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PARAMETER DRIFTING IN THE SECOND ORDER APPROXIMATED MODEL

Abstract:

We investigate possible drifting of structural parameters in an estimated small open economy DSGE model. We run a particle filter on the second-order approximated model. A nonlinear filtration is necessary for model agents to capture nontrivial influence of structural changes on model agents' behaviour. In our previous work, we found out that models designed for monetary policy analysis and forecasting of an economy that is undergoing structural changes must include time-varying parameters. These parameters can be either structural parameters, or other exogenous processes (technologies) showing the specific characteristics of individual sectors. From the perspective of monetary policy analysis and forecasting, it seems more convenient to assume that the structural parameters are stable and use sectoral technologies owing to their aggregate form. In this work, we confirm the previous results, but on the second-order approximated model. We add that parameters capturing import intensity drift even in framework with added technologies. We also analyse the source of such drifting through decomposition of drifting parameters into observables. We found out that their drifting is mainly caused by export and import prices and exchange rate changes. We also confirm that the technology capturing Balassa Samuelson effect is, at least in its cyclical component, a reflection of time-varying import intensity parameters.

Abstrakt:

V práci zkoumáme možnou časovou proměnlivost strukturálních parametrů u DSGE modelů, které jsou aproximovány druhým řádem. V tomto případě je nutné použití nelineárního filtru, aby bylo možné zachytit netriviální vliv strukturálních změn na chování agentů v modelu. V naší předchozí práci jsme zjistili, že současné modely jsou často dovybaveny dalšími exogenními procesy (technologiemi), aby byly schopné replikovat pozorovaná data. Tyto technologie můžeme chápat jako časově proměnné parametry a umožňují zachytit některá specifika (sektorová a často v čase se měnící) v chování ekonomiky. Naší hypotézou bylo, že zavedené technologie jsou odrazem časové proměnlivosti strukturálních parametrů, např. technologie zachycující Balassa-Samuelsonův efekt je, minimálně ve své cyklické složce, odrazem pohybu parametrů dovozní náročnosti. Závěrem tvrdíme, že zavedení technologických procesù umožňuje zachytit časovou proměnlivost parametrů a přiblížit tak modely více k datům i bez nutnosti explicitní práce s časově proměnnými parametry a



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nelineárními filtracemi. V této práci potvrzujeme předtím dosažené výsledky, navíc provádíme analýzu zdrojů pohybu časově proměnných parametrů prostřednictvím dekompozice endogenních proměnných do pozorovaných časových řad. Zjistili jsme, že pohyb parametrů je nejvíce určen vývojem cen vývozů a dovozů a vývojem směnného kurzu.

Recenzoval: Ing. Jan Brůha, Ph.D. doc. Ing. Luboš Komárek, Ph.D., MSc., MBA

Introduction

The research of parameter estimation in DSGE models with rational expectations is the core of contemporary macroeconomics. Dynamic systems are viewed *as a laboratory where economic theories can be confronted with real data, and where researchers can find recommendations for policy makers* (see Fernández-Villaverde and Rubio-Ramírez, 2007). Institutions where DSGE models are used for monetary policy analysis include the Federal Reserve System in the USA (Erceg, Guerrieri and Gust, 2005), the European Central Bank (Christoffel, Coenen and Warne, 2007), the central bank of Canada (Murchison and Rennison, 2006), the central bank of Sweden (Adolfson et al., 2005), the central bank of Spain (Andrés, Burriel and Estrada, 2006), Reserve bank of New Zealand (Beneš et al., 2009) and Czech National Bank (Andrle et al. 2009). Similar research departments can also be found at prestigious European and American universities and research institutions.

We analyse the possible drifting of structural parameters in a complex DSGE model estimated on Czech data. To do this, we let structural parameters drift and subsequently identify their trajectories via a non-linear filtration method on the second-order approximation of the model. This issue might be more important for developing countries, because such economies go through frequent structural changes. Twenty years after the "Velvet Revolution", Czech economy remains on a convergence path towards the more developed countries of Western Europe. It has been hit by various shocks, some of which have generated structural changes. Fast liberalization of capital account of the Czech Balance of Payments before the entry of the Czech Republic to OECD, economic slump at the end of the 1990s or European Union (EU) entry are good examples. In this respect, a question naturally arises about the projection of these changes and shocks into the parameters of the DSGE model and the drift in those parameters over time.

