## CHAPTER 3 National Income: Where It Comes From and Where It Goes

## Questions for Review

- 1. The factors of production and the production technology determine the amount of output an economy can produce. The factors of production are the inputs used to produce goods and services: the most important factors are capital and labor. The production technology determines how much output can be produced from any given amounts of these inputs. An increase in one of the factors of production or an improvement in technology leads to an increase in the economy's output.
- 2. When a firm decides how much of a factor of production to hire or demand, it considers how this decision affects profits. For example, hiring an extra unit of labor increases output and therefore increases revenue; the firm compares this additional revenue to the additional cost from the higher wage bill. The additional revenue the firm receives depends on the marginal product of labor (MPL) and the price of the good produced (P). An additional unit of labor produces MPL units of additional output, which sells for P dollars per unit. Therefore, the additional revenue to the firm is  $P \times MPL$ . The cost of hiring the additional unit of labor is the wage W. Thus, this hiring decision has the following effect on profits:

$$\Delta$$
Profit =  $\Delta$ Revenue -  $\Delta$ Cost  
=  $(P \times MPL) - W$ .

If the additional revenue,  $P \times MPL$ , exceeds the cost (W) of hiring the additional unit of labor, then profit increases. The firm will hire labor until it is no longer profitable to do so—that is, until the MPL falls to the point where the change in profit is zero. In the equation above, the firm hires labor until  $\Delta$ profit = 0, which is when  $(P \times MPL) = W$ .

This condition can be rewritten as:

$$MPL = W/P$$
.

Therefore, a competitive profit-maximizing firm hires labor until the marginal product of labor equals the real wage. The same logic applies to the firm's decision regarding how much capital to hire: the firm will hire capital until the marginal product of capital equals the real rental price.

3. A production function has constant returns to scale if an equal percentage increase in all factors of production causes an increase in output of the same percentage. For example, if a firm increases its use of capital and labor by 50 percent, and output increases by 50 percent, then the production function has constant returns to scale.

If the production function has constant returns to scale, then total income (or equivalently, total output) in an economy of competitive profit-maximizing firms is divided between the return to labor,  $MPL \times L$ , and the return to capital,  $MPK \times K$ . That is, under constant returns to scale, economic profit is zero.

- 4. A Cobb-Douglas production function function has the form  $F(K,L) = AK^{\alpha}L^{1-\alpha}$ . The text showed that the parameter  $\alpha$  gives capital's share of income. (Since income equals output for the overall economy, it is also capital's share of output.) So if capital earns one-fourth of total income, then a = 0.25. Hence,  $F(K,L) = AK^{0.25}L^{0.75}$ .
- 5. Consumption depends positively on disposable income—the amount of income after all taxes have been paid. The higher disposable income is, the greater consumption is.

The quantity of investment goods demanded depends negatively on the real interest rate. For an investment to be profitable, its return must be greater than its cost. Because the real interest rate measures the cost of funds, a higher real interest rate makes it more costly to invest, so the demand for investment goods falls.

- 6. Government purchases are a measure of the dollar value of goods and services purchased directly by the government. For example, the government buys missiles and tanks, builds roads, and provides services such as air traffic control. All of these activities are part of GDP. Transfer payments are government payments to individuals that are not in exchange for goods or services. They are the opposite of taxes: taxes reduce household disposable income, whereas transfer payments increase it. Examples of transfer payments include Social Security payments to the elderly, unemployment insurance, and veterans' benefits.
- 7. Consumption, investment, and government purchases determine demand for the economy's output, whereas the factors of production and the production function determine the supply of output. The real interest rate adjusts to ensure that the demand for the economy's goods equals the supply. At the equilibrium interest rate, the demand for goods and services equals the supply.
- 8. When the government increases taxes, disposable income falls, and therefore consumption falls as well. The decrease in consumption equals the amount that taxes increase multiplied by the marginal propensity to consume (MPC). The higher the MPC is, the greater is the negative effect of the tax increase on consumption. Because output is fixed by the factors of production and the production technology, and government purchases have not changed, the decrease in consumption must be offset by an increase in investment. For investment to rise, the real interest rate must fall. Therefore, a tax increase leads to a decrease in consumption, an increase in investment, and a fall in the real interest rate.

