

INDEX MODELS

THE MARKOWITZ PROCEDURE introduced in the preceding chapter suffers from two drawbacks. First, the model requires a huge number of estimates to fill the covariance matrix. Second, the model does not provide any guideline to the forecasting of the security risk premiums that are essential to construct the efficient frontier of risky assets. Because past returns are unreliable guides to expected future returns, this drawback can be telling.

In this chapter we introduce index models that simplify estimation of the covariance matrix and greatly enhance the analysis of security risk premiums. By allowing us to explicitly decompose risk into systematic and firm-specific components, these models also shed considerable light on both the power and limits of diversification. Further, they allow us to measure these components of risk for particular securities and portfolios.

We begin the chapter by describing a single-factor security market and show how it can justify a single-index model of security returns. Once its properties are analyzed, we proceed to an extensive example

of estimation of the single-index model. We review the statistical properties of these estimates and show how they relate to the practical issues facing portfolio managers.

Despite the simplification they offer, index models remain true to the concepts of the efficient frontier and portfolio optimization. Empirically, index models are as valid as the assumption of normality of the rates of return on available securities. To the extent that short-term returns are well approximated by normal distributions, index models can be used to select optimal portfolios nearly as accurately as the Markowitz algorithm. Finally, we examine optimal risky portfolios constructed using the index model. While the principles are the same as those employed in the previous chapter, the properties of the portfolio are easier to derive and interpret in this context. We illustrate how to use the index model by constructing an optimal risky portfolio using a small sample of firms. This portfolio is compared to the corresponding portfolio constructed from the Markowitz model. We conclude with a discussion of several practical issues that arise when implementing the index model.

8.1 A SINGLE-FACTOR SECURITY MARKET

The Input List of the Markowitz Model

The success of a portfolio selection rule depends on the quality of the input list, that is, the estimates of expected security returns and the covariance matrix. In the long run, efficient portfolios will beat portfolios with less reliable input lists and consequently inferior reward-to-risk trade-offs.

Suppose your security analysts can thoroughly analyze 50 stocks. This means that your input list will include the following:

$$\begin{aligned} n &= 50 \text{ estimates of expected returns} \\ n &= 50 \text{ estimates of variances} \\ (n^2 - n)/2 &= \frac{1,225}{1,325} \text{ estimates of covariances} \\ &= 1,325 \text{ total estimates} \end{aligned}$$

This is a formidable task, particularly in light of the fact that a 50-security portfolio is relatively small. Doubling n to 100 will nearly quadruple the number of estimates to 5,150. If $n = 3,000$, roughly the number of NYSE stocks, we need more than 4.5 million estimates.

Another difficulty in applying the Markowitz model to portfolio optimization is that errors in the assessment or estimation of correlation coefficients can lead to nonsensical results. This can happen because some sets of correlation coefficients are mutually inconsistent, as the following example demonstrates:¹

Asset	Standard Deviation (%)	Correlation Matrix		
		A	B	C
A	20	1.00	0.90	0.90
B	20	0.90	1.00	0.00
C	20	0.90	0.00	1.00

Suppose that you construct a portfolio with weights: -1.00 ; 1.00 ; 1.00 , for assets A; B; C, respectively, and calculate the portfolio variance. You will find that the portfolio variance appears to be negative (-200). This of course is not possible because portfolio variances cannot be negative: we conclude that the inputs in the estimated correlation matrix must be mutually inconsistent. Of course, *true* correlation coefficients are always consistent.² But we do not know these true correlations and can only estimate them with some imprecision. Unfortunately, it is difficult to determine at a quick glance whether a correlation matrix is inconsistent, providing another motivation to seek a model that is easier to implement.

