List of integrals of exponential functions

The following is a list of integrals of exponential functions. For a complete list of Integral functions, please see the list of integrals.

Indefinite integrals

Indefinite integrals are antiderivative functions. A constant (the constant of integration) may be added to the right hand side of any of these formulas, but has been suppressed here in the interest of brevity.

$$\int e^x \, \mathrm{d}x = e^x$$

$$\int e^{cx} \, \mathrm{d}x = \frac{1}{c} e^{cx}$$

$$\int a^{cx} \, \mathrm{d}x = \frac{1}{c \cdot \ln a} a^{cx} \text{ for } a > 0,\ a \neq 1$$

$$\int x e^{cx} \, \mathrm{d}x = \frac{e^{cx}}{c^2}(cx - 1)$$

$$\int x^2 e^{cx} \, \mathrm{d}x = e^{cx}\left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right)$$

$$\int x^n e^{cx} \, \mathrm{d}x = \frac{1}{c} x^n e^{cx} - \frac{n}{c}\int x^{n-1} e^{cx} \mathrm{d}x = \left(\frac{\partial}{\partial c}\right)^n \frac{e^{cx}}{c}$$

$$\int \frac{e^{cx}}{x} \, \mathrm{d}x = \ln|x| + \sum_{n=1}^{\infty} \frac{(cx)^n}{n \cdot n!}$$

$$\int \frac{e^{cx}}{x^n} \, \mathrm{d}x = \frac{1}{n-1}\left(-\frac{e^{cx}}{x^{n-1}} + c\int \frac{e^{cx}}{x^{n-1}} \, \mathrm{d}x\right) \qquad (\text{for } n \neq 1)$$

$$\int e^{cx} \ln x \, \mathrm{d}x = \frac{1}{c} e^{cx} \ln|x| - \operatorname{Ei}(cx)$$

$$\int e^{cx} \sin bx \, \mathrm{d}x = \frac{e^{cx}}{c^2 + b^2}(c \sin bx - b \cos bx)$$

$$\int e^{cx} \cos bx \, \mathrm{d}x = \frac{e^{cx}}{c^2 + b^2}(c \cos bx + b \sin bx)$$

$$\int e^{cx} \sin^n x \, \mathrm{d}x = \frac{e^{cx} \sin^{n-1} x}{c^2 + n^2}(c \sin x - n \cos x) + \frac{n(n-1)}{c^2 + n^2}\int e^{cx} \sin^{n-2} x \, \mathrm{d}x$$

$$\int e^{cx} \cos^n x \, \mathrm{d}x = \frac{e^{cx} \cos^{n-1} x}{c^2 + n^2}(c \cos x + n \sin x) + \frac{n(n-1)}{c^2 + n^2}\int e^{cx} \cos^{n-2} x \, \mathrm{d}x$$

$$\int x e^{cx^2} \, \mathrm{d}x = \frac{1}{2c} e^{cx^2}$$

$$\int e^{-cx^2} \, \mathrm{d}x = \sqrt{\frac{\pi}{4c}} \operatorname{erf}(\sqrt{c}x) \text{(} \operatorname{erf} \text{is the Error function)}$$

$$\int x e^{-cx^2} \, \mathrm{d}x = -\frac{1}{2c} e^{-cx^2}$$

$$\int \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \, \mathrm{d}x = -\frac{1}{2}\left(\operatorname{erf} \frac{-x+\mu}{\sigma\sqrt{2}}\right)$$

$$\int e^{x^2}\,\mathrm{d}x = e^{x^2}\left(\sum_{j=0}^{n-1} c_{2j}\,\frac{1}{x^{2j+1}}\right) + (2n-1)c_{2n-2}\int \frac{e^{x^2}}{x^{2n}}\,\mathrm{d}x \quad \text{valid for } n>0,$$

where $c_{2j} = \frac{1\cdot 3\cdot 5\cdots(2j-1)}{2^{j+1}} = \frac{(2j)\,!}{j!\,2^{2j+1}}$.