## **Problems and Applications**

- 1. a. According to the neoclassical theory of distribution, the real wage equals the marginal product of labor. Because of diminishing returns to labor, an increase in the labor force causes the marginal product of labor to fall. Hence, the real wage falls.
  - b. The real rental price equals the marginal product of capital. If an earthquake destroys some of the capital stock (yet miraculously does not kill anyone and lower the labor force), the marginal product of capital rises and, hence, the real rental price rises.
  - c. If a technological advance improves the production function, this is likely to increase the marginal products of both capital and labor. Hence, the real wage and the real rental price both increase.
- 2. A production function has decreasing returns to scale if an equal percentage increase in all factors of production leads to a smaller percentage increase in output. For example, if we double the amounts of capital and labor, and output less than doubles, then the production function has decreasing returns to scale. This may happen if there is a fixed factor such as land in the production function, and this fixed factor becomes scarce as the economy grows larger.

A production function has increasing returns to scale if an equal percentage increase in all factors of production leads to a larger percentage increase in output. For example, if doubling inputs of capital and labor more than doubles output, then the production function has increasing returns to scale. This may happen if specialization of labor becomes greater as the population grows. For example, if only one worker builds a car, then it takes him a long time because he has to learn many different skills, and he must constantly change tasks and tools. But if many workers build a car, then each one can specialize in a particular task and become very fast at it.

3. a. A Cobb–Douglas production function has the form  $Y = AK^{\alpha}L^{1-\alpha}$ . The text showed that the marginal products for the Cobb–Douglas production function are:

$$MPL = (1 - \alpha)Y/L.$$

$$MPK = \alpha Y/K.$$

Competitive profit-maximizing firms hire labor until its marginal product equals the real wage, and hire capital until its marginal product equals the real rental rate. Using these facts and the above marginal products for the Cobb-Douglas production function, we find:

$$W/P = MPL = (1 - \alpha)Y/L.$$
  
 $R/P = MPK = \alpha Y/K.$ 

Rewriting this:

$$(W/P)L = MPL \times L = (1 - \alpha)Y.$$
  
 $(R/P)K = MPK \times K = \alpha Y.$ 

Note that the terms (W/P)L and (R/P)K are the wage bill and total return to capital, respectively. Given that the value of  $\alpha = 0.3$ , then the above formulas indicate that labor receives 70 percent of total output (or income), which is (1 - 0.3), and capital receives 30 percent of total output (or income).

b. To determine what happens to total output when the labor force increases by 10 percent, consider the formula for the Cobb–Douglas production function:

$$Y = AK^{\alpha}L^{1-\alpha}$$

Let  $Y_1$  equal the initial value of output and  $Y_2$  equal final output. We know that  $\alpha = 0.3$ . We also know that labor L increases by 10 percent:

$$Y_1 = AK^{0.3}L^{0.7}.$$
  
 $Y_2 = AK^{0.3}(1.1L)^{0.7}.$ 

Note that we multiplied L by 1.1 to reflect the 10-percent increase in the labor force.

To calculate the percentage change in output, divide  $Y_2$  by  $Y_1$ :

$$\frac{Y_2}{Y_1} = \frac{AK^{0.3}(1.1L)^{0.7}}{AK^{0.3}L^{0.7}}$$
$$= (1.1)^{0.7}$$
$$= 1.069.$$

That is, output increases by 6.9 percent.

To determine how the increase in the labor force affects the rental price of capital, consider the formula for the real rental price of capital R/P:

$$R/P = MPK = \alpha AK^{\alpha - 1}L^{1 - \alpha}.$$

We know that  $\alpha = 0.3$ . We also know that labor (L) increases by 10 percent. Let  $(R/P)_1$  equal the initial value of the rental price of capital, and  $(R/P)_2$  equal the final rental price of capital after the labor force increases by 10 percent. To find  $(R/P)_2$ , multiply L by 1.1 to reflect the 10-percent increase in the labor force:

$$(R/P)_1 = 0.3AK^{-0.7}L^{0.7}.$$
  
 $(R/P)_2 = 0.3AK^{-0.7}(1.1L)^{0.7}.$ 

The rental price increases by the ratio

$$\frac{(R/P)_2}{(R/P)_1} = \frac{0.3AK^{-0.7}(1.1L)^{0.7}}{0.3AK^{-0.7}L^{0.7}}$$
$$= (1.1)^{0.7}$$
$$= 1.069.$$

So the rental price increases by 6.9 percent.