Introducing a model that simplifies the way we describe the sources of security risk allows us to use a smaller, consistent set of estimates of risk parameters and risk premiums. The simplification emerges because positive covariances among security returns arise from common economic forces that affect the fortunes of most firms. Some examples of common economic factors are business cycles, interest rates, and the cost of natural resources. The unexpected changes in these variables cause, simultaneously, unexpected changes

¹We are grateful to Andrew Kaplin and Ravi Jagannathan, Kellogg Graduate School of Management, Northwestern University, for this example.

²The mathematical term for a correlation matrix that cannot generate negative portfolio variance is “positive definite.”

in the rates of return on the entire stock market. By decomposing uncertainty into these system-wide versus firm-specific sources, we vastly simplify the problem of estimating covariance and correlation.

Normality of Returns and Systematic Risk

We can always decompose the rate of return on any security, i , into the sum of its expected plus unanticipated components:

$$r_i = E(r_i) + e_i \quad (8.1)$$

where the unexpected return, e_i , has a mean of zero and a standard deviation of σ_i that measures the uncertainty about the security return.

When security returns can be well approximated by normal distributions that are correlated across securities, we say that they are *joint normally distributed*. This assumption alone implies that, at any time, security returns are driven by one or more common variables. When more than one variable drives normally distributed security returns, these returns are said to have a *multivariate normal distribution*. We begin with the simpler case where only one variable drives the joint normally distributed returns, resulting in a single-factor security market. Extension to the multivariate case is straightforward and is discussed in later chapters.

Suppose the common factor, m , that drives innovations in security returns is some macroeconomic variable that affects all firms. Then we can decompose the sources of uncertainty into uncertainty about the economy as a whole, which is captured by m , and uncertainty about the firm in particular, which is captured by e_i . In this case, we amend Equation 8.1 to accommodate two sources of variation in return:

$$r_i = E(r_i) + m + e_i \quad (8.2)$$

The macroeconomic factor, m , measures unanticipated macro surprises. As such, it has a mean of zero (over time, surprises will average out to zero), with standard deviation of σ_m . In contrast, e_i measures only the firm-specific surprise. Notice that m has no subscript because the same common factor affects all securities. Most important is the fact that m and e_i are uncorrelated, that is, because e_i is firm-specific, it is independent of shocks to the common factor that affect the entire economy. The variance of r_i thus arises from two uncorrelated sources, systematic and firm specific. Therefore,

$$\sigma_i^2 = \sigma_m^2 + \sigma^2(e_i) \quad (8.3)$$

The common factor, m , generates correlation across securities, because all securities will respond to the same macroeconomic news, while the firm-specific surprises, captured by e_i , are assumed to be uncorrelated across firms. Because m is also uncorrelated with any of the firm-specific surprises, the covariance between any two securities i and j is

$$\text{Cov}(r_i, r_j) = \text{Cov}(m + e_i, m + e_j) = \sigma_m^2 \quad (8.4)$$

Finally, we recognize that some securities will be more sensitive than others to macroeconomic shocks. For example, auto firms might respond more dramatically to changes in general economic conditions than pharmaceutical firms. We can capture this refinement by assigning each firm a sensitivity coefficient to macro conditions. Therefore, if we denote the sensitivity coefficient for firm i by the Greek letter beta, β_i , we modify Equation 8.2 to obtain the **single-factor model**:

$$r_i = E(r_i) + \beta_i m + e_i \quad (8.5)$$

Equation 8.5 tells us the systematic risk of security i is determined by its beta coefficient. “Cyclical” firms have greater sensitivity to the market and therefore higher systematic risk. The systematic risk of security i is $\beta_i^2\sigma_m^2$, and its total risk is

$$\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma^2(e_i) \quad (8.6)$$

The covariance between any pair of securities also is determined by their betas:

$$\text{Cov}(r_i, r_j) = \text{Cov}(\beta_i m + e_i, \beta_j m + e_j) = \beta_i \beta_j \sigma_m^2 \quad (8.7)$$

In terms of systematic risk and market exposure, this equation tells us that firms are close substitutes. Equivalent beta securities give equivalent market positions.