In our previous work¹ we found out that *models* designed for monetary policy analysis and forecasting of an economy that is undergoing structural changes must include time-varying parameters. These parameters can be either structural parameters or other exogenous processes (technologies) showing specific characteristics of individual sectors. From the perspective of monetary policy analysis and forecasting, it seems more convenient to assume that the structural parameters are stable and use sectoral technologies owing to their aggregate form. In this work, we would like to carry out the analysis on the second order approximated model.

¹See [40] for details.

2 Solution

This work deals with model construction. Consequently, the main methodology is constructivism.² We must always bear in mind the limits of this approach. Models are consistent structures, and so their analytical conclusions result from the assumptions we use to construct them. Positivism is then used to interpret the results and their implications for economic theory and economic policy.

Models are simplified descriptions of real systems. We create them in order to learn about principles that exist in reality. We want to analyse past decisions and predict the future. When evaluating modelling results, we should be aware of the assumptions of the model. All models are subjective from this point of view. The mainstream of economic theory teaches us that we should regard economic systems as dynamic systems that are capable of returning to equilibrium after being hit by stochastic shocks. Economic agents are viewed as being rational in their behaviour and expectations. The general modelling framework is therefore built from DSGE models with rational expectations.

2.1 Model Development

The model development is a crucial part of the process of time-varying parameter estimation. We need to have sufficiently rich and complex model to avoid capturing a possible model misspecification through time-varying parameters (see [16]). We start building the model in the paper [43]. The paper provides an analysis of the baseline New Keynesian DSGE model for a closed economy. The model is estimated with a Bayesian technique using quarterly Eurozone data. The estimation results are discussed and compared with related papers. We analyze the behaviour of the model via impulse responses to unanticipated shocks, which an analysis we carry out in three steps. First, in order to understand essential model mechanisms, we analyze the model behaviour without any rigidities. Then, we add separately real and nominal rigidities to investigate their impacts within the model. In our contribution [42], we analyze fully anticipated shocks in a medium-scale closed economy DSGE model and compare them to unanticipated shocks. An anticipated shock might bring about significant adjustments of agents' behaviour before the moment a shock hits the economy. From a practical policy side, modelling of anticipated shocks can be a useful tool for a policy forecasting since we admit relevance of rational expectations in reality.

Then we construct a suitable model and check its properties (see [41] and [40]). For our purposes, we need a sufficiently rich small open economy (SOE) model to fit the main Czech stylized facts. The model is based mainly on the framework of Burriel et al. [8] which we alter in several sectors (export, government, monetary policy) to be closer to the Czech data.³ Then, we incorporate several technology processes into the model, following Andrle et al. [2]. The technologies are designed directly for the stylized facts of the Czech economy.⁴

² Ullm's understanding of the scientific reflection as a construction without metaphysical existence, see [24].

³For the analysis, we use post-1996 data. The data set before 1996 is incomplete.

⁴Some sectors of the framework are modelled in a parsimonious way, as we do not aim to use the model for regular forecasting. In such case, the model would converge more closely to the CNB's g3 model developed for this objective.



Figure 1: Structure of the Model

2.2 Time-varying Parameter Estimation

Our motivation for time-varying parameter estimation is to identify changes in DSGE model structure related implicitly with changes in productivity between tradable and nontradable sectors, with deregulation, reexports, government transformation or changes of monetary regimes (e.g. transition of fixed exchange rate to floating in the Czech economy in 1997). We expect that it must be represented by an essential change of DSGE structure which is given by parameters values.⁵

The method of time-varying parameter estimation is described in details in [41]. After the Bayesian estimation and checks, we allow several structural parameters to drift in time. We extend the methods proposed in [16]. First, we run the Kalman filter on the first-order approximated model. This procedure is the two-step problem where the former consists of adding technologies into the framework whereas the latter in endogenizing deep parameters via AR processes. Adding technologies helps us to get the model to data. Endogenizing deep parameters then shows us a time-varying structure of the model. Second, we run a Particle filter on the second-order

⁵In works [45], [46] and [47], we find out that the method of time-varying parameter estimation by Bootstrap filter can be applied only to models which are well-determined. In case of overcompleteness or undercompleteness the estimation fails because of mutual interactions (overcompleteness) and an insufficient base (undercompleteness). Simply said, the mutual interactions bring about instability of the estimated paths of parameters and states; they differ in every repeated experiment with the same Bootstrap filter setting. On the other hand, the poor base causes that we are not able to sufficiently fit the data. The well-determined model is recognized as stabilized estimated paths of parameters and states in every repeated Bootstrap filter estimation.

approximated model.⁶ Nonlinear filtration is necessary to capture nontrivial influence of structural changes on model agents' behaviour.