To determine how the increase in the labor force affects the real wage, consider the formula for the real wage W/P:

$$W/P = MPL = (1 - \alpha)AK^{\alpha}L^{-\alpha}$$
.

We know that  $\alpha = 0.3$ . We also know that labor (L) increases by 10 percent. Let  $(W/P)_1$  equal the initial value of the real wage and  $(W/P)_2$  equal the final value of the real wage. To find  $(W/P)_2$ , multiply L by 1.1 to reflect the 10-percent increase in the labor force:

$$(W/P)_1 = (1 - 0.3)AK^{0.3}L^{-0.3}.$$
  
 $(W/P)_2 = (1 - 0.3)AK^{0.3}(1.1L)^{-0.3}.$ 

To calculate the percentage change in the real wage, divide  $(W/P)_2$  by  $(W/P)_1$ :

$$\begin{split} \frac{(W/P)_2}{(W/P)_1} &= \frac{(1-0.3)AK^{0.3}(1.1L)^{-0.3}}{(1-0.3)AK^{0.3}L^{-0.3}} \\ &= (1.1)^{-0.3} \\ &= 0.972. \end{split}$$

That is, the real wage falls by 2.8 percent. c. We can use the same logic as in part (b) to set

$$Y_1 = AK^{0.3}L^{0.7}.$$
  
 $Y_2 = A(1.1K)^{0.3}L^{0.7}.$ 

Therefore, we have:

$$\frac{Y_2}{Y_1} = \frac{A(1.1K)^{0.3}L^{0.7}}{AK^{0.3}L^{0.7}}$$
$$= (1.1)^{0.3}$$
$$= 1.029.$$

This equation shows that output increases by about 3 percent. Notice that  $\alpha$  < 0.5 means that proportional increases to capital will increase output by less than the same proportional increase to labor.

Again using the same logic as in part (b) for the change in the real rental price of capital:

$$\frac{(R/P)_2}{(R/P)_1} = \frac{0.3A(1.1K)^{-0.7}L^{0.7}}{0.3AK^{-0.7}L^{0.7}}$$
$$= (1.1)^{-0.7}$$
$$= 0.935.$$

The real rental price of capital falls by 6.5 percent because there are diminishing returns to capital; that is, when capital increases, its marginal product falls.

Finally, the change in the real wage is:

$$\frac{(W/P)_2}{(W/P)_1} = \frac{0.7A(1.1K)^{0.3}L^{-0.3}}{0.7AK^{0.3}L^{-0.3}}$$
$$(1.1)^{0.3}$$
$$= 1.029.$$

Hence, real wages increase by 2.9 percent because the added capital increases the marginal productivity of the existing workers. (Notice that the wage and output

have both increased by the same amount, leaving the labor share unchanged—a feature of Cobb–Douglas technologies.)

d. Using the same formula, we find that the change in output is:

$$\frac{Y_2}{Y_1} = \frac{(1.1A)K^{0.3}L^{0.7}}{AK^{0.3}L^{0.7}}$$
= 1.1.

This equation shows that output increases by 10 percent. Similarly, the rental price of capital and the real wage also increase by 10 percent:

$$\frac{(R/P)_2}{(R/P)_1} = \frac{0.3(1.1A)K^{-0.7}L^{0.7}}{0.3AK^{-0.7}L^{0.7}}$$
$$= 1.1.$$

$$\frac{(W/P)_2}{(W/P)_1} = \frac{0.7(1.1A)K^{0.3}L^{-0.3}}{0.7AK^{0.3}L^{-0.3}}$$
$$= 1.1.$$

4. Labor income is defined as

$$\frac{W}{P} \times L = \frac{WL}{P}$$
.