Up to this point we have used only statistical implications from the joint normality of security returns. Normality of security returns alone guarantees that portfolio returns are also normal (from the “stability” of the normal distribution discussed in Chapter 5) and that there is a linear relationship between security returns and the common factor. This greatly simplifies portfolio analysis. Statistical analysis, however, does not identify the common factor, nor does it specify how that factor might operate over a longer investment period. However, it seems plausible (and can be empirically verified) that the variance of the common factor usually changes relatively slowly through time, as do the variances of individual securities and the covariances among them. We seek a variable that can proxy for this common factor. To be useful, this variable must be observable, so we can estimate its volatility as well as the sensitivity of individual securities returns to variation in its value.

8.2 THE SINGLE-INDEX MODEL

A reasonable approach to making the single-factor model operational is to assert that the rate of return on a broad index of securities such as the S&P 500 is a valid proxy for the common macroeconomic factor. This approach leads to an equation similar to the single-factor model, which is called a **single-index model** because it uses the market index to proxy for the common factor.

The Regression Equation of the Single-Index Model

Because the S&P 500 is a portfolio of stocks whose prices and rates of return can be observed, we have a considerable amount of past data with which to estimate systematic risk. We denote the market index by M , with excess return of $R_M = r_M - r_f$, and standard deviation of σ_M . Because the index model is linear, we can estimate the sensitivity (or beta) coefficient of a security on the index using a single-variable linear regression. We regress the excess return of a security, $R_i = r_i - r_f$, on the excess return of the index, R_M . To estimate the regression, we collect a historical sample of paired observations, $R_i(t)$ and $R_M(t)$, where t denotes the date of each pair of observations (e.g., the excess returns on the stock and the index in a particular month).³ The **regression equation** is

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t) \quad (8.8)$$

The intercept of this equation (denoted by the Greek letter alpha, or α) is the security’s expected excess return *when the market excess return is zero*. The slope coefficient, β_i , is

³Practitioners often use a “modified” index model that is similar to Equation 8.8 but that uses total rather than excess returns. This practice is most common when daily data are used. In this case the rate of return on bills is on the order of only about .01% per day, so total and excess returns are almost indistinguishable.

the security beta. Beta is the security's sensitivity to the index: it is the amount by which the security return tends to increase or decrease for every 1% increase or decrease in the return on the index. e_i is the zero-mean, firm-specific surprise in the security return in time t , also called the **residual**.

The Expected Return–Beta Relationship

Because $E(e_i) = 0$, if we take the expected value of $E(R_i)$ in Equation 8.8, we obtain the expected return–beta relationship of the single-index model:

$$E(R_i) = \alpha_i + \beta_i E(R_M) \quad (8.9)$$

The second term in Equation 8.9 tells us that part of a security's risk premium is due to the risk premium of the index. The market risk premium is multiplied by the relative sensitivity, or beta, of the individual security. We call this the *systematic* risk premium because it derives from the risk premium that characterizes the entire market, which proxies for the condition of the full economy or economic system.

The remainder of the risk premium is given by the first term in the equation, α . Alpha is a *nonmarket* premium. For example, α may be large if you think a security is underpriced and therefore offers an attractive expected return. Later on, we will see that when security prices are in equilibrium, such attractive opportunities ought to be competed away, in which case α will be driven to zero. But for now, let's assume that each security analyst comes up with his or her own estimates of alpha. If managers believe that they can do a superior job of security analysis, then they will be confident in their ability to find stocks with nonzero values of alpha.

We will see shortly that the index model decomposition of an individual security's risk premium to market and nonmarket components greatly clarifies and simplifies the operation of macroeconomic and security analysis within an investment company.

