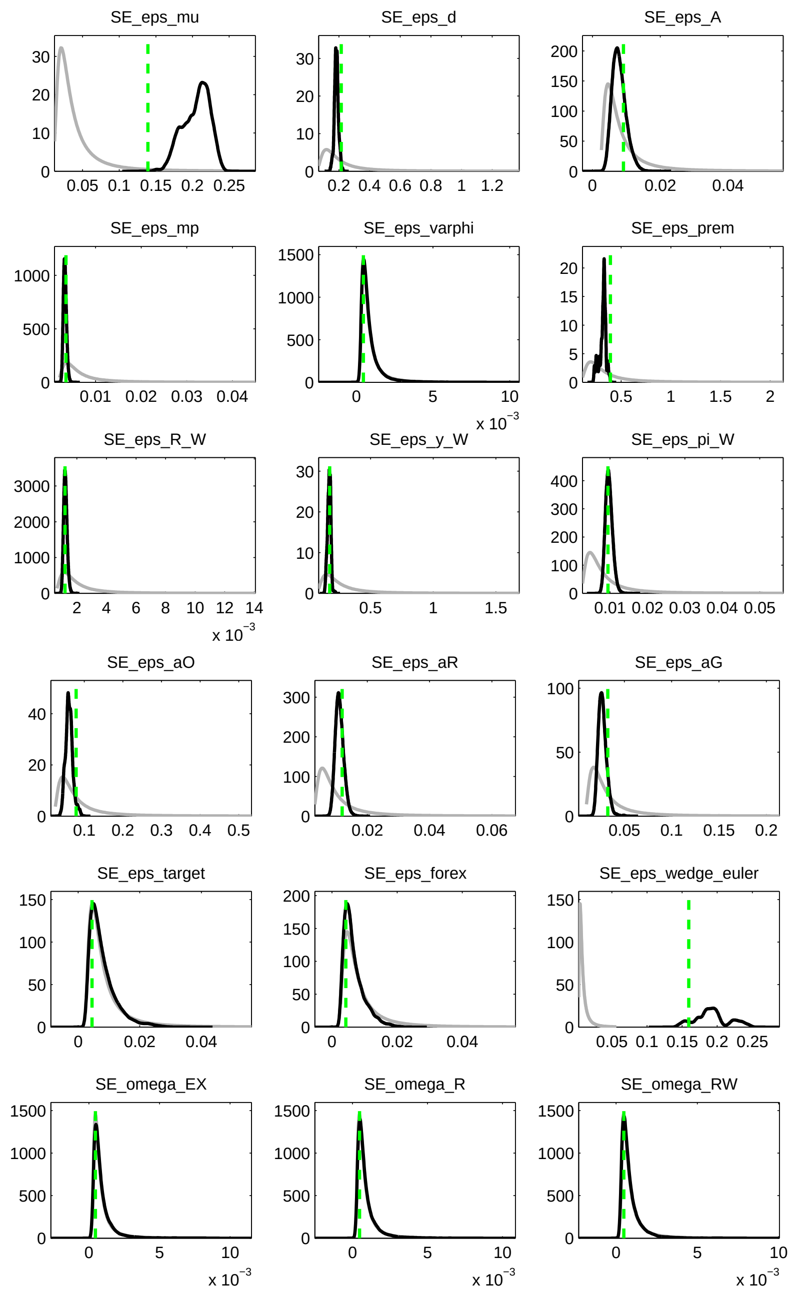

Figure 8: Posterior distributions

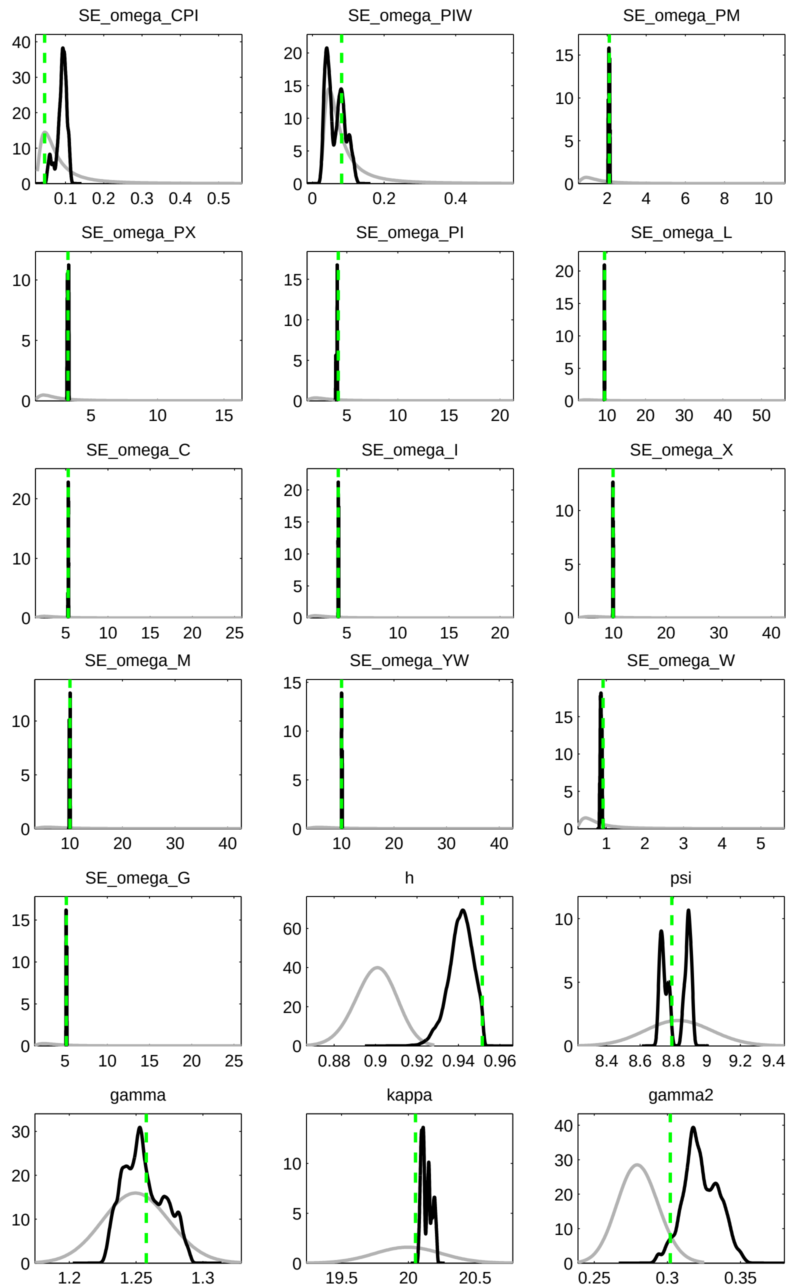

Figure 9: Posterior distributions

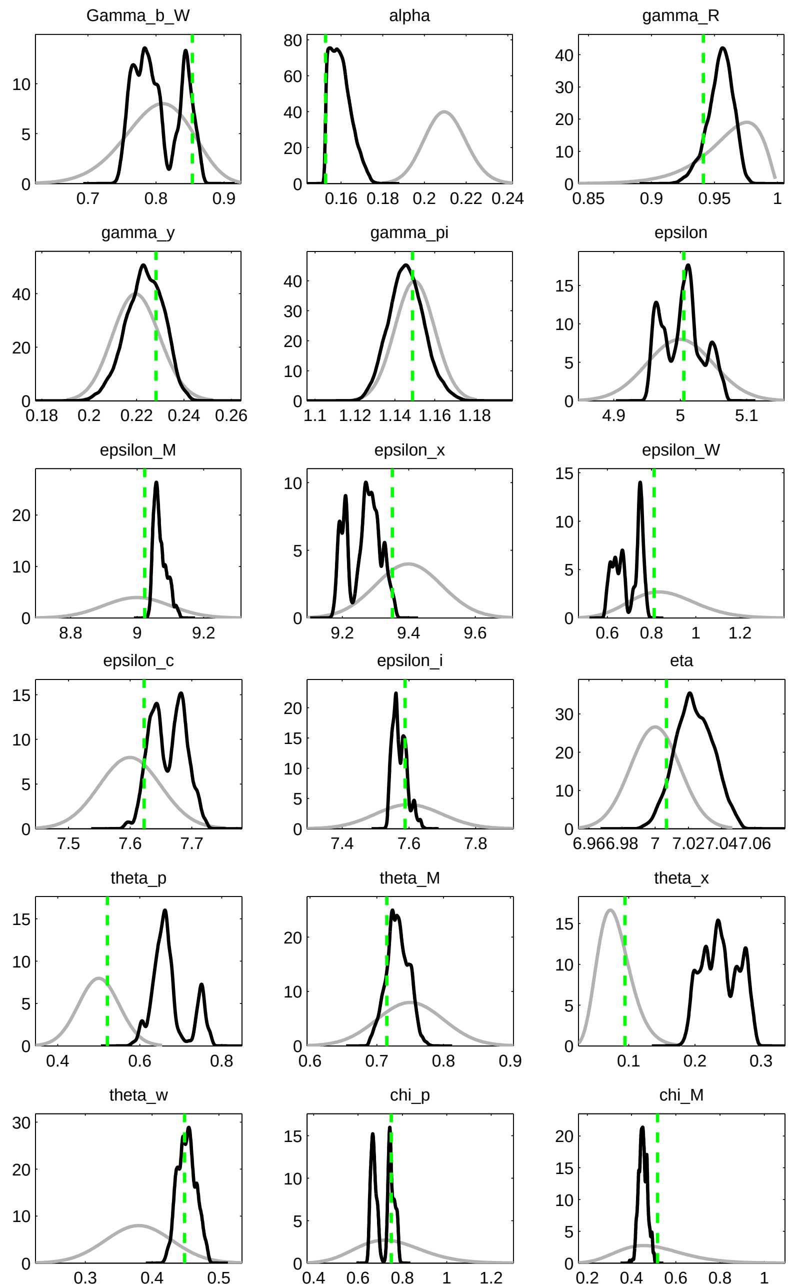

Figure 10: Posterior distributions

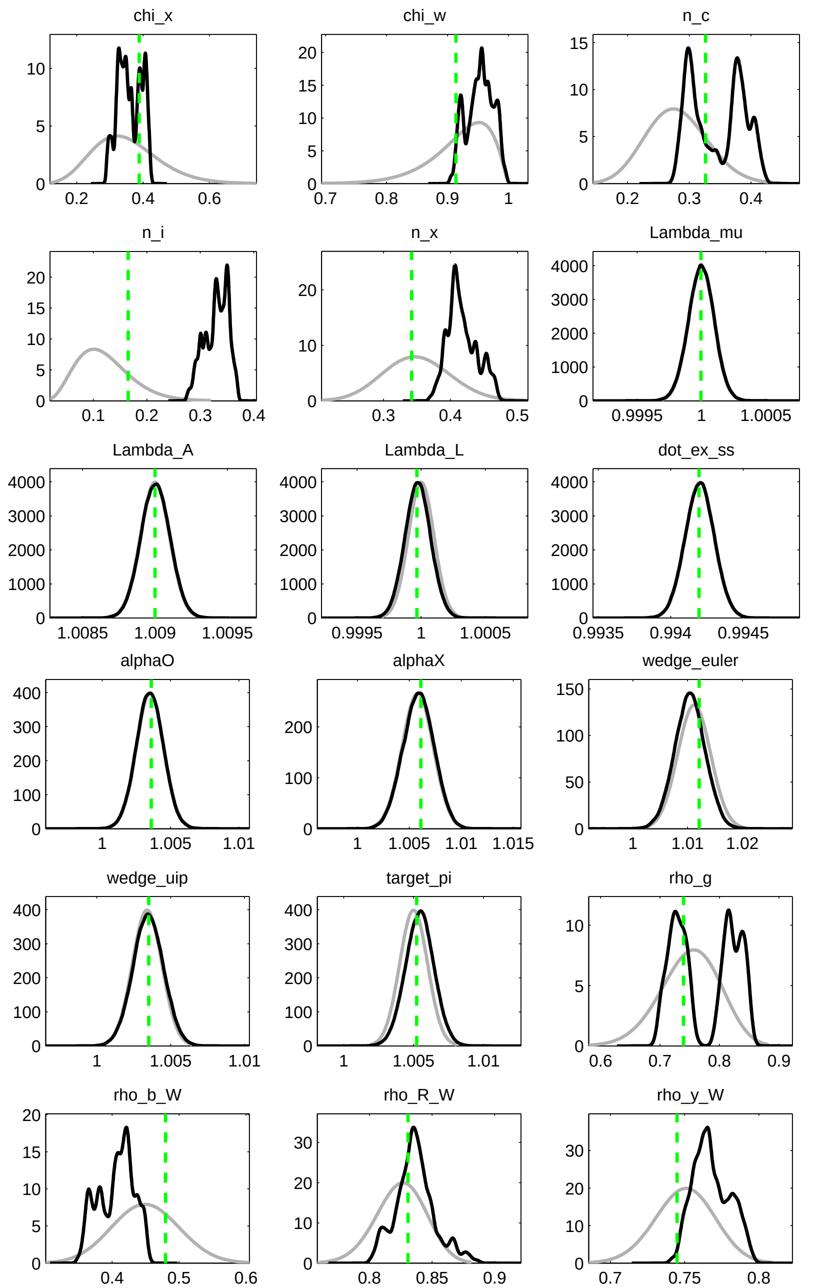

Figure 11: Posterior distributions

Table 3: Estimated Parameters

Parameter		**Prior Mean**	**Dist**	**Posterior Mean**	**Lower and Upper Bound of a 90 % HPD interval**
AR coefs of shocks					
Intertemp. preferences.	ρ_d	0.550	beta	0.5370	(0.5163 , 0.5675)
Hours preferences.	ρ_φ	0.400	-	-	-
Public consumption	ρ_g	0.750	beta	0.7758	(0.7107 , 0.8488)
Foreign prices	$\rho_{\pi W}$	0.300	beta	0.2992	(0.2839 , 0.3161)
Foreign demand	ρ_{yW}	0.750	beta	0.7680	(0.7489 , 0.7884)
World interest rate	ρ_{RW}	0.825	beta	0.8365	(0.8060 , 0.8600)
Foreign debt	ρ_{bW}	0.450	beta	0.4061	(0.3605 , 0.4439)
Regulated prices	ρ_{aR}	0.300	-	-	-
General tech.	ρ_A	0.750	-	-	-
Export specific tech.	ρ_{aX}	0.200	-	-	-
Wedge euler	ρ_{euler}	0.500	-	-	-
Wedge forex	ρ_{forex}	0.600	-	-	-
Standard devs of shocks					
Invest. spec. tech.	σ_μ	0.045	invg	0.2048	(0.1747 , 0.2324)
General tech.	σ_A	0.010	invg	0.0078	(0.0045 , 0.0110)
Intertemp. preferences	σ_d	0.250	invg	0.1831	(0.1634 , 0.2087)
Hours preferences	σ_φ	0.001	invg	0.0009	(0.0002 , 0.0015)
Monetary policy	σ_m	0.008	invg	0.0033	(0.0027 , 0.0038)
Foreign prices	$\sigma_{\pi W}$	0.010	invg	0.0097	(0.0082 , 0.0112)
Foreign demand	σ_{yW}	0.310	invg	0.1681	(0.1456 , 0.1883)
World interest rate	σ_{RW}	0.003	invg	0.0012	(0.0010 , 0.0014)
Premium	σ_{prem}	0.400	invg	0.3140	(0.2454 , 0.3652)
Openness	σ_{aO}	0.095	invg	0.0609	(0.0448 , 0.0749)
Regulated prices	σ_{aR}	0.012	invg	0.0111	(0.0089 , 0.0133)
Government specific	σ_{aG}	0.038	invg	0.0269	(0.0199 , 0.0337)
Population	σ_L	0.0001	-	-	-
Government	σ_g	0.0001	-	-	-
Export specific	σ_{aX}	0.0001	-	-	-
Target	σ_{target}	0.0100	-	-	-
UIP	σ_{uip}	0.0001	-	-	-
Wedge forex	σ_{forex}	0.0100	-	-	-
Wedge euler	σ_{euler}	0.0100	-	-	-
Std of measurement errors					
Exchange rate	σ_{EX}	0.001	invg	0.0009	(0.0002 , 0.0016)
Domestic interest rate	σ_R	0.001	invg	0.0009	(0.0002 , 0.0016)
Foreign interest rate	σ_{RW}	0.001	invg	0.0008	(0.0002 , 0.0016)
Domestic inflation	σ_{CPI}	0.100	invg	0.0900	(0.0657 , 0.1128)
Foreign inflation	σ_{PIW}	0.100	invg	0.0643	(0.0295 , 0.1045)
Import prices inflation	σ_{PM}	2.000	invg	2.0983	(2.0442 , 2.1381)
Export prices inflation	σ_{PX}	3.000	invg	3.2641	(3.1952 , 3.3376)
Investment prices inflation	σ_{PI}	4.000	invg	4.0216	(3.9240 , 4.0849)
Population	σ_L	10.000	invg	9.3616	(9.3189 , 9.4103)
Consumption	σ_C	5.000	invg	5.3204	(5.2871 , 5.3603)
Investment	σ_I	4.000	invg	4.1811	(4.1423 , 4.2135)
Export	σ_X	10.000	invg	9.9154	(9.8412 , 9.9854)
Import	σ_M	10.000	invg	9.9734	(9.8564 , 10.0801)
Foreign demand	σ_{YW}	10.000	invg	10.0867	(10.0176 , 10.1802)