First, we carry out a simulation study to test a Particle filter. Then we filter technologies which can also be seen as time-varying parameters and filter structural parameters which were inverted to model variables. We are interested in mutual invertibility of these two possible understanding of time-varying parameters. The introduction of the trade openness and export-specific technology into the model might serve as an example. Is filtration of these technologies not only reflection of time variability of the import intensity parameters?

2.2.1 Simulation Study

We thus perform a simulation study to verify the functionality of the methodology. This study consists of several steps. First of all, we simulate data using the estimated model, and simultaneously set an arbitrary parameter as a time variable (we choose parameter n_x for the simulation study).

$$n_{x,t} = \rho_{n_x} n_{x,t-1} + (1 - \rho_{n_x}) n_{x,ss} + \xi_t^{n_x}.$$

As in the first exercise, we carry out a time-varying parameter estimation allowing the drifting of parameters when these movements are unanticipated by agents in the model. The functional form of the linearized model is obtained via the Dynare Toolbox⁷.

$$y_t = y_s + Ayh_{t-1} + Bu_t$$

where ys is the steady state value of y and $yh_t = y_t - ys$.

The next step consists of a time-varying parameter estimation allowing the drifting which is anticipated by model agents due to the higher order approximation. In such case, one needs to use a nonlinear filter because the model structure is nonlinear as well. We use the Particle filter.⁸ For obtaining the second order approximation, we again employ the Dynare Toolbox.

The second order approximation is

$$y_t = y_s + 0.5\Delta^2 + Ayh_{t-1} + Bu_t + 0.5C(y_{t-1} \otimes y_{t-1}) + 0.5D(u_t \otimes u_t) + E(y_{t-1} \otimes u_t)$$

where y_s is the steady state value of y, $yh_t = y_t - y_s$, and Δ^2 is the shift effect of the variance of future shocks.

From the simulation, we know the exact trajectory of parameter $n_{x,t}$. Through non-linear filter, we try to find the parameter path. We chose several types of time variability. Firstly, the parameter was chosen as a constant ($\rho_{n_x} = 0, \sigma_{\xi_t^{n_x}} = 0$), but the filter was set for time-varying parameter ($\rho_{n_x} = 0, \sigma_{\xi_t^{n_x}} \neq 0$). From a comparison of simulation and filtering it can be seen that the filter did not detect temporal variability. Second, time-varying parameter was simulated as a random deviation from long-term

⁶The particle filter is very similar to the Bootstrap filter. These algorithms represent the same realizations of Monte Carlo method. However, the Particle filter software is more user friendly for the second-order approximated models.

⁷See dynare manual [13]

⁸See [1] and [46].



(steady state) value ($\rho_{n_x} = 0$, $\sigma_{\xi_t^{n_x}} \neq 0$). Result for 5000 samples in each time step is again depicted in Fig. 2.

The same exercise was done for model technologies. Experiment results indicate good identification of the regulated prices technology and trade openness technology.

2.2.2 Structural Parameters

For the time-varying parameter estimation, we need to choose candidate parameters for the drifting. Our first guess comes from the incentive experiment $(n_x, n_c, n_i, \rho_g, \alpha)$ and from the Bayesian estimation. Figure 3 shows parameters posterior distributions of which are considerably bimodal⁹. The candidate is particularly the parameter for the import intensity of export because we can expect that the openness of the Czech economy was changing during the analyzed period.¹⁰

An exercise is performed in the following procedure. Gradually, each parameter is separately controlled as time-varying, and then its model filtrations are compared¹¹. We compared the Kalman filter and particle filters (see [1]), while in the case of particle filter, we still choose between the first and second approximations of the model. Be-

⁹Bimodality can also signify problems in Bayesian estimation of the model.

¹⁰In fact, we did not resist the temptation and tried to estimate all parameters as time-varying.