Risk and Covariance in the Single-Index Model

Remember that one of the problems with the Markowitz model is the overwhelming number of parameter estimates required to implement it. Now we will see that the index model simplification vastly reduces the number of parameters that must be estimated. Equation 8.8 yields the systematic and firm-specific components of the overall risk of each security, and the covariance between any pair of securities. Both variances and covariances are determined by the security betas and the properties of the market index:

$$\begin{aligned} \text{Total risk} &= \text{Systematic risk} + \text{Firm-specific risk} \\ \sigma_i^2 &= \beta_i^2 \sigma_M^2 + \sigma^2(e_i) \\ \text{Covariance} &= \text{Product of betas} \times \text{Market index risk} \\ \text{Cov}(r_i, r_j) &= \beta_i \beta_j \sigma_M^2 \\ \text{Correlation} &= \text{Product of correlations with the market index} \\ \text{Corr}(r_i, r_j) &= \frac{\beta_i \beta_j \sigma_M^2}{\sigma_i \sigma_j} = \frac{\beta_i \sigma_M^2 \beta_j \sigma_M^2}{\sigma_i \sigma_M \sigma_j \sigma_M} = \text{Corr}(r_i, r_M) \times \text{Corr}(r_j, r_M) \end{aligned} \quad (8.10)$$

Equations 8.9 and 8.10 imply that the set of parameter estimates needed for the single-index model consists of only α , β , and $\sigma(e)$ for the individual securities, plus the risk premium and variance of the market index.

CONCEPT
CHECK

1

The data below describe a three-stock financial market that satisfies the single-index model.

Stock	Capitalization	Beta	Mean Excess Return	Standard Deviation
A	\$3,000	1.0	10%	40
B	\$1,940	0.2	2	30
C	\$1,360	1.7	17	50

The standard deviation of the market index portfolio is 25%.

- What is the mean excess return of the index portfolio?
- What is the covariance between stock A and stock B?
- What is the covariance between stock B and the index?
- Break down the variance of stock B into its systematic and firm-specific components.

The Set of Estimates Needed for the Single-Index Model

We summarize the results for the single-index model in the table below.

	Symbol
1. The stock's expected return if the market is neutral, that is, if the market's excess return, $r_M - r_f$, is zero	α_i
2. The component of return due to movements in the overall market; β_i is the security's responsiveness to market movements	$\beta_i(r_M - r_f)$
3. The unexpected component of return due to unexpected events that are relevant only to this security (firm specific)	e_i
4. The variance attributable to the uncertainty of the common macroeconomic factor	$\beta_i^2 \sigma_M^2$
5. The variance attributable to firm-specific uncertainty	$\sigma^2(e_i)$

These calculations show that if we have:

- n estimates of the extra-market expected excess returns, α_i
- n estimates of the sensitivity coefficients, β_i
- n estimates of the firm-specific variances, $\sigma^2(e_i)$
- 1 estimate for the market risk premium, $E(R_M)$
- 1 estimate for the variance of the (common) macroeconomic factor, σ_M^2

then these $(3n + 2)$ estimates will enable us to prepare the entire input list for this single-index security universe. Thus for a 50-security portfolio we will need 152 estimates rather than 1,325; for the entire New York Stock Exchange, about 3,000 securities, we will need 9,002 estimates rather than approximately 4.5 million!

It is easy to see why the index model is such a useful abstraction. For large universes of securities, the number of estimates required for the Markowitz procedure using the index model is only a small fraction of what otherwise would be needed.

Another advantage is less obvious but equally important. The index model abstraction is crucial for specialization of effort in security analysis. If a covariance term had to be calculated directly for each security pair, then security analysts could not specialize by industry. For example, if one group were to specialize in the computer industry and another

in the auto industry, who would have the common background to estimate the covariance *between* IBM and GM? Neither group would have the deep understanding of other industries necessary to make an informed judgment of co-movements among industries. In contrast, the index model suggests a simple way to compute covariances. Covariances among securities are due to the influence of the single common factor, represented by the market index return, and can be easily estimated using the regression Equation 8.8 on (p. 247).