Nominal wages	σ_W	1.000	invg	0.8639	(0.8339 , 0.8987)
Government spending	σ_G	5.000	invg	5.1637	(5.1101 , 5.2234)

5.5. Impulse responses

This section presents the behaviour of the model[27]. All shocks are unanticipated, positive and have one standard deviation size. The model is simulated with the Dynare Toolbox [8]. The figures are in the Appendix where impulse responses of the standard model (left panels) are compared with impulse responses of the second-order version of the model (right panels).

Figure 5 presents the **technology shock** ξ_t^A. A positive total factor productivity (TFP) shock results in positive reactions of investment, consumption and exports. Imports increase as well, partly because of a considerable import intensities of other sectors. Wages react positively to the technology shock as well. A higher productivity decreases marginal costs implying lower inflation. The reaction of inflation to the shock is not instantaneous because of the price stickiness. The nominal exchange rate appreciates. A central bank decreases its interest rate as a reaction of lower inflation and anti-inflation pressures from the appreciation. The reaction of the economy to the **investment-specific technological shock** ξ_t^μ (Figure 13) is similar with moderate impact on the consumption.

The reaction of the model to the **population shock** ξ_t^L (Figure 14) implies an increase of consumption, exports and imports. The growth rate of wages falls because new workers lower wage pressures.[28] The reaction of inflation and interest rate is negligible. The **labour supply shock** ξ_t^φ (Figure 15) decreases hours worked resulting in higher wages and lower consumption and investment. The exchange rate depreciates and thus, net exports increase. The reaction of prices and the central bank's interest rate is positive as a response to higher inflation pressures from the import prices (via a depreciation).

With nominal rigidities, the one-time **monetary policy shock** ξ_t^m (Figure 16) cannot spill over to the one-time decrease of the inflation. The nominal and real (because of price stickiness) interest rates rise implying a fall of consumption, investment, hours worked, and real wages. The exchange rate appreciates as a reaction to the positive inflation differential with a strong impact on net exports. The positive shock to **real government spending** ξ_t^g (Figure 17) depreciates nominal exchange rate resulting in higher net exports. Private consumption and investment are crowded out by the positive government spending. The depreciation causes inflation pressures from import prices and the central bank reacts by increasing interest rates. The extent of interest rates increase is of limited importance for the economy since the inflation pressures are relatively small.

