

MATHEMATICS TEST (variant A)

For each of the following tasks, choose the correct item (the only one item is correct) and its corresponding code (a, b, c, d, or e) write on the answer sheet.

Task 1. Find the greatest number of the following numbers:

- a) 2^{10} b) $\log(100^{10})$ c) 10^2 d) $\binom{10}{2}$ e) $1 + 2 + 3 + \dots + 10$

Task 2. Simplify the expression

$$\frac{2xy - y^2}{xy^2 - 4x^3}.$$

- a) $\frac{y}{2x^2 - y}$ b) $-\frac{y}{x(2x + y)}$ c) $-\frac{1}{2x + y}$ d) $-\frac{y}{x}(2x - y)$ e) $\frac{x}{2x^2 - y^2}$

Task 3. Find the simple form of the expression

$$\left(\frac{a}{b} - \frac{b}{a}\right) : \left(\frac{b}{a} - \frac{a}{b}\right).$$

- a) $\frac{a}{b}$ b) $\frac{b}{a}$ c) a d) b e) -1

Task 4. Let $i^2 = -1$. Which of the following numbers is a real number?

- a) $\sqrt{-16}$ b) $\sqrt[3]{i}$ c) $1 - i^7$ d) $\frac{1 - i}{1 + i}$ e) $\frac{(1 - i)^2}{(1 + i)^2}$

Task 5. What is the domain of the function

$$y = \frac{e^{\sin(1/x)}}{\sqrt[3]{1 - x}} ?$$

- a) $(0, 1)$ b) $(-\infty, 1)$ c) $(-\infty, 0) \cup (0, \infty)$ d) $(-\infty, 1) \cup (1, \infty)$ e) $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

Task 6. The function $y = \left(\frac{1}{a}\right)^{x-a}$ decreases on its domain for the parameter:

- a) $a \in (0, 1) \cup (1, \infty)$ b) $a \in (0, 1)$ c) $a \in (0, \infty)$ d) $a \in (1, \infty)$ e) $a = 1$

Task 7. Only one of the following function is odd. Which one?

- a) $y = x(x^4 + 1)$ b) $y = \tan(x + 1)$ c) $y = \sqrt{x^2 + 1}$ d) $y = \log(x^2 + x + 1)$ e) $y = x \sin x$

Task 8. For which values of the parameter a does the graph of function $y = \left(a - \frac{1}{a}\right)x^2 + 8x + 9a$ have some (nonempty) intersection with axis x ?

- a) $a \in (-\infty, -5/3] \cup [5/3, \infty)$ b) $a \in [-5/3, 0) \cup (0, 5/3]$ c) $a \in [-1, 0) \cup [1, \infty)$ d) $a \in (-\infty, 25/9]$ e) $a \in [25/9, \infty)$

Task 9. In the plain there are 7 points such that no three of them are collinear. How many different triangles can we construct? (The vertices of these triangles are considered points.)

- a) 21 b) 28 c) 35 d) 42 e) 210

Task 10. We have two boxes. In every box there are 100 numbers from 1 to 100. We randomly pick one number from every box. What is the probability that the product of these number will be an even number?

- a) 1/100 b) 1/10 c) 1/4 d) 1/2 e) 3/4

Task 11. Indian king decided to reward the inventor of the chess game as follows. He said: "I put one grain of wheat on the first field of the chessboard, two grains of wheat on the second field, four grains on the third field and so on. Thus, for each following field, I double the grains from the previous field. All wheat this way placed on the chessboard will be yours." How many grains does inventor receive?

- a) 64^2 b) $2^{64} - 1$ c) 2^{64} d) $2^{64} + 1$ e) $64!$

Task 12. Peter has thrown 10 pairs of white and 10 pairs of black socks of the same size in a box. Randomly he takes out (without looking) some of them. How many socks must he take to be sure that he has two pairs of the same color?

- a) 4 b) 5 c) 6 d) 7 e) 8

Task 13. How many solutions are there for the equation $|x - 3| = -3 - |x|$ in domain \mathbb{R} ?

- a) 3 b) 2 c) 1 d) infinitely many solutions e) no solution

Task 14. How many solutions are there for the equation $\sqrt{x+1} = \sqrt{3x+7} - 2$ in domain \mathbb{R} ?

- a) 3 b) 2 c) 1 d) infinitely many solutions e) no solution

Task 15. By increasing the side of the square, its area increases by 69 %. By what percentage did the side of the square increase?

- a) by 13 % b) by 23 % c) by 30 % d) by 33 % e) by 69 %

Task 16. What is the equation of the line passing through the points $A = [-1, 4]$ a $B = [1, 3]$?

- a) $-x + 2y - 5 = 0$ b) $x + 2y - 7 = 0$ c) $-2x + y - 1 = 0$ d) $2x - y + 6 = 0$ e) $x + 2y + 7 = 0$

Task 17. Let c be the size of the longest side of right-angled isosceles triangle. What is the circumference of this triangle?

- a) $2c$ b) $\sqrt{3}c$ c) $\sqrt{2}c$ d) $(1 + \sqrt{2})c$ e) $\left(\frac{1 + \sqrt{2}}{\sqrt{2}}\right)c$

Task 18. Let $u = (1, 1, 2)$ and $v = (2, 3, 5)$ be vectors in the space. Find the vector w , which is perpendicular to both vectors u and v .

- a) $w = (-1, 1, -1)$ b) $w = (1, -1, -1)$ c) $w = (1, 1, -1)$ d) $w = (1, -1, 1)$ e) $w = (1, 1, 1)$

Task 19. The height of cylinder is the same as its radius. Moreover, the value of the volume of the cylinder is the same as the value of its surface (considered in cm^3 , resp. cm^2). What is the size of the radius?

- a) 1 cm b) 2 cm c) 3 cm d) 4 cm e) this problem has no solution

Task 20. A circle has the equation $2x^2 - 8x + 2y^2 + 12y - 24 = 0$. What is its centre C ?

- a) $C = [-2, 3]$ b) $C = [2, -3]$ c) $C = [3, -2]$ d) $C = [-3, 2]$ e) $C = [2, 2]$