¹¹Technologies are switched on.



cause the particle filtrations are very time-consuming¹², we came to non-linear filtering after the linear filtration had promised an interesting result. The results are reported in Figure 4 and Figure 5. The nonlinear estimation confirms significant movements in imports intensity parameters. It is especially during 2002 and 2003 that we do identify a big increase of such parameters which indicate an increase of domestic component in producing consumption and export. Other parameters seem to be stable over time.

¹²The trajectories on Figure 5 are computed averages among 10 rerunned non-linear filtrations, where each non-linear filtration has 6000 samples for each time step.



Figure 4: Parameters suspected to be time-varying, but they are not





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To complete the analysis, we need a tool for finding out which observables are responsible for the parameter drifting. The first idea about how to verify the accuracy of filtering dwells in confrontation of the development of time-varying parameters with the data, i.e. to find the true development of import-intensive exports, and compare it with n_x trajectories, for example. Unfortunately, the Czech Statistical Office has no such time series, and furthermore, published papers on this topic do not exist either. Therefore, we decided to construct time series very simply as the share of real exports to real imports in the case of n_x parameter n_c . These time series are of course only a rough approximation of the parameters because they always consider all imports and not only that portion that was used in the production of final goods. We put up with the fact that at least the direction is intuitive, i.e. if there are increases of the import intensity, both time series have to decline, which also corresponds to the definition of parameters in the model, i.e. the lower the parameter, the higher the share of imports.

Moreover, we choose a traditional tool like endogenous variables decompositions into observables. The Figure 6 presents decompositions of deviations of time-varying parameters from their steady state into observables using linear¹³ approximation of the model. Both decompositions show great influence of the exchange rate (obs_EX). Its effect is intuitive, higher than the steady state appreciation (1999-2002 and 2004-2006) and implies lower rate parameters n_x and n_c , i.e. the share of imports in the production of exports and consumption is increasing, because imports are cheaper and vice versa. We observed strong depreciation in 1997 (Q2 6.5, Q3 4, Q3 2.5), 1999 (Q1 5.6, Q2 1.2), 2002 (Q4 2.0, Q1 2003 2.5), 2008 (Q4 5.0, Q1 2009 8.5) and related increases in n_x and n_c in these periods. Another interesting observation is that the export prices (obs_PX) and import prices (obs_PM) compensate each other in a decomposition of a specific parameter.

¹³Individual filtrations are not additive because of a nonlinear world in the case of the second order approximated model



Figure 6: Decomposition of the import intensity into observables

In the case of the second order approximated model, we employ a simple correlation analysis which shows lead and lag correlations between the drifting and observed time series. We find out a negative correlation between current exchange rate and current import intensity parameters. Higher domestic component of consumption and export implies lower import and thus positive net foreign assets, and an appreciated exchange rate. This finding is also in line with negative correlation between import and intensity parameters (mainly in case of the second order approximation). Moreover, we find out a positive correlation between future exchange rate movements and intensity parameters. The depreciation anticipation, in this case, is a strong incentive for consumption and export goods producers to increase the domestic component.

Our hypothesis that established technologies are merely a reflection of variability of structural parameters was tested in two experiments. To avoid mutual interaction, we switched on the export specific technology and switched off time-varying import intensity parameters in the first experiment; in the second experiment it was vice versa. The comparison of trajectories can be seen in Figure 7. We conclude that technology capturing Balass Samuelson effect is, at least in its cyclical component, a reflection of time-varying import intensity parameters.



Figure 7: Comparison of export specific technology and import intensity parameter in the second-order-approximated model

2.3 Monetary Policy Relevance

Our results indicate that the central banks can use sufficiently rich DSGE models for a long period without frequent recalibrations, even during later phases of economic transformation. For the case of the Czech Republic, we show that structural parameters are stable in the model with added exogenous technology processes. We can consider these technologies to be time-varying processes which capture the gradually changing behaviour of an economy. The stability of the model parameters allows to use the core model for several years till the substantial structural break occurs, or the economy's convergence moves forward.

However, if the central bank expects structural changes in the future, this assumption should be incorporated into their forecasts. In this section, we will therefore evaluate the relevance of the dissertation for monetary policy. We currently have the tools to incorporate expert assumptions into the forecasts. In this case, it is necessary to analyze the situation when the central bank is assuming a structural change. To analyze the impulse responses to anticipated shocks, as in the work [42], represents a good start.