The positive shock to **foreign demand** $\xi_t^{y_w}$ (Figure 18) increases volume of exports accompanied by an increase of imports. The nominal exchange rate appreciates instantaneously implying pressures on lower inflation.[29] The central banks

[27]Impulse responses are expressed as deviations from steady state in percentage of q-o-q growths. The shocks are unanticipated and their sizes are five standard deviations to see differences between the first and the second approximated models.

[28]Note that the effects of population growth in the model are very moderate since the population growth and its economic impacts are inconsiderable with respect to some other Eurozone countries. The exception might be the last expansion of the Czech economy before the financial crisis where there was a high inflow of foreign workers. On the other hand, it should be add that the labour force time series is very volatile.

[29]The nominal exchange rate appreciation is actually an increase of prices of export goods since

decreases interest rates to bring future inflation back to the target. The positive shock to the **foreign interest rates** $\xi_t^{pi_w}$ (Figure 19) causes, ceteris paribus, a negative interest rate differential with inflationary pressures from the exchange rate depreciation. The central bank raises its interest rate with negative consequences for domestic consumption and investment. The depreciation causes an increase of exports since export goods are cheaper in foreign markets. The increase of imports is very low in comparison with exports since higher imports for exports are lowered by lower imports for investment and consumption. The positive one standard error shock to **foreign inflation** $\xi_t^{R_w}$ (Figure 20) leads to an appropriate level shift in foreign prices. Our economy protects itself against higher imported inflation via the exchange rate appreciation. The reaction of the central bank depends on the magnitude of the appreciation. In this model, the instantaneous appreciation is high enough and the central bank decreases its interest rate with the stimulus for the consumption and investment.[30]

The positive **uncovered interest parity** ξ_t^{uip} (UIP) shock (Figure 21) depreciates the nominal exchange rate. A central bank raises its interest rate as a reaction to increased inflation pressures from import prices. Net exports increase as a reaction to the depreciated exchange rate while consumption and investment decrease because of higher domestic interest rates. The similar impulse responses are after the positive **forex shock** ξ_t^{forex} (Figure 22) and a positive shock to the **debt elastic premium** (Figure 23) ξ_t^{prem}.

After the positive **regulated prices shock** ξ_t^{aR} (Figure 24), the headline inflation increases since regulated prices inflation is higher. A central bank increases its interest rates to decrease the net inflation below target bringing the headline inflation to the target in the future. Consumption and investment fall. The nominal exchange rate depreciates and net exports increase. The **intertemporal preference shock** ξ_t^d is shown in Figure 25. Consumption, output, and wages rise whereas the investment expenditures decrease. The inflation is above target because of higher demand pressures and a central bank raises its interest rate. The nominal exchange rate depreciates and thus allows an increase of net exports.

According to Andrle et al. [2], export specific technology makes domestic intermediate goods more effective in the production of exports (a wedge between export and import deflators and the GDP deflator). Thus, the positive shock to the **export specific technology** ξ_t^{aX} (Figure 26) increases net exports. The exchange rate appreciates. The headline inflation increases as a result of inflation pressures stemming from the nontradables sector (Harrod-Ballassa-Samuelson effect). The reaction of domestic interest rate depends on the relative size of inflationary pressures form higher inflation and anti-inflationary pressures of the appreciation. In the model, the appreciation is strong enough to force a central bank to decrease interest rates. Consumption and investment react positively to the central bank's reaction.

The positive **kappa wedge euler shock** ξ_t^{euler} (Figure 27) increases future inflation, decreases current interest rates, and appreciates the nominal exchange rate. As a reaction, net exports decrease. The consumption and investment decrease

foreign prices are exogenous.

[30] In a reverse case when the appreciation is not strong enough, the central bank increases the interest rate as a reaction to the inflationary pressures from higher import prices.

since the shock increases the shadow value of wealth. On the whole, the shock has similar impulse responses as reverse preference shock.

Impulse responses comparison between first and second order approximated models delivers practically no substantial difference when comparing one standard deviation shocks. That is why we show the comparison of five standard deviation shocks in the Appendix. We can see that reactions are not so strong in the case of the second order approximation because of precautionary behaviour (risk stems into policy functions). Another important difference embodies in the shift of steady state. Increases in steady states are mostly caused by the neutral technology shock.

Table 4: Δ^2 - shift effect

Variable	**Shift (%)**
Investment deflator	0.10
Export deflator	0.98
Import deflator	0.21
Nominal wages	-0.05
Hours worked	-0.17
Exchange rate	-1.11
Consumption growth	-0.31
Investment growth	-0.44
Export growth	-1.47
Import growth	-1.64
Foreign demand growth	0.00
Foreign inflation growth	0.00
Interest rate	-0.00
Foreign interest rate	-0.00
CPI inflation	0.09
Government spending growth	-0.31