Labor's share of income is defined as

$$\left(\frac{WL}{P}\right)/Y = \frac{WL}{PY}.$$

If this ratio is about constant at, say, a value of 0.7, then it must be the case that W/P = 0.7\*Y/L. This means that the real wage is roughly proportional to labor productivity. Hence, any trend in labor productivity must be matched by an equal trend in real wages—otherwise, labor's share would deviate from 0.7. Thus, the first fact (a constant labor share) implies the second fact (the trend in real wages closely tracks the trend in labor productivity)

- 5. a. According to the neoclassical theory, technical progress that increases the marginal product of farmers causes their real wage to rise.
  - b. The real wage for farmers is measured as units of farm output per worker. The real wage is  $W/P_E$ , and this is equal to ( $\sqrt{\frac{w}{w}}$ )( $\sqrt{\frac{w}{w}}$ ).
  - c. If the marginal productivity of barbers is unchanged, then their real wage is unchanged.
  - d. The real wage for barbers is measured as haircuts per worker. The real wage is  $W/P_B$ , and this is equal to (\$/worker)/(\$/haircut).
  - e. If workers can move freely between being farmers and being barbers, then they must be paid the same wage *W* in each sector.
  - f. If the nominal wage W is the same in both sectors, but the real wage in terms of farm goods is greater than the real wage in terms of haircuts, then the price of haircuts must have risen relative to the price of farm goods. We know that W/P = MPL so that  $W = P \times MPL$ . This means that  $P_FMPL_F = P_HMPL_B$ , given that the nominal wages are the same. Since the marginal product of labor for barbers has not changed and the marginal product of labor for farmers has risen, the price of a haircut must have risen relative to the price of the farm output. If we put it in growth-rate terms, then the growth of the farm price + the growth of the marginal product of the farm labor = the growth of the haircut price.
  - g. Both groups benefit from technological progress in farming.

6. a. The marginal product of labor *MPL* is found by differentiating the production function with respect to labor:

$$\begin{split} MPL &= \frac{dY}{dL} \\ &= \frac{1}{3} \, K^{1/3} H^{1/3} L^{-2/3}. \end{split}$$

An increase in human capital will increase the marginal product of labor because more human capital makes all the existing labor more productive.

b. The marginal product of human capital *MPH* is found by differentiating the production function with respect to human capital:

$$MPH = \frac{dY}{dH}$$

$$= \frac{1}{3} K^{1/3} L^{1/3} H^{-2/3}.$$

An increase in human capital will decrease the marginal product of human capital because there are diminishing returns.

c. The labor share of output is the proportion of output that goes to labor. The total amount of output that goes to labor is the real wage (which, under perfect competition, equals the marginal product of labor) times the quantity of labor. This quantity is divided by the total amount of output to compute the labor share:

Labor Share = 
$$\frac{(\frac{1}{3}K^{1/3}H^{1/3}L^{\cdot 2/3})L}{K^{1/3}H^{1/3}L^{1/3}}$$
 = 
$$\frac{1}{3}.$$

We can use the same logic to find the human capital share:

Human Capital Share = 
$$\frac{\left(\frac{1}{3}K^{1/3}L^{1/3}H^{-2/3}\right)H}{K^{1/3}H^{1/3}L^{1/3}}$$
 = 
$$\frac{1}{3}$$
, so labor gets one-third of the output, and human capital gets one-third of the output.

so labor gets one-third of the output, and human capital gets one-third of the output. Since workers own their human capital (we hope!), it will appear that labor gets two-thirds of output.

d. The ratio of the skilled wage to the unskilled wage is:

$$\begin{split} \frac{W_{\text{skilled}}}{W_{\text{unskilled}}} &= \frac{MPL + MPH}{MPL} \\ &= \frac{\frac{1}{3}K^{1/3}L^{-2/3}H^{1/3} + \frac{1}{3}K^{1/3}L^{1/3}H^{-2/3}}{\frac{1}{3}K^{1/3}L^{-2/3}H^{1/3}} \\ &= 1 + \frac{L}{H} \,. \end{split}$$

Notice that the ratio is always greater than 1 because skilled workers get paid more than unskilled workers. Also, when H increases this ratio falls because the diminishing returns to human capital lower its return, while at the same time increasing the marginal product of unskilled workers.

e. If more college scholarships increase *H*, then it does lead to a more egalitarian society. The policy lowers the returns to education, decreasing the gap between the wages of more and less educated workers. More importantly, the policy even raises the absolute wage of unskilled workers because their marginal product rises when the number of skilled workers rises.