Reactions of the Monetary Authority to an unexpected export-specific technology is described in the Appendix well enough. Recall that the export-specific technology captures the evolution of relative productivity in tradable sector to productivity in non-tradeable sector. A positive shock to this technology must necessarily lead to a rate hike despite the strong appreciation. A similar response also brings the shock to the import intensity parameter. In the event that the monetary authority is expecting a decrease in import intensity of exports, its response should be in the direction of lower rates, as shown by the expected shock to the export-specific technology and to the import intensity parameter. Contemporary low rates should offset the disinflationary pressures resulting from the current exchange rate strengthening. Appreciation of the exchange rate in turn will partly compensate for inflation at the moment of the shock realization (see Figures 8 and 9).



Figure 8: Export specific technology shock

Figure 9: Import intensity of exports parameter shock



Conclusion

We investigated possible drifting of structural parameters in an estimated small open economy DSGE model. We run a particle filter on the second-order approximated model. Nonlinear filtration is necessary for model agents to capture nontrivial influence of structural changes on model agents' behaviour. In our previous work, we found out that models designed for monetary policy analysis and forecasting of an economy that is undergoing structural changes must include time-varying parameters. These parameters can be either structural parameters or other exogenous processes (technologies) showing the specific characteristics of individual sectors. From the perspective of monetary policy analysis and forecasting, it seems more convenient to assume that the structural parameters are stable, and use sectoral technologies owing to their aggregate form. In this work, we confirmed the previous results but on the secondorder approximated model. We added to it that parameters capturing import intensity drift even in framework with added technologies. We also analysed the source of such drifting through decomposition of drifting parameters into observables. We found out that their drifting is mainly caused by export and import prices and exchange rate changes. We also confirmed that the technology capturing Balassa-Samuelson effect is, at least in its cyclical component, a reflection of time-varying import intensity parameters.

We carried out a comparison of first and second order approximated models. The biggest differences represent a positive investment technology shock that increases consumption in the case of an overheated economy, a positive foreign inflation shock that increases nominal wages and consumption in the case of economy in crisis, and positive exchange rate shocks that decrease nominal wages and consumption in case of an economy in crisis. We also explained how these results can be used for monetary policy analysis through impulse responses analysis of anticipated shocks.

Because the work deals with many issues, it offers a lot of questions and unsolved problems. In my opinion, we have not exhausted all the opportunities offered by the second order approximation. For example, it would be nice to check whether the identification of the economy's position in the business cycle would be more accurate in times of crisis in the case of the second order approximated model. Let us remember that the bigger a distance from the steady state, the worse is the approximation, and during the crisis, we were far away. It also appears that the third order approximation of a model is better suited to capture nontrivial influence of structural changes (changes of structural parameters) on model agents' behaviour. The new version of the toolbox DYNARE already offers third-order approximation as standard, and therefore it will be less difficult to verify this thesis.

The model itself offers another major challenge. It is a very simple but comprehensive form of a small open economy. In future research we will have to focus on better incorporating of labour market, government and financial frictions.

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Appendix

The Appendix presents behaviour of the model. Impulse responses are expressed as deviations from the steady state in percentage points of q-o-q growths, except for interest rates which are expressed as deviations from the steady state in percentage points.¹⁴ The shocks are of one-standard-deviation size and unanticipated. An impulse responses simulation of the first-order approximated model is easy because it does not depend on the position of the economy in the business cycle. In case of the second-order approximated model, three different impulse responses are presented: 2+ for overheated economy behaviour (Q3 2008), 2- for the economy behaviour in crisis (Q1 2009) and 2 for the economy in the 'second-order' steady-state.

¹⁴For example, one-standard-deviation neutral technology shock increases foreign demand from 2.25% q-o-q growth steady state value to 3.25% q-o-q growth. See Figure 10.