The simplification derived from the index model assumption is, however, not without cost. The “cost” of the model lies in the restrictions it places on the structure of asset return uncertainty. The classification of uncertainty into a simple dichotomy—macro versus micro risk—oversimplifies sources of real-world uncertainty and misses some important sources of dependence in stock returns. For example, this dichotomy rules out industry events, events that may affect many firms within an industry without substantially affecting the broad macroeconomy.

This last point is potentially important. Imagine that the single-index model is perfectly accurate, except that the residuals of two stocks, say, British Petroleum (BP) and Royal Dutch Shell, are correlated. The index model will ignore this correlation (it will assume it is zero), while the Markowitz algorithm (which accounts for the full covariance between every pair of stocks) will automatically take the residual correlation into account when minimizing portfolio variance. If the universe of securities from which we must construct the optimal portfolio is small, the two models will yield substantively different optimal portfolios. The portfolio of the Markowitz algorithm will place a smaller weight on both BP and Shell (because their mutual covariance reduces their diversification value), resulting in a portfolio with lower variance. Conversely, when correlation among residuals is negative, the index model will ignore the potential diversification value of these securities. The resulting “optimal” portfolio will place too little weight on these securities, resulting in an unnecessarily high variance.

The optimal portfolio derived from the single-index model therefore can be significantly inferior to that of the full-covariance (Markowitz) model when stocks with correlated residuals have large alpha values and account for a large fraction of the portfolio.

If many pairs of the covered stocks exhibit residual correlation, it is possible that a *multi-index* model, which includes additional factors to capture those extra sources of cross-security correlation, would be better suited for portfolio analysis and construction. We will demonstrate the effect of correlated residuals in the spreadsheet example in this chapter, and discuss multi-index models in later chapters.

CONCEPT CHECK

2

Suppose that the index model for the excess returns of stocks *A* and *B* is estimated with the following results:

$$R_A = 1.0\% + .9R_M + e_A$$

$$R_B = -2.0\% + 1.1R_M + e_B$$

$$\sigma_M = 20\%$$

$$\sigma(e_A) = 30\%$$

$$\sigma(e_B) = 10\%$$

Find the standard deviation of each stock and the covariance between them.

The Index Model and Diversification

The index model, first suggested by Sharpe,⁴ also offers insight into portfolio diversification. Suppose that we choose an equally weighted portfolio of *n* securities. The excess rate of return on each security is given by

$$R_i = \alpha_i + \beta_i R_M + e_i$$

⁴William F. Sharpe, “A Simplified Model of Portfolio Analysis,” *Management Science*, January 1963.

Similarly, we can write the excess return on the portfolio of stocks as

$$R_p = \alpha_p + \beta_p R_M + e_p \quad (8.11)$$

We now show that, as the number of stocks included in this portfolio increases, the part of the portfolio risk attributable to nonmarket factors becomes ever smaller. This part of the risk is diversified away. In contrast, market risk remains, regardless of the number of firms combined into the portfolio.

To understand these results, note that the excess rate of return on this equally weighted portfolio, for which each portfolio weight $w_i = 1/n$, is

$$\begin{aligned} R_p &= \sum_{i=1}^n w_i R_i = \frac{1}{n} \sum_{i=1}^n R_i = \frac{1}{n} \sum_{i=1}^n (\alpha_i + \beta_i R_M + e_i) \\ &= \frac{1}{n} \sum_{i=1}^n \alpha_i + \left(\frac{1}{n} \sum_{i=1}^n \beta_i \right) R_M + \frac{1}{n} \sum_{i=1}^n e_i \end{aligned} \quad (8.12)$$