7. The effect of a government tax increase of \$100 billion on (a) public saving, (b) private saving, and (c) national saving can be analyzed by using the following relationships:

National Saving = [Private Saving] + [Public Saving]  
= 
$$[Y - T - C(Y - T)] + [T - G]$$
  
=  $Y - C(Y - T) - G$ .

- a. **Public Saving**—The tax increase causes a 1-for-1 increase in public saving. *T* increases by \$100 billion and, therefore, public saving increases by \$100 billion.
- b. **Private Saving**—The increase in taxes decreases disposable income, Y T, by \$100 billion. Since the marginal propensity to consume (MPC) is 0.6, consumption falls by  $0.6 \times $100$  billion, or \$60 billion. Hence,

$$\Delta$$
Private Saving =  $-\$100b - 0.6 (-\$100b) = -\$40b$ .

Private saving falls \$40 billion.

c. **National Saving**—Because national saving is the sum of private and public saving, we can conclude that the \$100 billion tax increase leads to a \$60 billion increase in national saving.

Another way to see this is by using the third equation for national saving expressed above, that national saving equals Y - C(Y - T) - G. The \$100 billion tax increase reduces disposable income and causes consumption to fall by \$60 billion. Since neither G nor Y changes, national saving thus rises by \$60 billion.

d. **Investment**—To determine the effect of the tax increase on investment, recall the national accounts identity:

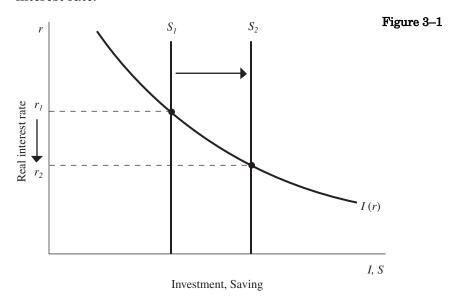
$$Y = C(Y - T) + I(r) + G.$$

Rearranging, we find

$$Y - C(Y - T) - G = I(r).$$

The left-hand side of this equation is national saving, so the equation just says that national saving equals investment. Since national saving increases by \$60 billion, investment must also increase by \$60 billion.

How does this increase in investment take place? We know that investment depends on the real interest rate. For investment to rise, the real interest rate must fall. Figure 3–1 illustrates saving and investment as a function of the real interest rate.



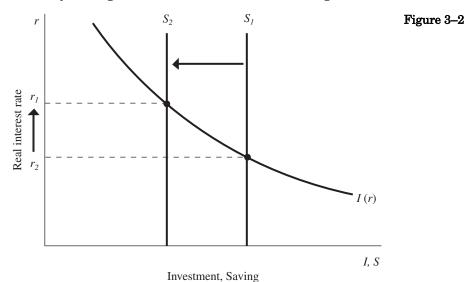
The tax increase causes national saving to rise, so the supply curve for loanable funds shifts to the right. The equilibrium real interest rate falls, and investment rises.

8. If consumers increase the amount that they consume today, then private saving and, therefore, national saving will fall. We know this from the definition of national saving:

National Saving = [Private Saving] + [Public Saving]  
= 
$$[Y - T - C(Y - T)] + [T - G]$$
.

An increase in consumption decreases private saving, so national saving falls.

Figure 3–2 illustrates saving and investment as a function of the real interest rate. If national saving decreases, the supply curve for loanable funds shifts to the left, thereby raising the real interest rate and reducing investment.



9. a. Private saving is the amount of disposable income, Y - T, that is not consumed:

$$S^{\text{private}} = Y - T - C$$
  
= 5,000 - 1,000 - (250 + 0.75(5,000 - 1,000))  
= 750.

Public saving is the amount of taxes the government has left over after it makes its purchases:

$$S^{\text{public}} = T - G$$
  
= 1,000 - 1,000  
= 0.