Figure 10 shows a positive **neutral technology** shock ξ_t^A of one standard deviation size. The neutral technology shock is considered to be a shock to the productivity of both the foreign economy and the small open domestic economy (information symmetry). Therefore, the shock leads to growth of foreign GDP and thus to demand growth for domestic production. The increased demand for domestic production strengthens the domestic currency in comparison to the foreign currency. Stronger exchange rate allows goods to be imported more cheaply and in greater quantity. This supports production in sectors which use imports as an input, i.e. production of consumption (both private and governmental) and investment goods. Exports fall as a result of the stronger exchange rate, but this effect is outweighed by a higher foreign demand. Nominal wages respond to higher productivity with higher-than-steady-state growth. Wages represent a cost item of all manufacturing sectors in the model, i.e. the consumption, investment and export goods manufacturing sectors. Higher wage growth is outweighed by lower import prices which are also the prices of inputs in manufacturing sectors. Consequently, consumer price inflation is below its steady-state level and the monetary authority responds by lowering interest rates.



Figure 11: Investment - specific technology

The response of the model economy to an **investment-specific technology** shock ξ^{μ}_{t} can be seen in Figure 11. This technology, together with the previous technology, makes up the aggregate technology, which is used to stationarize the model equations. For the same reason as for the neutral technology shock, the shock to investmentspecific technology has an international dimension given perfect transmission of information, and thus will be reflected in growth of foreign GDP and even in growth of foreign demand for domestic goods. This again leads to higher-than-steady-state exchange rate appreciation, although in contrast to the previous technology shock, this is a one-off appreciation, and in other periods the rate of appreciation is lower than the steady state. Growth rates of wages and price of capital¹⁵ in response to higher productivity are above the steady state, however, in this case these the effects as cost items are greater than the effect of the one-off appreciation, which implies CPI inflation above its steady state. Because the reaction of the monetary authority is forward-looking, interest rates will be increased so that the inflation target is hit at the monetary policy horizon. This shock, of course, increases real investment and then indirectly affects imports and exports. Note that in case of an overheated economy, a positive shock increases consumption.

¹⁵Change of the price of capital depends on the setting of the capital adjustment costs.



Figure 12: Population growth shock

Shock to population growth ξ_t^L , analysed in Figure 12, is another shock included in the model. This variable is also used to stationarize the model, i.e. it converts all variables to variables expressed in terms of per capita. The arrival of new workers implies growth of consumption, investment and imports, and that is because the growing population requires more consumption and higher investment, thus putting upward pressure on imports. On the contrary, wage growth pressure is lower than in the steady state because the newcomers are competitors for existing employees. The downward pressure on wages implies a lower consumer price inflation, which might, ceteris paribus, lead to lower-than-steady-state rates. In this case, however, the demand effect of newcomers is very strong, and the monetary authority is forced to prevent an overheating of the economy at the cost of failing to achieve the inflation target.



A shock to the **labour supply** ξ_t^{φ} , as depicted in Figure 13, reduces labour supply, implying a reduction in hours worked. It results, ceteris paribus, in higher-than-steady-state wages. Domestic production of intermediate goods, inputs of which are labour and capital, is reduced. The lower domestic production of intermediate goods is an input in production of final consumption (private and government) and investment goods, and is only partially substituted in the production of these goods by imported goods. Higher imports lead to a balance of payments deficit, which implies a weakening exchange rate. This supports exporters. Depreciation leads to a cost increase because imports enter the final production of all sectors with the exception of the government consumption goods production sector. Higher wages and import prices imply higher-than-steady-state consumer goods price inflation and an appropriate response of the monetary authority in the direction of higher rates. We add that the labour supply shock reduces the number of hours worked at all wage levels, and therefore works very much like a shock to population growth (see Burriel et al. in [8]).



Assuming nominal rigidities, effectiveness of a monetary policy shock is nontrivial. A one-off increase in rates, therefore, does not merely reduce inflation and has a zero effect on real variables. The response of the model economy to an unexpected **monetary policy shock** ξ_t^m is shown in Figure 14. A one-off rate increase leads, ceteris paribus, to exchange rate appreciation in accordance with the uncovered interest parity condition. This leads to a high current account deficit because a stronger exchange rate will allow more and cheaper goods to be imported, which, however, is a disadvantage for exporters. Higher interest rates reduce domestic production of intermediate goods, and, despite higher imports, production of consumer products (household and government) and investment goods falls below its steady state level. Reduction in production of domestic intermediate goods also results in a decrease in hours worked and a corresponding decline in nominal wages. Lower nominal wages, together with the stronger exchange rate, lead to lower consumer goods price inflation, so the initial intention of the monetary authority is realized.