Comparing Equations 8.11 and 8.12, we see that the portfolio has a sensitivity to the market given by

$$\beta_p = \frac{1}{n} \sum_{i=1}^n \beta_i \quad (8.13)$$

which is the average of the individual β_i s. It has a nonmarket return component of

$$\alpha_p = \frac{1}{n} \sum_{i=1}^n \alpha_i \quad (8.14)$$

which is the average of the individual alphas, plus the zero mean variable

$$e_p = \frac{1}{n} \sum_{i=1}^n e_i \quad (8.15)$$

which is the average of the firm-specific components. Hence the portfolio's variance is

$$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma^2(e_p) \quad (8.16)$$

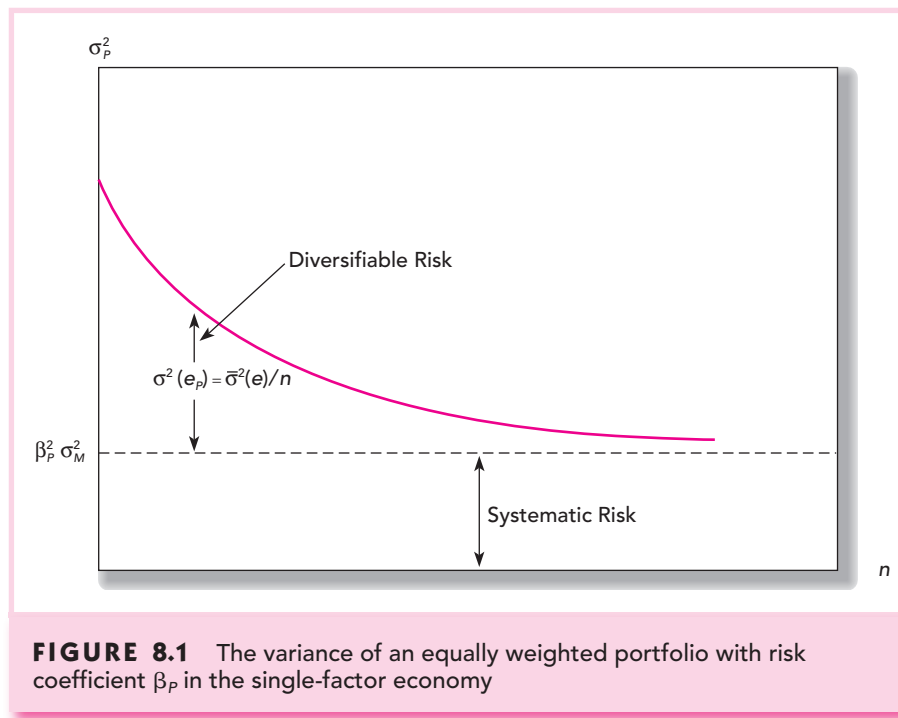
The systematic risk component of the portfolio variance, which we defined as the component that depends on marketwide movements, is $\beta_p^2 \sigma_M^2$ and depends on the sensitivity coefficients of the individual securities. This part of the risk depends on portfolio beta and σ_M^2 and will persist regardless of the extent of portfolio diversification. No matter how many stocks are held, their common exposure to the market will be reflected in portfolio systematic risk.⁵

In contrast, the nonsystematic component of the portfolio variance is $\sigma^2(e_p)$ and is attributable to firm-specific components, e_i . Because these e_i s are independent, and all have zero expected value, the law of averages can be applied to conclude that as more and more stocks are added to the portfolio, the firm-specific components tend to cancel out, resulting in ever-smaller nonmarket risk. Such risk is thus termed *diversifiable*. To see this more rigorously, examine the formula for the variance of the equally weighted "portfolio" of firm-specific components. Because the e_i s are uncorrelated,

$$\sigma^2(e_p) = \sum_{i=1}^n \left(\frac{1}{n} \right)^2 \sigma^2(e_i) = \frac{1}{n} \bar{\sigma}^2(e) \quad (8.17)$$

where $\bar{\sigma}^2(e)$ is the average of the firm-specific variances. Because this average is independent of n , when n gets large, $\sigma^2(e_p)$ becomes negligible.

⁵Of course, one can construct a portfolio with zero systematic risk by mixing negative β and positive β assets. The point of our discussion is that the vast majority of securities have a positive β , implying that well-diversified portfolios with small holdings in large numbers of assets will indeed have positive systematic risk.