Total saving is the sum of private saving and public saving:

$$S = S^{\text{private}} + S^{\text{public}}$$
  
= 750 + 0  
= 750.

b. The equilibrium interest rate is the value of *r* that clears the market for loanable funds. We already know that national saving is 750, so we just need to set it equal to investment:

$$S = I$$
  
 $750 = 1,000 - 50r$ 

Solving this equation for r, we find:

$$r = 5\%$$
.

c. When the government increases its spending, private saving remains the same as before (notice that G does not appear in the  $S^{\text{private}}$  above) while government saving decreases. Putting the new G into the equations above:

$$S^{\text{private}} = 750$$
  
 $S^{\text{public}} = T - G$   
 $= 1,000 - 1,250$   
 $= -250.$ 

Thus,

$$S = S^{\text{private}} + S^{\text{public}}$$
  
= 750 + (-250)  
= 500.

d. Once again the equilibrium interest rate clears the market for loanable funds:

$$S = I$$
  
 $500 = 1,000 - 50r$ 

Solving this equation for r, we find:

$$r = 10\%$$
.

10. To determine the effect on investment of an equal increase in both taxes and government spending, consider the national income accounts identity for national saving:

National Saving = [Private Saving] + [Public Saving]  
= 
$$[Y - T - C(Y - T)] + [T - G]$$
.

We know that Y is fixed by the factors of production. We also know that the change in consumption equals the marginal propensity to consume (MPC) times the change in disposable income. This tells us that

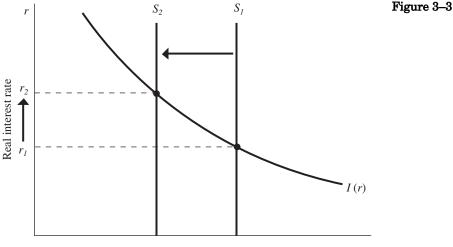
$$\Delta \text{National Saving} = \left[ -\Delta T - (MPC \times (-\Delta T)) \right] + \left[ \Delta T - \Delta G \right]$$

$$= \left[ -\Delta T + (MPC \times \Delta T) \right] + 0$$

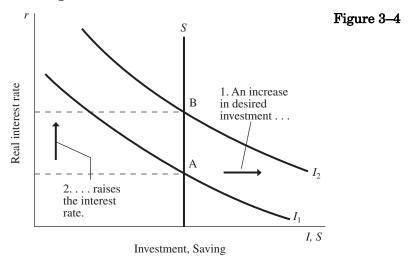
$$= (MPC - 1) \Delta T.$$

The above expression tells us that the impact on saving of an equal increase in T and G depends on the size of the marginal propensity to consume. The closer the MPC is to 1, the smaller is the fall in saving. For example, if the MPC equals 1, then the fall in consumption equals the rise in government purchases, so national saving [Y-C(Y-T)-G] is unchanged. The closer the MPC is to 0 (and therefore the larger is the amount saved rather than spent for a one-dollar change in disposable income), the greater is the impact on saving. Because we assume that the MPC is less than 1, we expect that national saving falls in response to an equal increase in taxes and government spending.

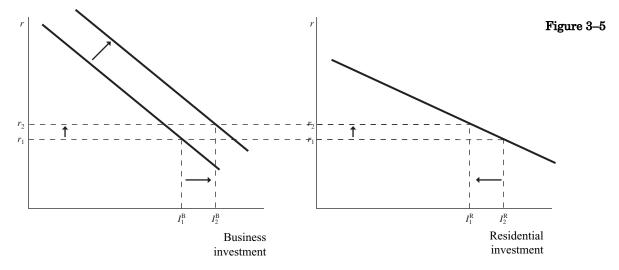
The reduction in saving means that the supply of loanable funds curve shifts to the left in Figure 3–3. The real interest rate rises, and investment falls.



- 11. a. The demand curve for business investment shifts out to the right because the subsidy increases the number of profitable investment opportunities for any given interest rate. The demand curve for residential investment remains unchanged.
  - b. The total demand curve for investment in the economy shifts out to the right since it represents the sum of business investment, which shifts out to the right, and residential investment, which is unchanged. As a result the real interest rate rises as in Figure 3–4.

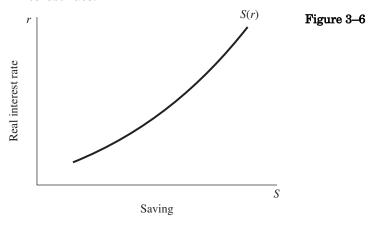


c. The total quantity of investment does not change because it is constrained by the inelastic supply of savings. The investment tax credit leads to a rise in business investment, but an offsetting fall in residential investment. That is, the higher interest rate means that residential investment falls (a movement along the curve), whereas the rightward shift of the business investment curve leads business investment to rise by an equal amount. Figure 3–5 shows this change. Note that  $I_1^{\rm B} + I_1^{\rm R} = I_2^{\rm B} + I_2^{\rm R} = \overline{S}$ .