Figure 15: Real government consumption shock

An increase in **real government consumption** caused by shock ξ_t^g is shown in Figure 15. The reaction of model agents to the growth in government spending is simplified in order to reflect the response of agents in the real economy as much as possible. In particular, higher-than-steady-state government spending is viewed as a demand shock which leads to crowding out of private consumption and private investment through a reduction of inputs in each sector. Shortage of domestic intermediate goods (government final consumption is produced exclusively from domestic intermediate goods) leads to higher imports. This results, ceteris paribus, in depreciation of currency, because the balance of payments goes into a significant deficit. Exports are partially supported, but not enough (crowding out of exports). Depreciation, coupled with higher wages, leads to increased costs in all production sectors, and thus implies a higher-than-steady-state consumer price inflation. This leads to the appropriate response of the monetary authority, namely an increase in interest rates, which is the likely reaction of the monetary authority when government spending is higher than steady state.



An increase in **foreign demand** realized by the shock $\xi_t^{y^W}$ is illustrated in Figure 16. First, we must say that foreign countries are modelled exogenously (not even as a VAR model), so a shock to foreign demand (a shock to foreign GDP growth) does not lead to an increase in foreign inflation and foreign rates. A single shock to foreign demand leads mainly to higher-than-steady-state exchange rate appreciation through a positive trade balance. Stronger exchange rate is detrimental for exporters, but this effect is outweighed by the direct effect of higher foreign demand. Due to the strong appreciation, consumer price inflation declines below the inflation target, as imports as inputs in the production of consumer goods get much cheaper. The reaction of the monetary authority is intuitive again – there is a reduction in interest rates which leads to depreciation pressure on the exchange rate in order to steer inflation to the target at the monetary policy horizon. The lower rates stimulate production in other sectors (consumption and investment) but only very slightly. There will also be moderate growth in nominal wages, but the growth in wages as a cost item cannot offset the impact of the appreciation.



Figure 17: Foreign interest rates shock

Higher-than-steady-state **foreign interest rates** are realized by the shock $\xi_t^{R^W}$, whose model implications are captured by Figure 17. The logics of uncovered interest rate parity implies that an increase in foreign rates leads to a widening differential between interest rates in the domestic and foreign economy. This automatically leads to upward pressure on domestic interest rates and significant exchange rate depreciation in the domestic economy. This weakening of the exchange rate supports domestic exporters and dampens imports. Since the export sector is highly import-intensive, production in other manufacturing sectors, i.e. consumption and investment, will be muted. Missing imports will be partially substituted by domestic intermediate goods. This will lead to growth of hours worked and also to growth of nominal wages, but the rise in domestic production of intermediate goods will initially be unable to completely offset the outflow of imports to the export-producing sector. Together with higher nominal wages, depreciation will lead to growth of prices in the final consumption goods production sector. The reaction of the monetary authority – higher nominal rates – is in line with the anticipated higher inflation.



The response of the domestic economy to an increase in **foreign inflation** above its steady state value is realized with shock $\xi_t^{\pi^W}$ and shown in Figure 18. According to the model equations showing the identity between inflation in import prices in the domestic currency and the product of inflation of foreign prices in foreign currency and the exchange rate, the endogenous exchange rate must respond by appreciating strongly. Economically, this relationship could be interpreted as the reaction of the domestic economy to growth in foreign demand as a result of increased competitiveness of the domestic economy. The domestic economy thus avoids an overheating due to increased foreign demand and imported inflation through a strengthening of the exchange rate. In this case, the exchange rate more than offsets the increase in foreign prices in foreign currency and implies a reduction in monetary policy rates so that consumer price inflation hits the inflation target at the monetary policy horizon.¹⁶ Note that a positive shock increases nominal wages and consumption in the case of the economy in crisis.

¹⁶Note that if the appreciation was not so strong, the effect of higher international prices could outweigh the effect of the stronger exchange rate and import prices in the domestic currency would rise above their steady state level. In this case, consumer price inflation would rise above its steady state level, and the monetary authority would have to raise rates.