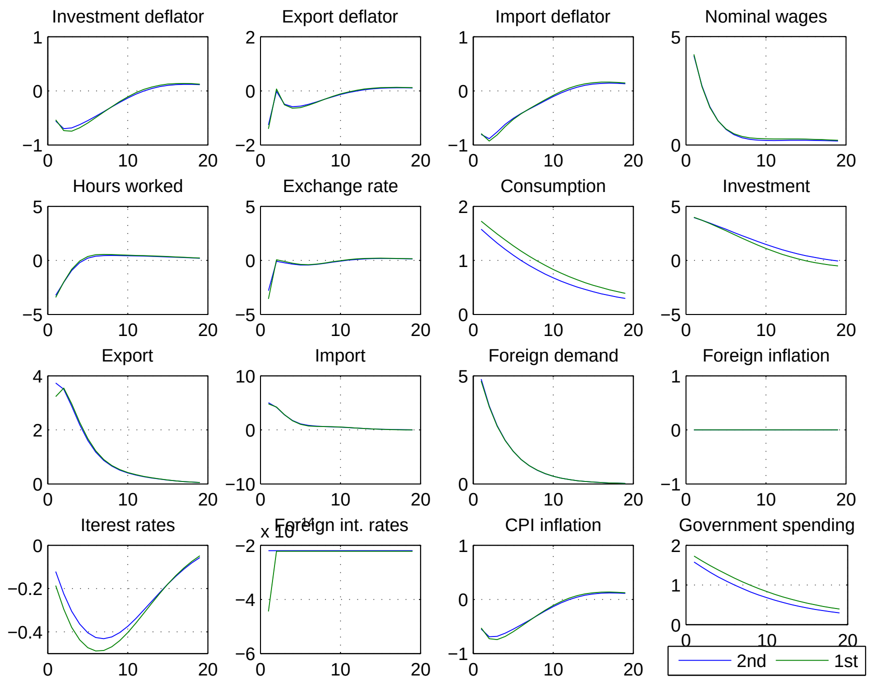

Figure 12: Technology shock

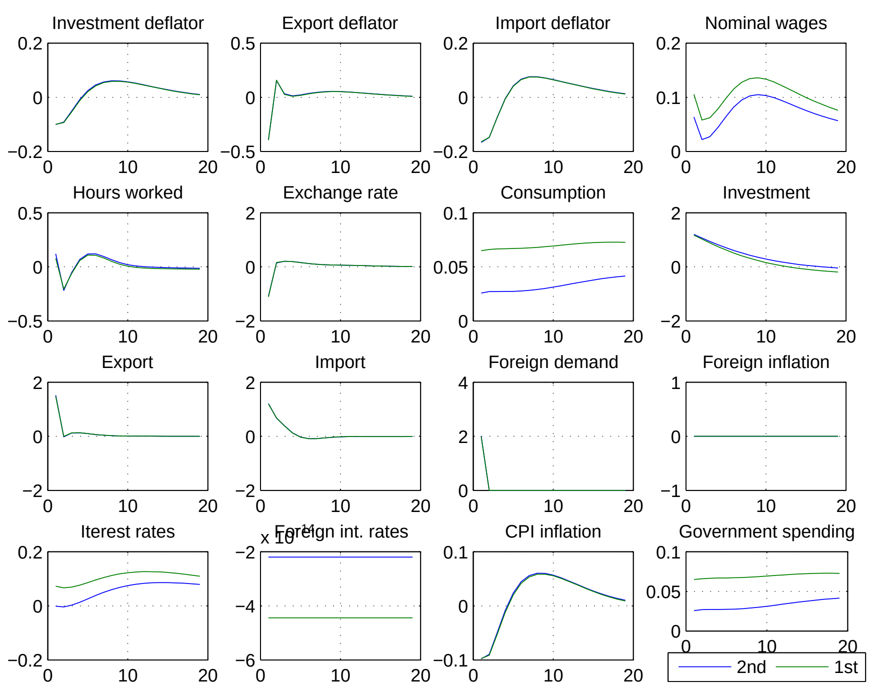

Figure 13: Investment specific technology shock

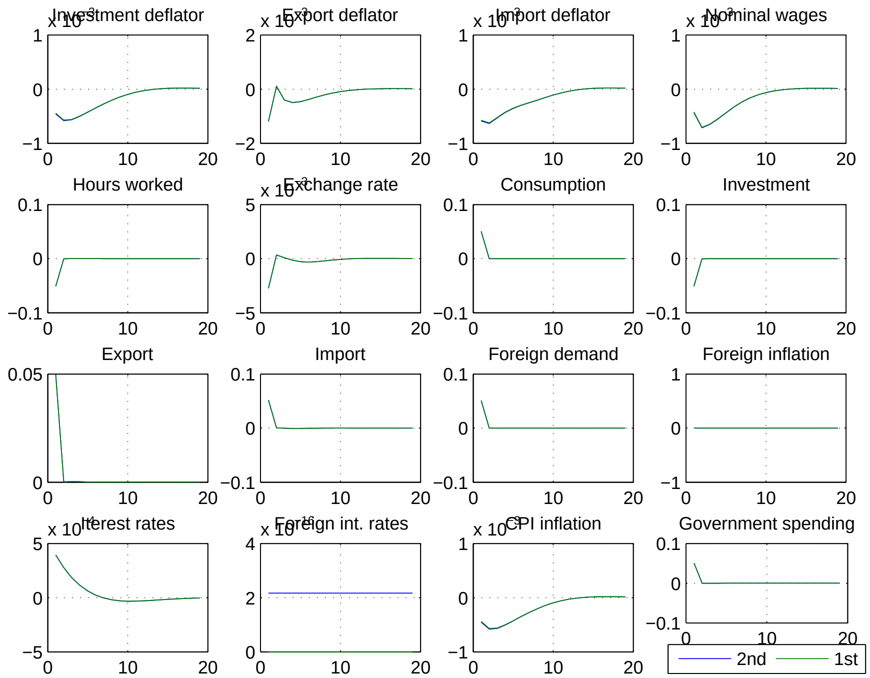

Figure 14: Population shock

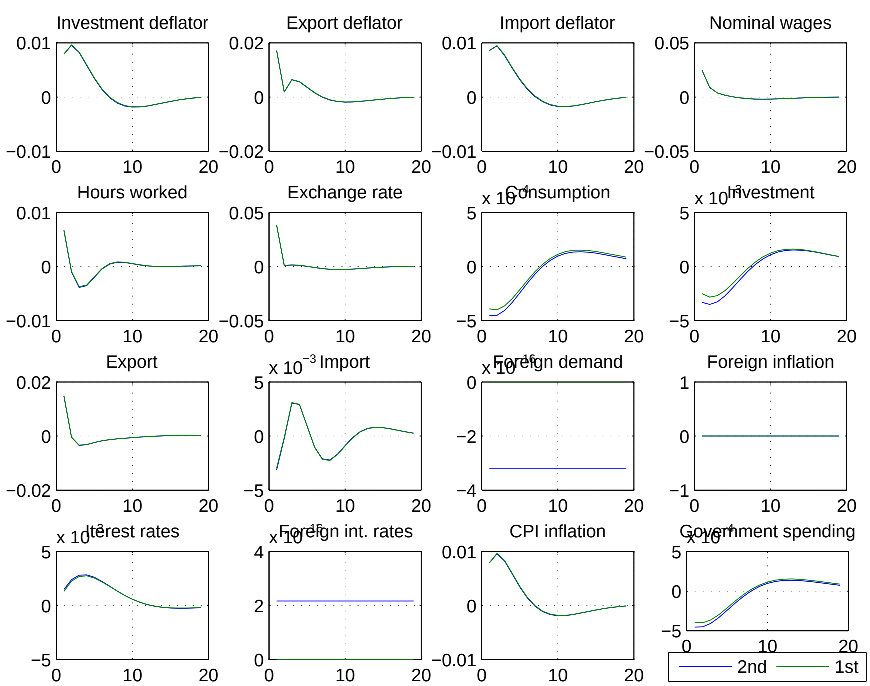

Figure 15: Labor supply shock

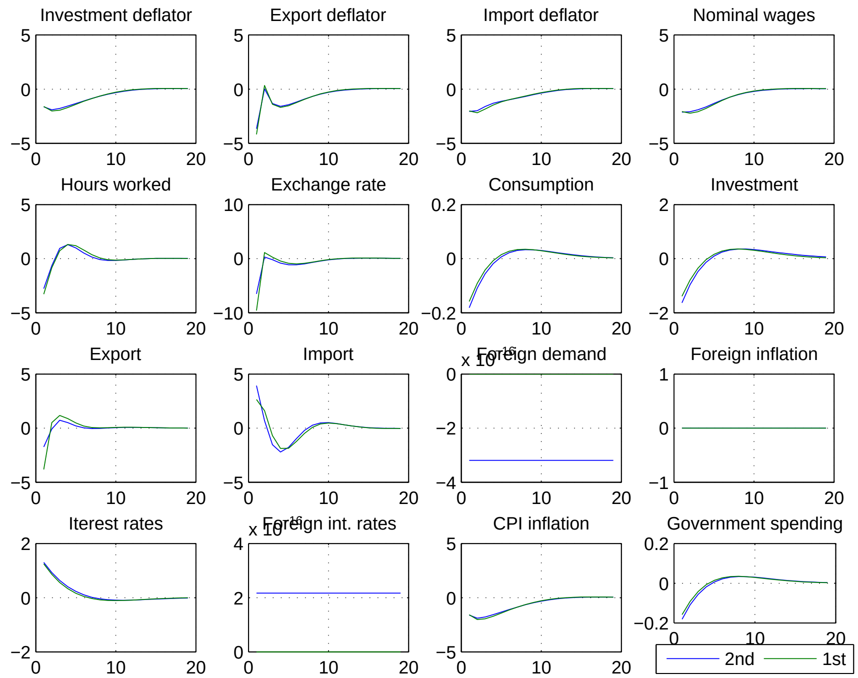

Figure 16: Monetary policy shock

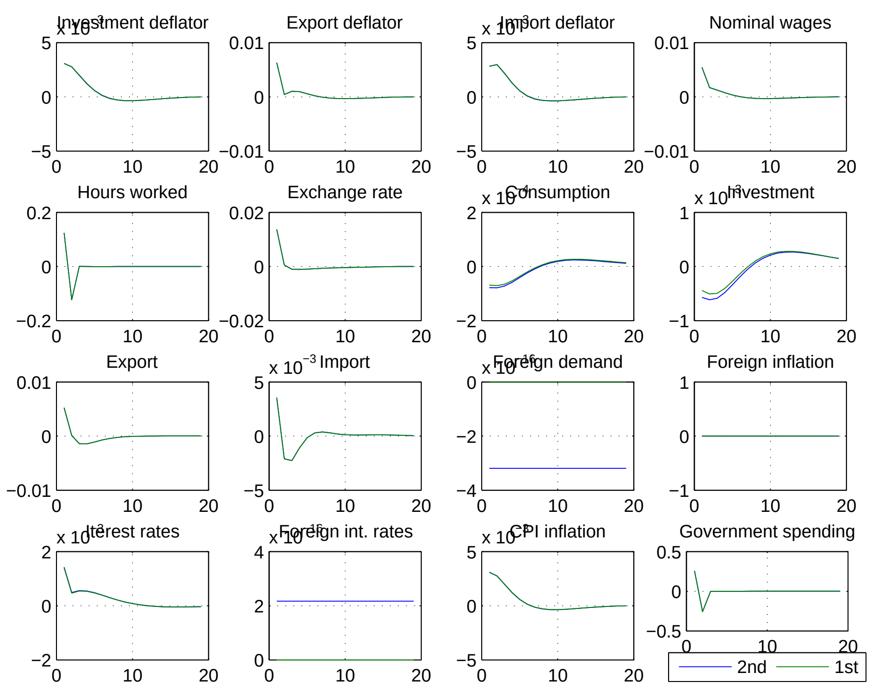

Figure 17: Government real consumption shock

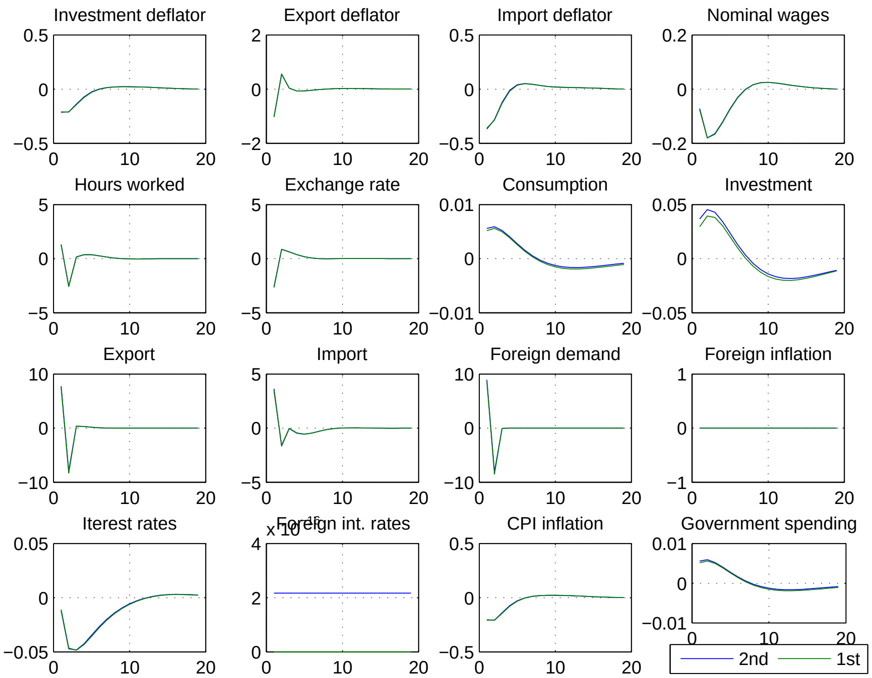

Figure 18: Foreign demand shock

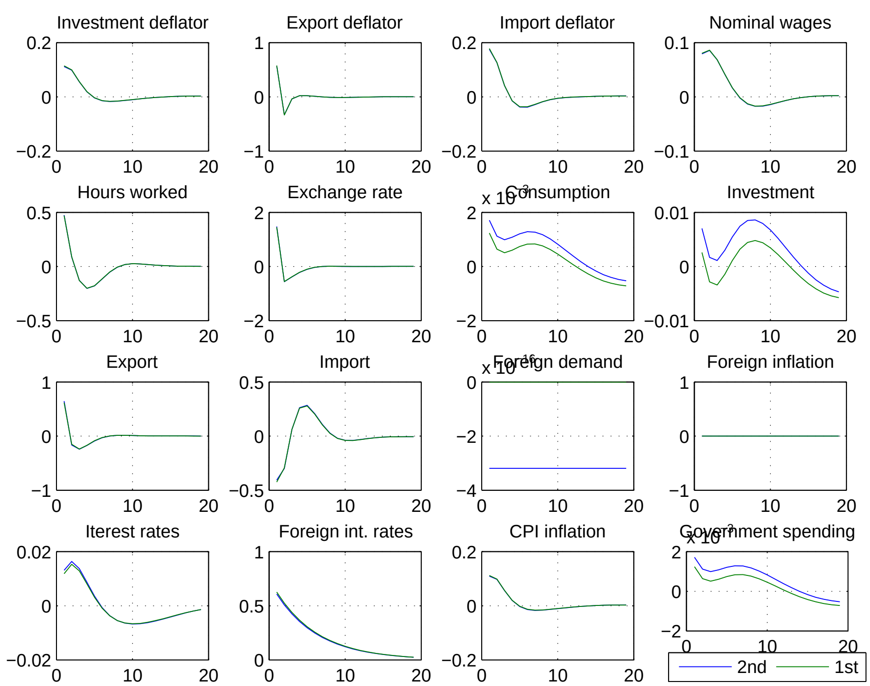

Figure 19: Foreign interest rate shock

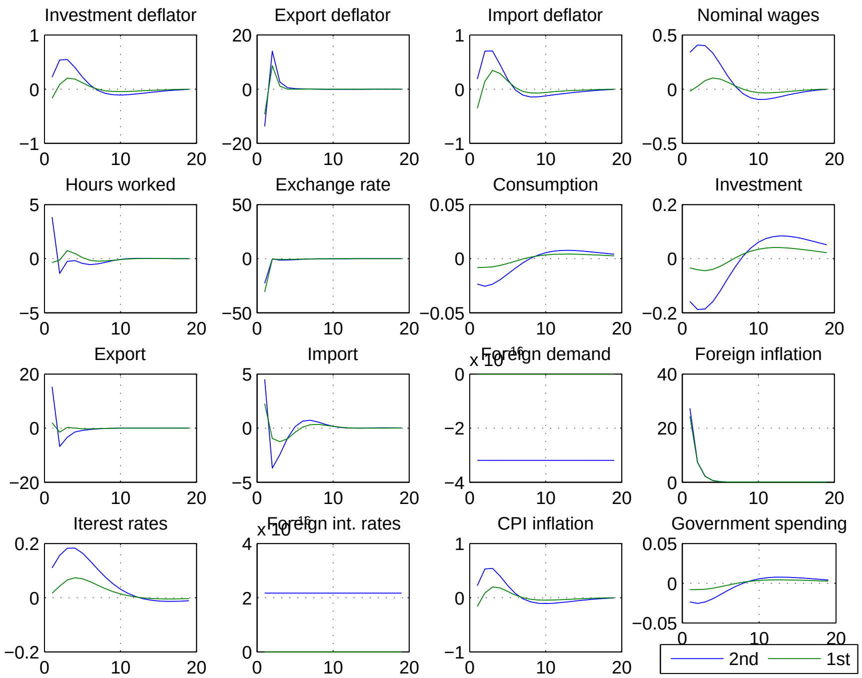

Figure 20: Foreign prices shock

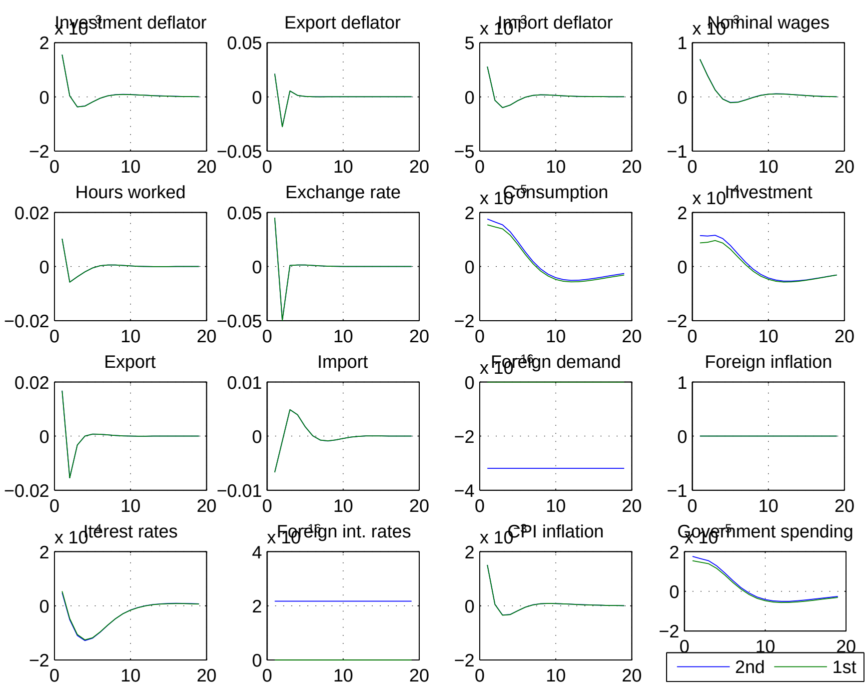

Figure 21: UIP shock

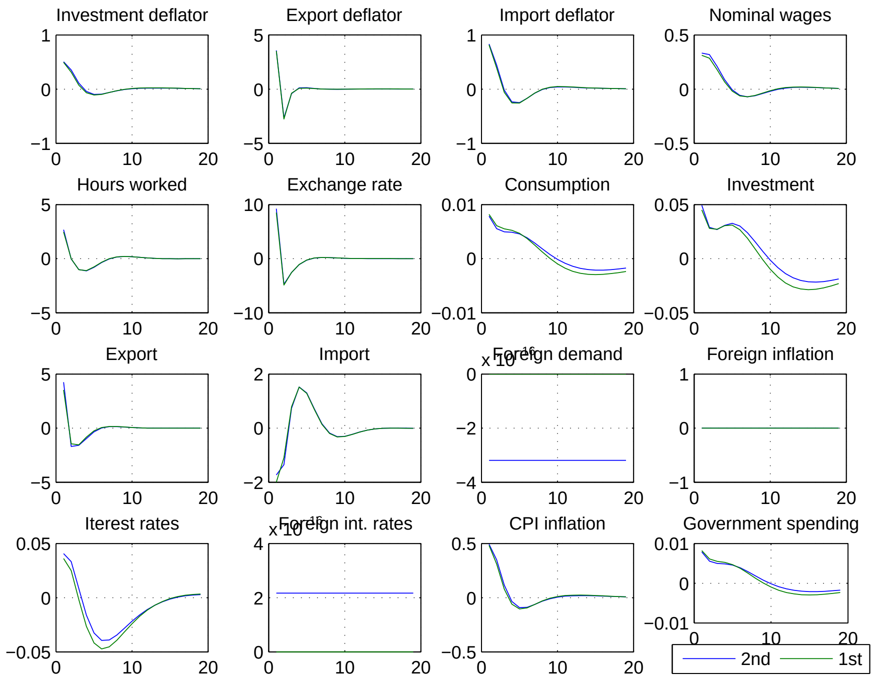

Figure 22: Forex shock

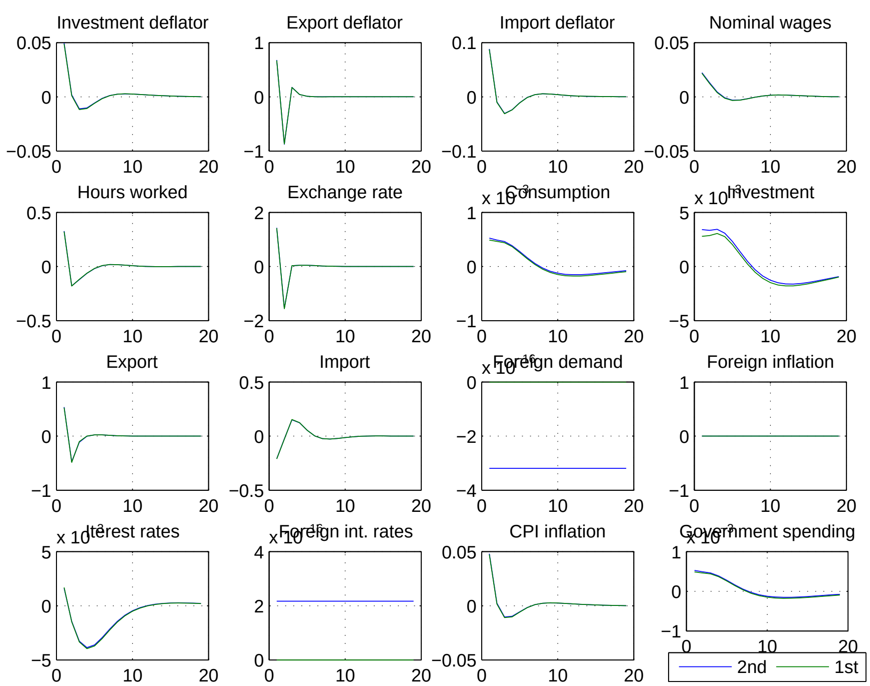

Figure 23: Premium shock

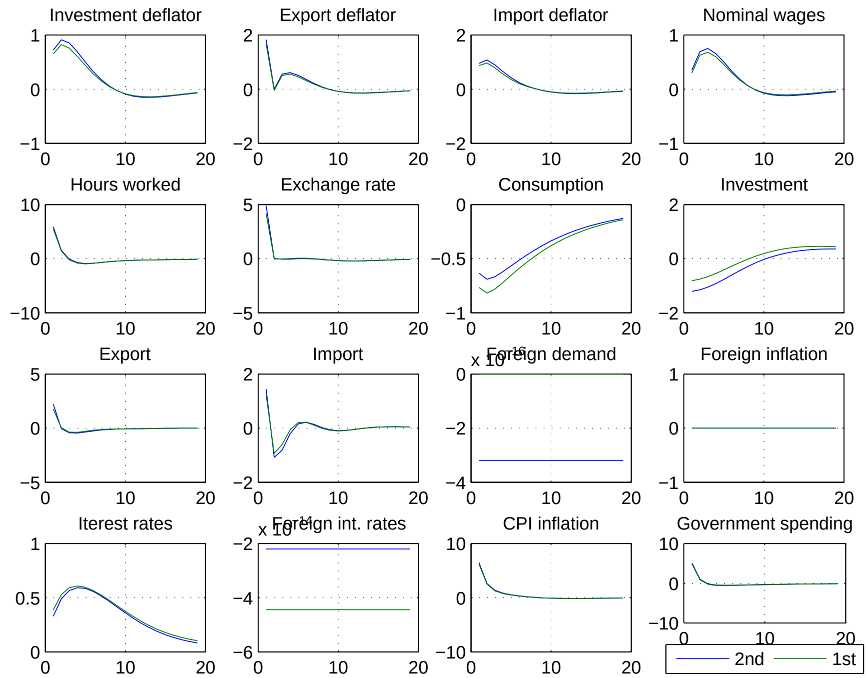

Figure 24: Regulated prices shock

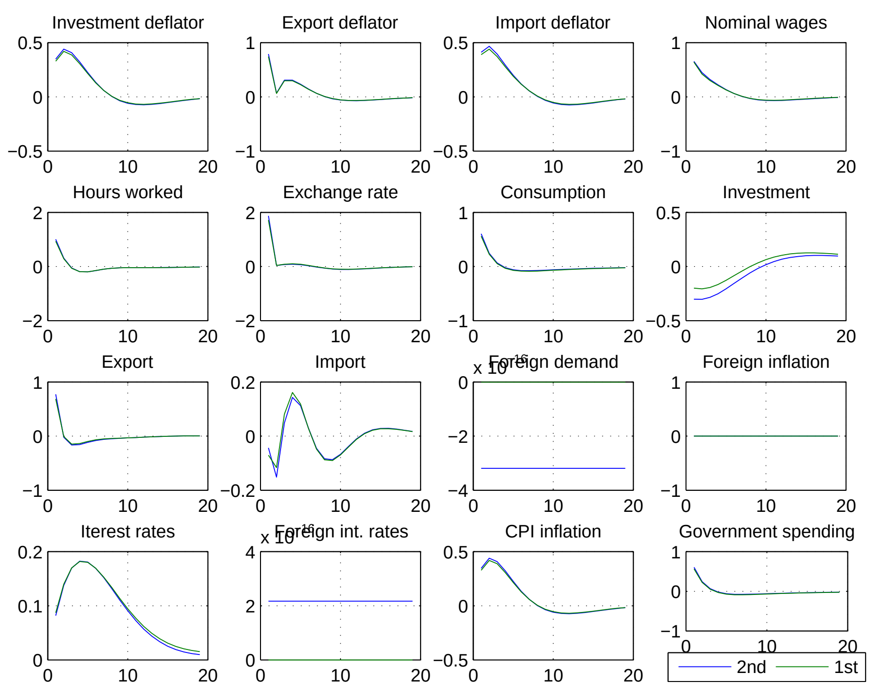

Figure 25: Preference shock

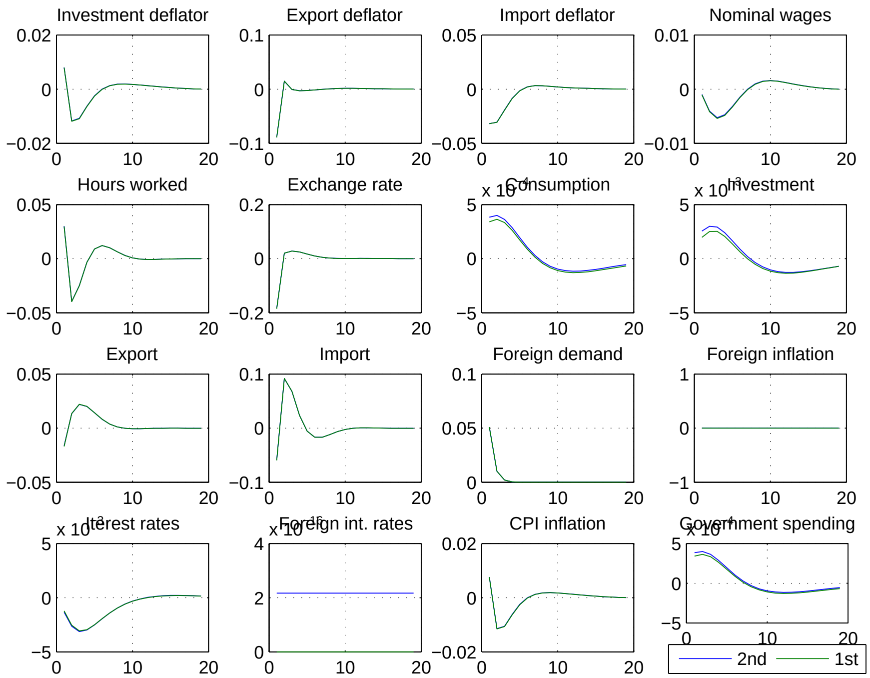

Figure 26: Export specific technology shock

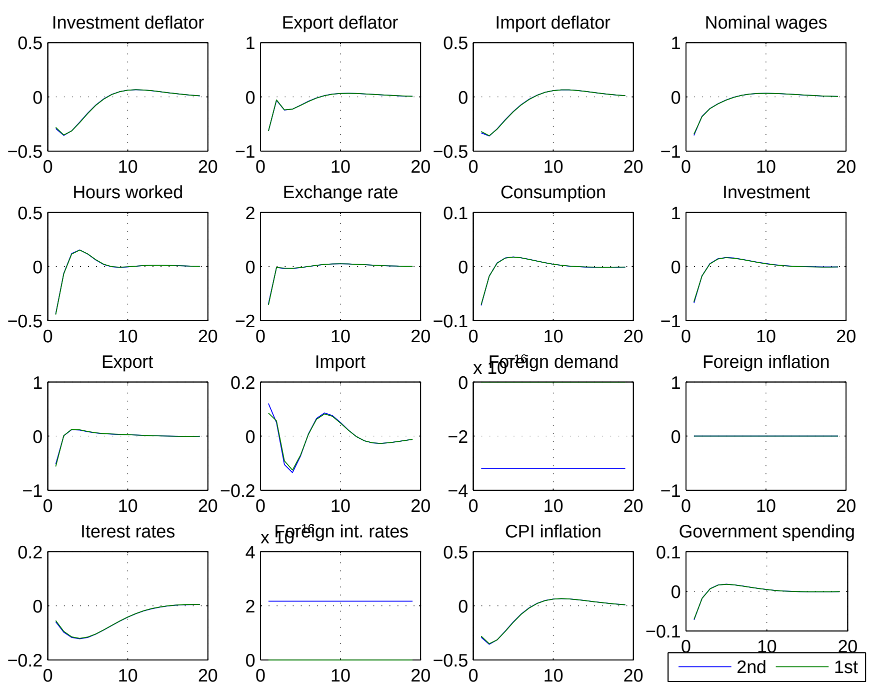

Figure 27: Wedge euler shock

5.6. Model Verification

In Section 3, we present results of model estimation. The estimation can be understood as a tool for ensuring the model consistence with data. Because the estimation itself is not sufficient enough, we need to employ additional tools to test a model quality. This section presents some model