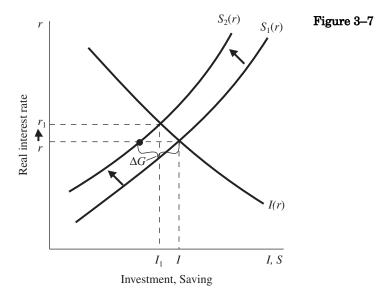
12. In this chapter, we concluded that an increase in government expenditures reduces national saving and raises the interest rate; it therefore crowds out investment by the full amount of the increase in government expenditure. Similarly, a tax cut increases disposable income and hence consumption; this increase in consumption translates into a fall in national saving—again, it crowds out investment by the full amount of the increase in consumption.

If consumption depends on the interest rate, then these conclusions about fiscal policy are modified somewhat. If consumption depends on the interest rate, then so does saving. The higher the interest rate, the greater the return to saving. Hence, it seems reasonable to think that an increase in the interest rate might increase saving and reduce consumption. Figure 3–6 shows saving as an increasing function of the interest rate.



Chapter 3

Consider what happens when government purchases increase. At any given level of the interest rate, national saving falls by the change in government purchases, as shown in Figure 3–7. The figure shows that if the saving function slopes upward, investment falls by less than the amount that government purchases rises by; this happens because consumption falls and saving increases in response to the higher interest rate. Hence, the more responsive consumption is to the interest rate, the less government purchases crowd out investment.



13. a. Figure 3–8 shows the case where the demand for loanable funds is stable but the supply of funds (the saving schedule) fluctuates perhaps reflecting temporary shocks to income, changes in government spending, or changes in consumer confidence. In this case, when interest rates fall, investment rises; when interest rates rise, investment falls. We would expect a negative correlation between investment and interest rates.

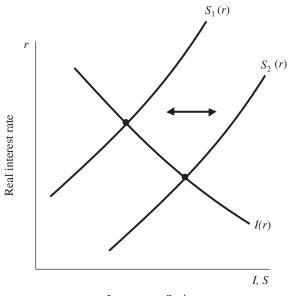
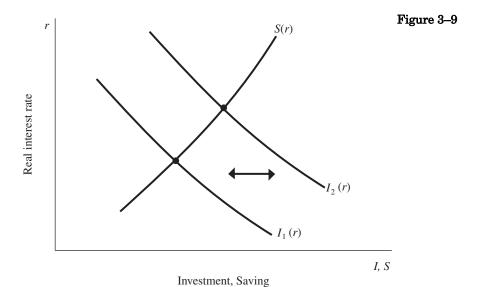


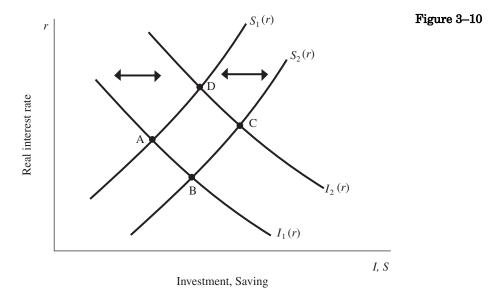
Figure 3-8

Investment, Saving

b. Figure 3-9 shows the case where the supply of loanable funds (saving) does not respond to the interest rate. Also suppose that this curve is stable, whereas the demand for loanable funds varies, perhaps reflecting fluctuations in firms' expectations about the marginal product of capital. We would now find a positive correlation between investment and the interest rate—when demand for funds rises, this pushes up the interest rate, so we see investment increase and the real interest rate increase at the same time.



If both curves shift, we might generate a scatter plot as in Figure 3–10, where the economy fluctuates among points A, B, C, and D. Depending on how often the economy is at each of these points, we might find little clear relationship between investment and interest rates.



d. Situation (c) seems fairly reasonable—both the supply of and demand for loanable funds fluctuate over time in response to changes in the economy.