Figure 19: Debt elastic premium shock

A one-period shock to the **debt elastic premium** is realized by shock ξ_t^{prem} and is captured by Figure 19. This shock can be viewed as a fundamental shock to the exchange rate, i.e. in the case of a positive shock it leads to depreciation of the exchange rate. Weakening of the exchange rate obviously favours domestic exporters exporting to foreign countries, so we observe an increase in domestic exports. On the other hand, imports are significantly more expensive, so they fall below their steady-state growth. Because exports are set as very import-intensive in the domestic economy, imports are replaced by domestic intermediate goods as inputs, i.e. substituted by goods produced by labour and capital. This leads to an increase in hours worked and also to growth of nominal wages. The considerably weakened exchange rate, combined with the growth of nominal wages, leads to a rise in consumer price inflation, which forces the monetary authority to increase interest rates so that inflation will hit the inflation target at the monetary policy horizon. Note that a positive shock decreases nominal wages and consumption in the case of the economy in the crisis.



Figure 20: Exchange rate shock - persistent



Figure 22 captures an unanticipated one-period positive shock to the growth of **regulated prices**, which is implemented by exogenous variable ξ_t^{aR} . This shock directly increases consumer price inflation. Consumer basket is composed of both non-regulated goods and goods prices of which are heavily regulated (rents and energy prices being good examples for the Czech Republic). The monetary authority must respond by raising interest rates so that inflation will hit the inflation target. Government consumption will increase because the income from the increased regulated prices is income of the government. Since the only input into production of government consumption goods are domestic intermediate goods, domestic intermediate goods are replaced by imports in other production sectors but not sufficiently so that they could maintain real consumption, investment and exports at their steady state growth levels (crowding-out effect). Balance of payments deficit increases, which implies exchange rate depreciation. We should add that regulated prices are approximated by an exogenous process in this model, i.e. there is no link to the observed regulated prices.



Figure 23: Intertemporal preference shock

An increase in **intertemporal preference** implemented by shock ξ_t^d is shown in Figure 23. This demand shock increases private consumption and government consumption in particular. By contrast, investment activity will be crowded out. The increased demand for inputs in consumption sectors leads to substitution of domestic intermediate goods for imports. As a result of the increased imports, balance of payments falls into deficit and exchange rate depreciates, which partly boosts export activity. On the other hand, growth in demand for domestic intermediate goods leads to an increase in demand for inputs in this sector, i.e. for labour and capital. It therefore increases number of hours worked and nominal wages. Depreciation, combined with a higher nominal wage, increases inflation in prices of final consumption, to which the monetary authority reacts by increasing monetary policy rates. We should add that this shock alongside with the shock to government consumption are the only domestic demand shocks in the model. The difference between them lies in the crowding-out effect. While the shock to government consumption also crowds out private domestic consumption, the domestic demand shock crowds out only private investment.



Figure 24: Export specific technology shock

Growth in **export-specific technology** realized by shock ξ_t^{aX} is shown in Figure 24.¹⁷ Growth in this technology increases productivity in tradable goods production sector relative to non-tradable goods production sector, but the less productive workers in the non-tradable sector will demand an increase in wages so that the wage gap between the tradable and non-tradable sectors in the domestic economy does not increase. This, of course, leads to inflation because the wage growth in the non-tradable sector is not supported by a corresponding increase in productivity. On the other hand, the exchange rate appreciates because the tradable sectors are able to produce more goods at lower cost, so the balance of payments moves into surplus, which implies a strengthening of the exchange rate. The response of the monetary authority is ambiguous and depends on whether the effect of exchange rate or higher wages prevails at the monetary policy horizon. In our case, the effect of stronger exchange rate prevails and the monetary authority sets lower rates.

¹⁷This technology should capture the relative effectiveness of the tradable sector w.r.t. the non-tradable sector in the domestic economy. This effect is known as the Harrod-Balassa-Samuelson effect. See more in Andrle et al. in [2].



In transition economies, there is a difference between real output growth and real interest rates. In the Czech Republic, for example, real GDP growth is around 5%, while real interest rates are hovering around 1% (the difference between 3% nominal rates and 2% inflation). This difference is due to poorly developed financial markets in emerging economies. An increase in this difference in the **Euler equation** is simulated by a positive shock ξ_t^{euler} and the response of domestic economy is shown in Figure 25. This shock mainly reduces domestic nominal interest rates and increases future inflation so that the difference between real economic output and real interest rates widens. At the same time, it increases the shadow price of wealth in the economy, which results in decline in domestic consumption (private and government) and private investment.