applications for the Czech economy and other important tools to check model properties and its forecasting performance. We focus mainly on data filtering and forecasting since these are important criteria how to evaluate the model. Moreover, we use structural shock decompositions, decompositions of endogenous variables into observations or forecast decompositions into individual factors with respect to the steady state.[31]

By means of data filtration, we estimate and analyze past realizations of structural shocks that lie behind the evolution of observable time series.[32] Analyzing the decomposition of structural shocks allows us to assess the current state of the economy and interpret observed economic data.

We do not aim to explain the overall evolution of observable time series. Instead, we allow for measurement errors in the model implemented on levels (thus we have trends in the model). Such setting is able to capture middle-term and possibly long-term dynamics without information noise. ME can be understood as permanent judgments for the model filtration. The size of each error differs according its precise measurement, frequency of revisions, methodology changes etc. Thus, interest rates, exchange rate, or inflations are in fact measured without errors (or with a small sizes). National accounts data, on the contrary, with considerable errors. To sum up, a model-consistent data filtration (subtracting of noise from observables) should improve an analytic message of data since the model would be able to preserve fundamental intra-temporal as well as inter-temporal links among variables.

After data filtration, we proceed to forecasting with the model. We carry out forecasting exercises via simple model simulations conditioned on exogenized foreign variables.[33]

The data filtration and model forecast are shown in Figures 28 - 31.

To evaluate model performance, we carry out various decompositions:

- A structural shock decomposition serves for comparing our intuition with model filtration. Model endogenous variables can be decomposed into individual structural shocks and thus we should be able to observe which structural shocks are responsible for a deviation of a given variable from the steady state in each period. An example of this tool is shown in Figure 32 which presents a decomposition of implied aggregate technology. The Figure indicates a dominant role of investment-specific technology over a TFP technology in the model. This result can be also seen from the model filtration since there is a downward trend of real investment from 2006Q1 to 2009 whereas the filtered consumption is stable. This analysis points out a potential shortage

[31] The detailed description and discussion of these tools can be found in Andrle [2].

[32] In line with Andrle et al. [2], we use a version of diffuse Kalman smoother since our measurement series may not be stationary.

[33] In other words, the values of foreign variables are fixed. Moreover, we assume that trajectories of foreign variables are anticipated.