$$\int \underbrace{x^{x^{\cdot^{\cdot^{x}}}}}_{m} dx = \sum_{n=0}^{m} \frac{(-1)^n(n+1)^{n-1}}{n!}\Gamma(n+1,-\ln x) + \sum_{n=m+1}^{\infty}(-1)^n a_{mn}\Gamma(n+1,-\ln x) \qquad (\text{for } x>0)$$

where $a_{mn} = \begin{cases} 1 & \text{if } n=0, \\ \frac{1}{n!} & \text{if } m=1, \\ \frac{1}{n}\sum_{j=1}^{n} j a_{m,n-j}a_{m-1,j-1} & \text{otherwise} \end{cases}$

and $\Gamma(x,y)$ is the Gamma Function

$$\int \frac{1}{ae^{\lambda x}+b}\,\mathrm{d}x = \frac{x}{b} - \frac{1}{b\lambda}\ln\left(ae^{\lambda x}+b\right)$$ when $b \neq 0$, $\lambda \neq 0$, and $ae^{\lambda x}+b>0$.

$$\int \frac{e^{2\lambda x}}{ae^{\lambda x}+b}\,\mathrm{d}x = \frac{1}{a^2\lambda}\left[ae^{\lambda x}+b-b\ln\left(ae^{\lambda x}+b\right)\right]$$ when $a \neq 0$, $\lambda \neq 0$, and $ae^{\lambda x}+b>0$.

Definite integrals

$$\int_0^1 e^{x\cdot\ln a+(1-x)\cdot\ln b}\,\mathrm{d}x = \int_0^1 \left(\frac{a}{b}\right)^x\cdot b\,\mathrm{d}x = \int_0^1 a^x\cdot b^{1-x}\,\mathrm{d}x = \frac{a-b}{\ln a - \ln b}$$ for

$a>0,\ b>0,\ a\neq b$, which is the logarithmic mean

$$\int_0^\infty e^{ax}\,\mathrm{d}x = \frac{1}{a}(a<0)$$

$$\int_0^\infty e^{-ax^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a}} \quad (a>0)$$ (the Gaussian integral)

$$\int_{-\infty}^\infty e^{-ax^2}\,\mathrm{d}x = \sqrt{\frac{\pi}{a}} \quad (a>0)$$

$$\int_{-\infty}^\infty e^{-ax^2}e^{-2bx}\,\mathrm{d}x = \sqrt{\frac{\pi}{a}}e^{\frac{b^2}{a}} \quad (a>0)$$ (see Integral of a Gaussian function)

$$\int_{-\infty}^\infty xe^{-a(x-b)^2}\,\mathrm{d}x = b\sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^\infty x^2e^{-ax^2}\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a^3}} \quad (a>0)$$

$$\int_0^\infty x^n e^{-ax^2}\,\mathrm{d}x = \begin{cases} \frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)/a^{\frac{n+1}{2}} & (n>-1, a>0) \\ \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}} & (n=2k, k \text{ integer}, a>0) \\ \frac{k!}{2a^{k+1}} & (n=2k+1, k \text{ integer}, a>0) \end{cases}$$ (!! is the double factorial)

$$\int_0^\infty x^n e^{-ax}\,\mathrm{d}x = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}} & (n>-1, a>0) \\ \frac{n!}{a^{n+1}} & (n=0,1,2,\ldots, a>0) \end{cases}$$

$$\int_0^\infty e^{-ax}\sin bx\,\mathrm{d}x = \frac{b}{a^2+b^2} \quad (a>0)$$

$$\int_0^\infty e^{-ax}\cos bx\,\mathrm{d}x = \frac{a}{a^2+b^2} \quad (a>0)$$

$$\int_0^{\infty} xe^{-ax} \sin bx \, \mathrm{d}x = \frac{2ab}{(a^2+b^2)^2} \quad (a>0)$$

$$\int_0^{\infty} xe^{-ax} \cos bx \, \mathrm{d}x = \frac{a^2-b^2}{(a^2+b^2)^2} \quad (a>0)$$

$\int_0^{2\pi} e^{x\cos\theta} d\theta = 2\pi I_0(x)$ (I_0 is the modified Bessel function of the first kind)

$$\int_0^{2\pi} e^{x\cos\theta + y\sin\theta} d\theta = 2\pi I_0\left(\sqrt{x^2+y^2}\right)$$

References

- Wolfram Mathematica Online Integrator (http://integrals.wolfram.com/index.jsp)
- V. H. Moll, The Integrals in Gradshteyn and Ryzhik (http://www.math.tulane.edu/~vhm/Table.html)

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