

MATHEMATICS TEST (variant A)

For each of the following tasks, choose the correct item (the only one item is correct) and its corresponding code (a, b, c, d, or e) write on the answer sheet.

Task 1. Find an equivalent form of the expression

$$\frac{1}{x + \sqrt{1 + x^2}}.$$

- a) $x^{-1} + (1 + x^2)^{-1/2}$ b) $\left[x + (1 + x^2)^{1/2} \right]^{-1}$ c) $x + (1 + x^2)^{-1}$ d) $\frac{1}{1 + 2x}$ e) $x - \sqrt{1 + x^2}$

Task 2. For $t \geq 0$ find an equivalent form of the expression

$$\left(t^{\frac{1}{3}} + t^{\frac{1}{6}} \right) \cdot \left(t^{\frac{1}{3}} - t^{\frac{1}{6}} \right).$$

- a) $\sqrt[3]{t}$ b) $\sqrt[3]{t^2} - \sqrt[3]{t}$ c) \sqrt{t} d) 1 e) $\left(\sqrt[3]{t} - \sqrt[6]{t} \right)^2$

Task 3. Find an equivalent form of the expression

$$\operatorname{tg} x + \operatorname{cotg} x.$$

- a) $\sin x \cos x$ b) $\frac{1}{\sin x + \cos x}$ c) 1 d) $\frac{2}{\sin 2x}$ e) $\frac{1}{\sin x} + \frac{1}{\cos x}$

Task 4. Let $i^2 = -1$. Find a simple form of

$$\frac{i^{2019}}{i - 1}.$$

- a) $(i - 1)/2$ b) $1/2$ c) $1 - i$ d) $\sqrt{2}/2$ e) $(1 + i)/2$

Task 5. What is the smallest period of the function $y = \operatorname{tg} 4x$?

- a) 4π b) 2π c) π d) $\pi/2$ e) $\pi/4$

Task 6. The composition $F(x) = (f \circ g \circ h)(x) = f(g(h(x)))$ of functions $f(x) = \sqrt{x + 1}$, $g(x) = x^2 - 1$ and $h(x) = \sin x$ is:

- a) $F(x) = \sqrt{\sin(x^2 + 1) + 1}$ b) $F(x) = \sin \sqrt{x^2 - 1}$ c) $F(x) = \sin x$ d) $F(x) = \sin |x|$ e) $F(x) = |\sin x|$

Task 7. What is the domain of the function

$$y = \frac{1}{\ln(1 - |x|)} ?$$

- a) $(-\infty, -1) \cup (1, \infty)$ b) All reals c) $(-1, 1)$ d) $(-1, 0) \cup (0, 1)$ e) $(0, \infty)$

Task 8. The function $y = \frac{1-x}{x}$ is:

- a) even b) odd c) increasing for positive x d) positive for positive x e) bounded for large x

Task 9. At the birthday celebration, 45 clinks with wine glasses were heard. How many people were there? (We assume that each person clinks each other person once.)

- a) 7 b) 8 c) 9 d) 10 e) 11

Task 10. The probability of some car accident in Brno during the day is 90% every day. What is the probability that in two days in a row, there will be just one day without car accident?

- a) 9% b) 10% c) 12% d) 16% e) 18%

Task 11. What is the sum of all integers from -50 to 100 ?

- a) 2655 b) 3200 c) 5050 d) 3775 e) 3025

Task 12. What is the solution of the inequality $\ln(1 - 2x) \geq 0$?

- a) $x \in (-\infty, \infty)$ b) $x > 0$ c) $x \leq 0$ d) $x \in (0, 1]$ e) $x \geq 1$

Task 13. What is the solution of the equation $\frac{(n-1)!}{(n-3)!} = 2\binom{9}{7}$ for $n \in \mathbb{N}$?

- a) $n = 6$ b) $n = 7$ c) $n = 8$ d) $n = 9$ e) $n = 10$

Task 14. How many solutions are there for the equation $\cos^2 x = 2 + \sin^2 x$ in domain \mathbb{R} ?

- a) 1 b) 2 c) 3 d) infinitely many solutions e) no solution

Task 15. Let $0 < a < b$. Decide which of the following statements is true.

- a) $a^b < b^a$ b) $a\%$ from $b > b\%$ from a c) $a\%$ from $b = b\%$ from a d) $a\%$ from $b < b\%$ from a e) none

Task 16. The area of the rectangle $ABCD$ is 30 cm^2 . Let E be the midpoint of the side AB , F be the midpoint of the side BC , G be the midpoint of the side CD and H be the midpoint of the side AD . What is the area of the quadrilateral $EFGH$?

- a) 20 cm^2 b) 15 cm^2 c) 10 cm^2 d) 7.5 cm^2 e) not possible to determine

Task 17. Let ABC be a triangle with sides $a = 3$, $b = 4$, $c = 5$. The smallest interior angle α of ABC satisfies:

- a) $\sin \alpha = \frac{3}{3+4+5}$ b) $\sin \alpha = \frac{4}{5}$ c) $\sin \alpha = \frac{3}{4}$ d) $\sin \alpha = \frac{3}{5}$ e) $\sin \alpha = \frac{1}{5}$

Task 18. Let $u = (1, 2, 1)$ and $v = (2, 1, 3)$ be vectors in the space. Find the vector w , which is perpendicular to both vectors u and v .

- a) $w = (1, -3, 5)$ b) $w = (5, -3, -1)$ c) $w = (5, -1, -3)$ d) $w = (-3, 3, 1)$ e) $w = (-3, -1, 5)$

Task 19. What is the equation of the line passing through the points $A = [1, 7]$ and $B = [-1, 3]$?

- a) $x+2y-15=0$ b) $-x+2y-13=0$ c) $2x+y-9=0$ d) $-2x+y-9=0$ e) $2x-y+5=0$

Task 20. From the following equations, choose the equation of the parabola.

- a) $x^2 - y^2 + 8y - 12 = 0$ b) $x^2 + y^2 + 8y - 12 = 0$ c) $x + y^2 + 8y - 12 = 0$ d) $x + y + 8y - 12 = 0$ e) $(x-4)(y-2) = 0$