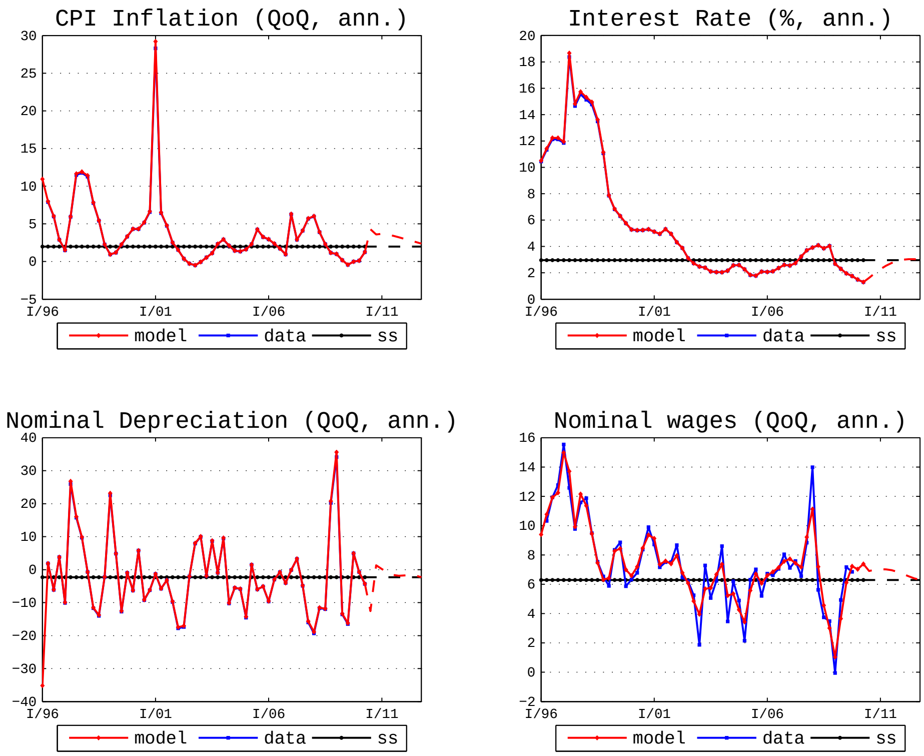

Figure 28: Filtration and Forecast

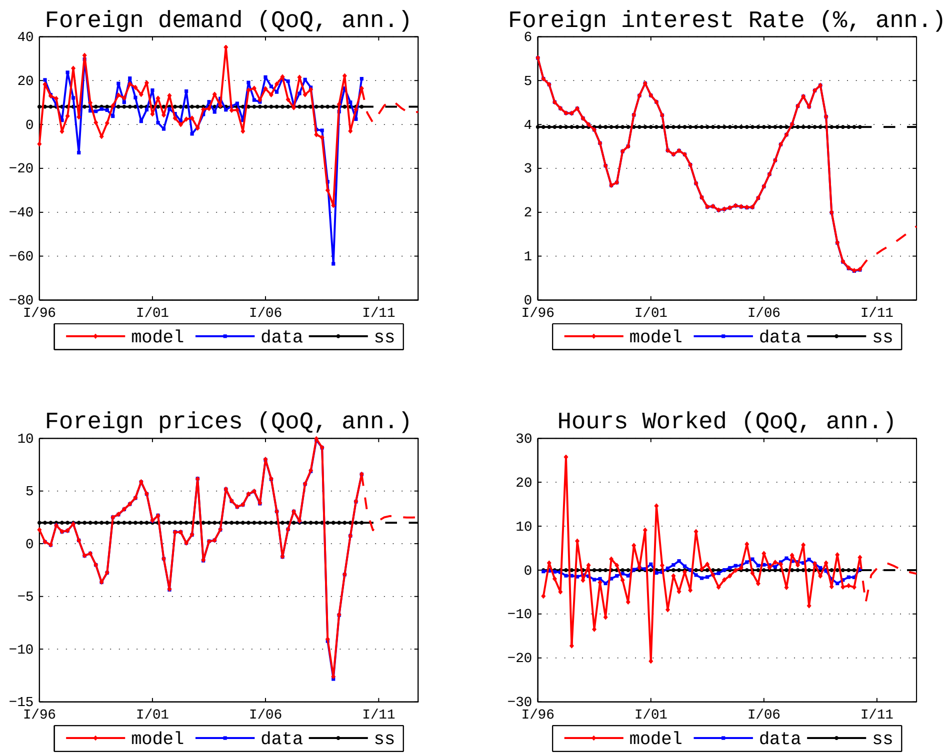

Figure 29: Filtration and Forecast

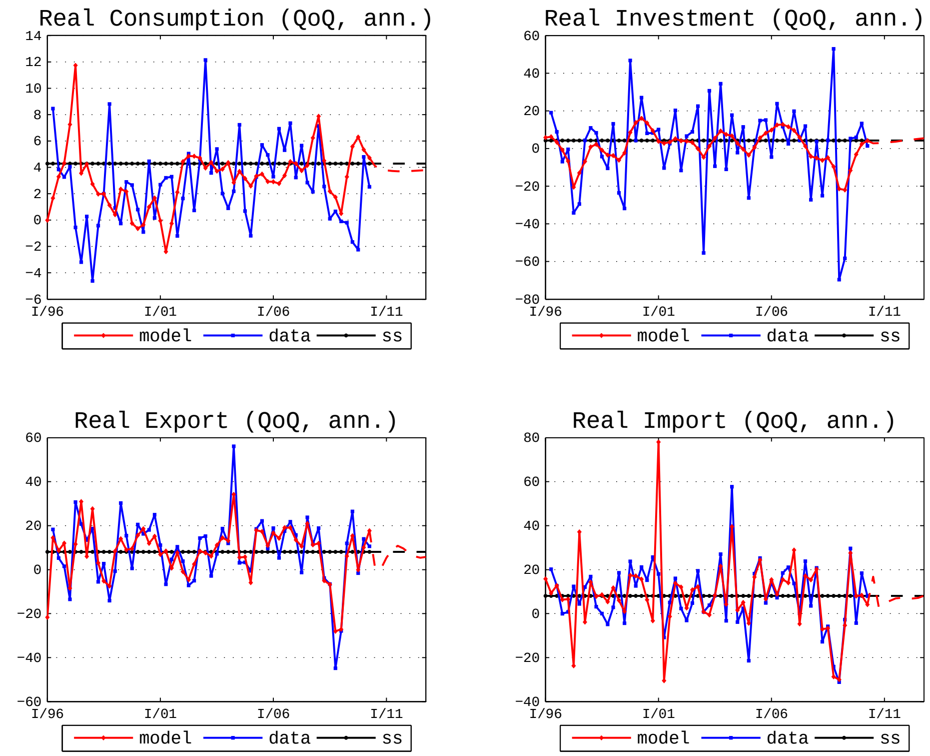

Figure 30: Filtration and Forecast

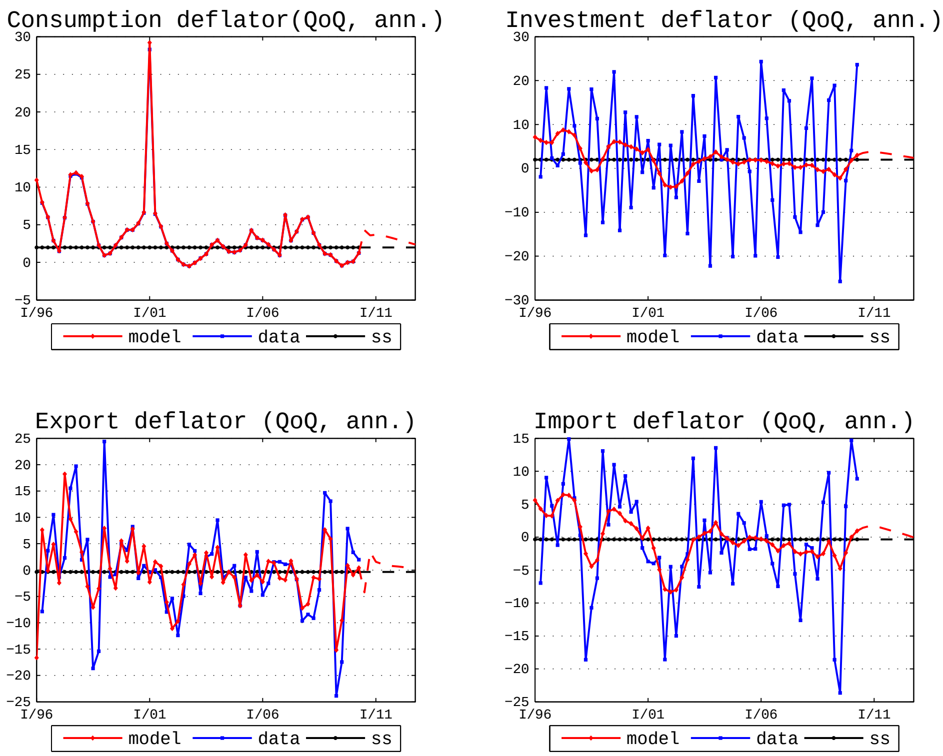

Figure 31: Filtration and Forecast

of the model since the filtered series is below its steady state. This example greatly shows how this type of analysis is necessary.

- The decomposition of an endogenous variable's deviation from its steady state into individual observables is used to evaluate which observation changes (and their size) contribute to changes of a model filtration. We can also evaluate contributions of new period observations.

- The decomposition of model forecasts shows factors that are deviating the forecasted variables from their steady-states. The Figure 33 shows the domestic interest rate forecast decomposition from the steady state into individual factors. This deviation is mainly given by low foreign interest rates in the Europe. This influence is only partly compensated by setting of initial conditions.

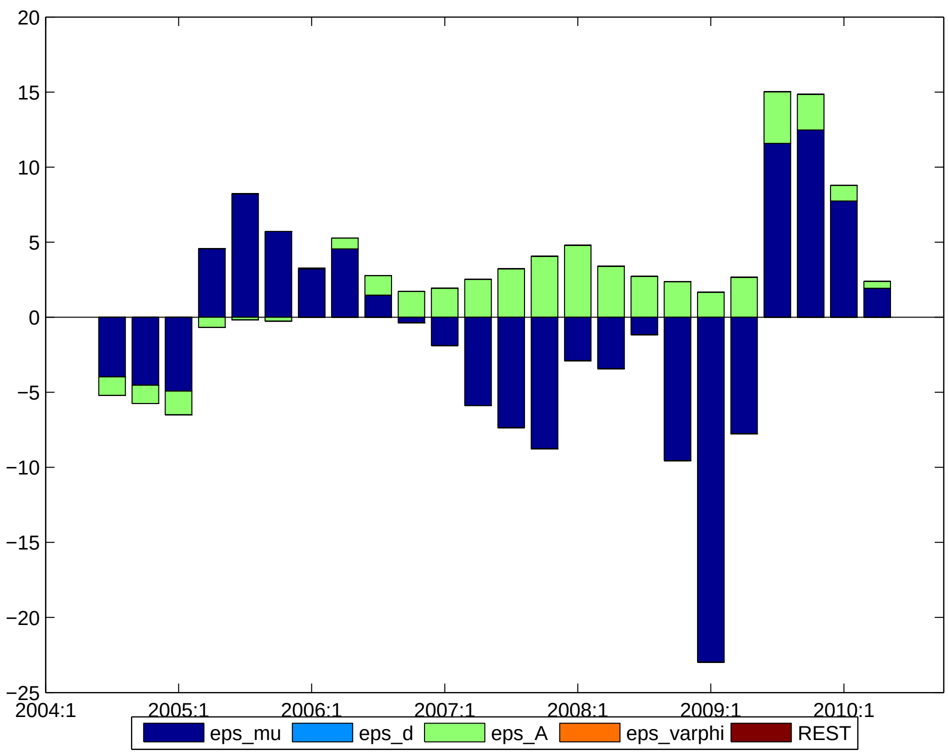

Figure 32: Decomposition of Implied Aggregate Technology Growth

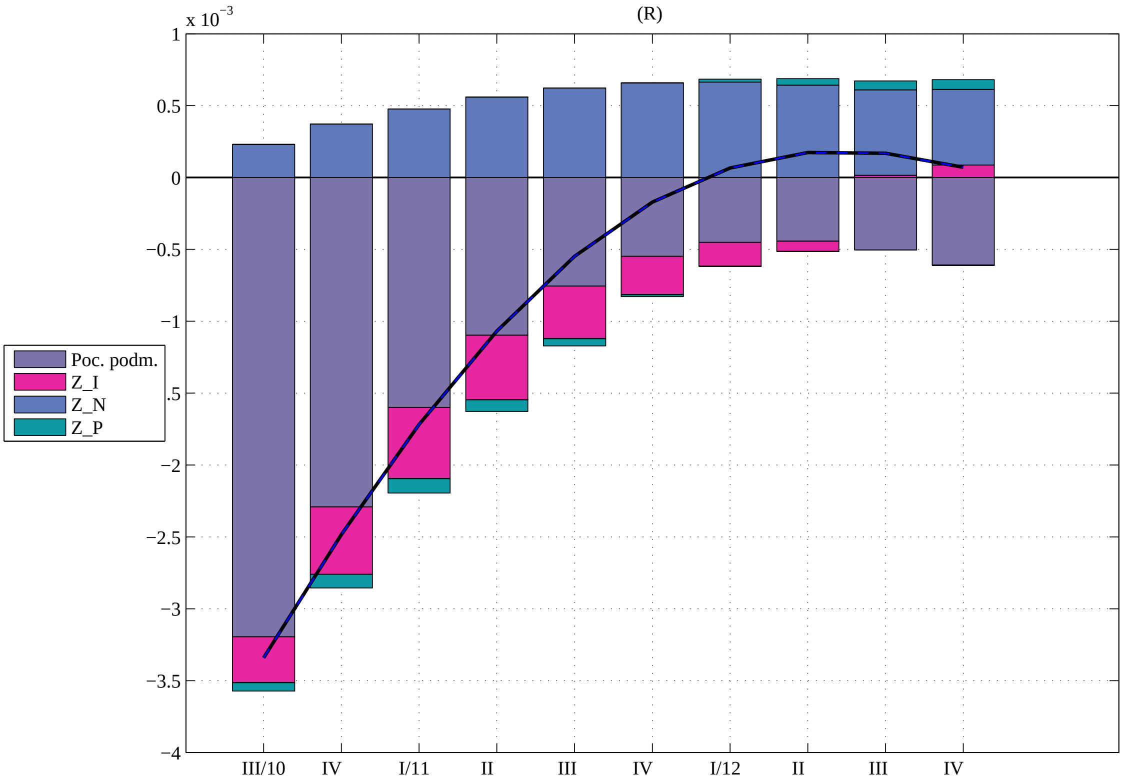

Figure 33: Decomposition of Interest Rate Forecast

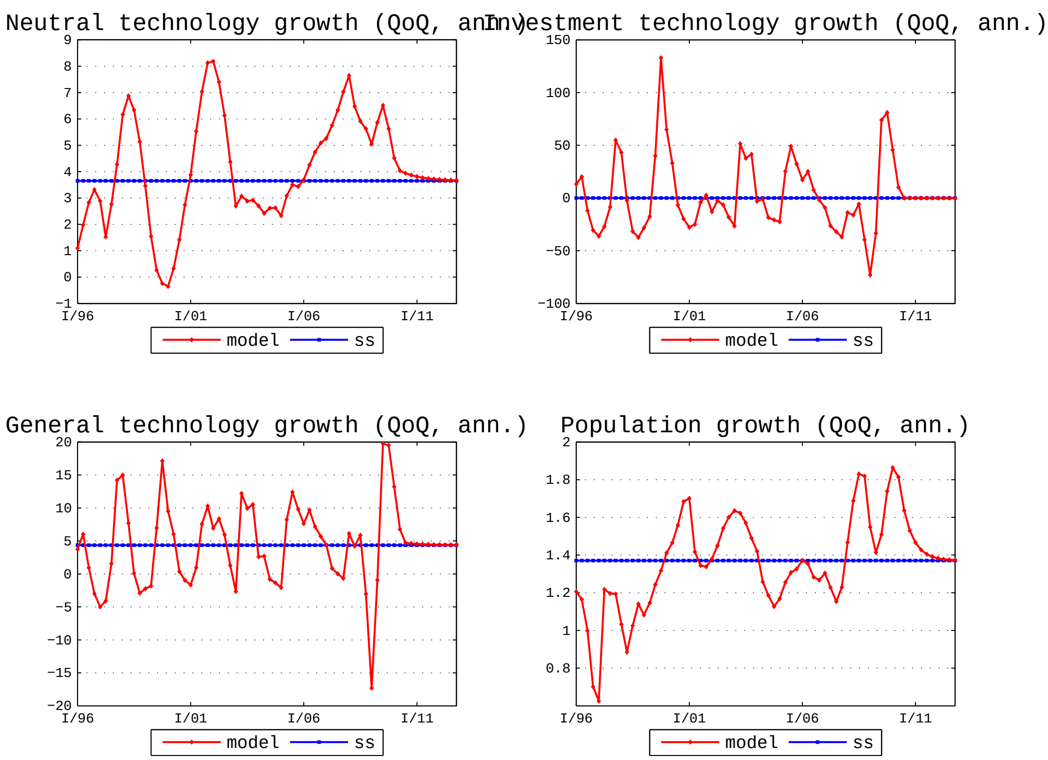

Figure 34: Main technologies

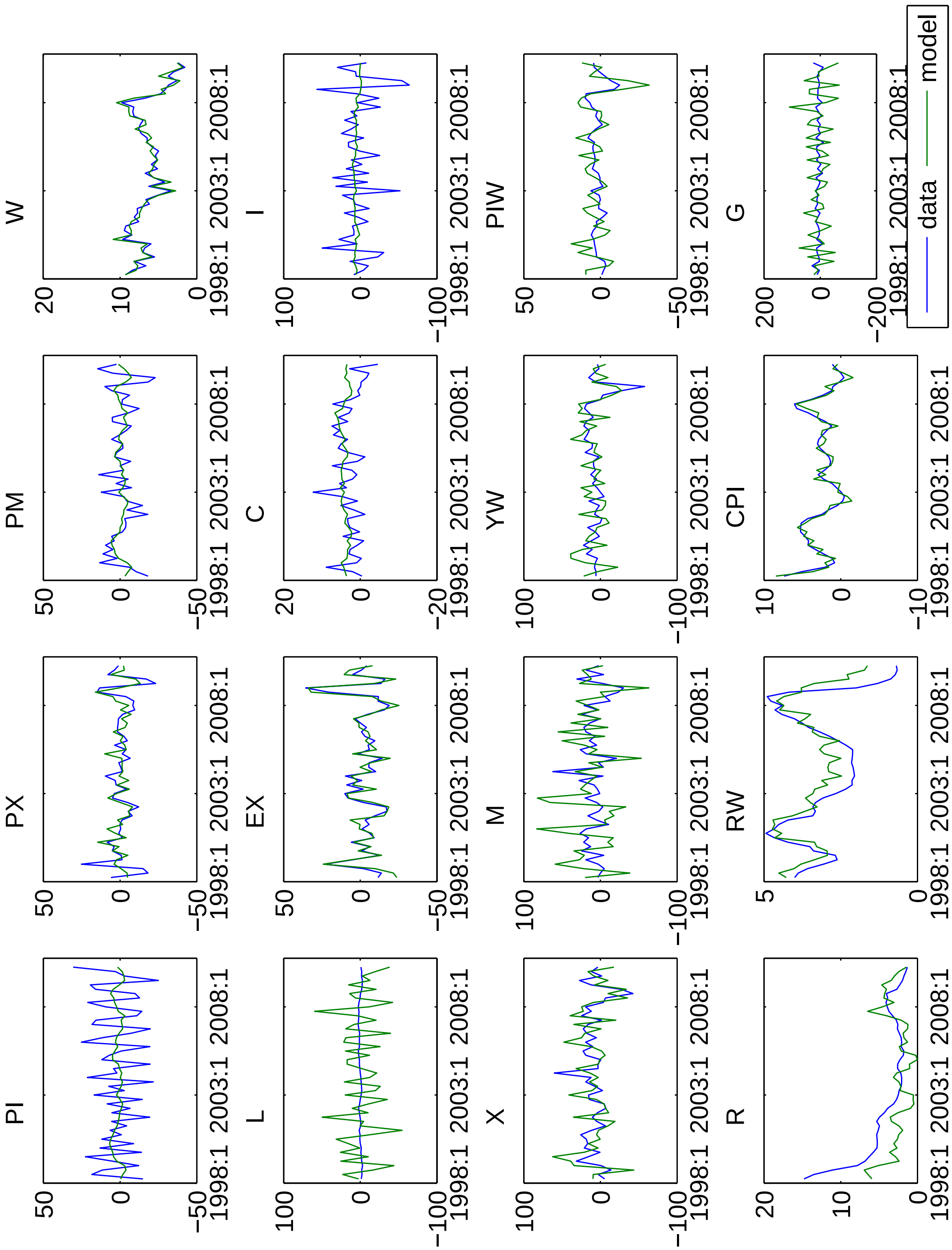

Figure 35: Nonlinear filter