To summarize, as diversification increases, the total variance of a portfolio approaches the systematic variance, defined as the variance of the market factor multiplied by the square of the portfolio sensitivity coefficient, β_p^2 . This is shown in Figure 8.1.

Figure 8.1 shows that as more and more securities are combined into a portfolio, the portfolio variance decreases because of the diversification of firm-specific risk. However, the power of diversification is limited. Even for very large n , part of the risk remains because of the exposure of virtually all assets to the common, or market, factor. Therefore, this systematic risk is said to be nondiversifiable.

**CONCEPT
CHECK
3**

Reconsider the two stocks in Concept Check 2. Suppose we form an equally weighted portfolio of A and B. What will be the nonsystematic standard deviation of that portfolio?

This analysis is borne out by empirical evidence. We saw the effect of portfolio diversification on portfolio standard deviations in Figure 7.2. These empirical results are similar to the theoretical graph presented here in Figure 8.1.

8.3

ESTIMATING THE SINGLE-INDEX MODEL

Armed with the theoretical underpinnings of the single-index model, we now provide an extended example that begins with estimation of the regression equation (8.8) and continues through to the estimation of the full covariance matrix of security returns.

To keep the presentation manageable, we focus on only six large U.S. corporations: Hewlett-Packard and Dell from the information technology (IT) sector of the S&P 500, Target and Wal-Mart from the retailing sector, and British Petroleum and Royal Dutch Shell from the energy sector.

We work with monthly observations of rates of return for the six stocks, the S&P 500 portfolio, and T-bills over the period April 2001 to March 2006 (60 observations). As a first step, the excess returns on the seven risky assets are computed. We start with a detailed look at the preparation of the input list for Hewlett-Packard (HP), and then proceed to display the entire input list. Later in the chapter, we will show how these estimates can be used to construct the optimal risky portfolio.

The Security Characteristic Line for Hewlett-Packard

The index model regression Equation 8.8 (on p. 247), restated for Hewlett-Packard (HP) is

$$R_{HP}(t) = \alpha_{HP} + \beta_{HP}R_{S\&P\ 500}(t) + e_{HP}(t)$$

The equation describes the (linear) dependence of HP's excess return on changes in the state of the economy as represented by the excess returns of the S&P 500 index portfolio. The regression estimates describe a straight line with intercept α_{HP} and slope β_{HP} , which we call the **security characteristic line (SCL)** for HP.

Figure 8.2 shows a graph of the excess returns on HP and the S&P 500 portfolio over the 60-month period from April 2001 to March 2006. The graph shows that HP returns generally follow those of the index, but with much larger swings. Indeed, the annualized standard deviation of the excess return on the S&P 500 portfolio over the period was 13.58%, while that of HP was 38.17%. The swings in HP's excess returns suggest a greater-than-average sensitivity to the index, that is, a beta greater than 1.0.

The relationship between the returns of HP and the S&P 500 is made clearer by the **scatter diagram** in Figure 8.3, where the regression line is drawn through the scatter. The vertical distance of each point from the regression line is the value of HP's residual, $e_{HP}(t)$, corresponding to that particular date. The rates in Figures 8.2 and 8.3 are not annualized, and the scatter diagram shows monthly swings of over $\pm 30\%$ for HP, but returns in the range of -11% to 8.5% for the S&P 500. The regression analysis output obtained by using Excel is shown in Table 8.1.

The Explanatory Power of the SCL for HP

Considering the top panel of Table 8.1 first, we see that the correlation of HP with the S&P 500 is quite high (.7238), telling us that HP tracks changes in the returns of the S&P 500 fairly closely. The R -square (.5239) tells us that variation in the S&P 500 excess returns explains about 52% of the variation in the HP series. The adjusted R -square (which is slightly smaller) corrects for an upward bias in R -square that arises because we use the fitted values of two parameters,⁶ the slope (beta) and intercept (alpha), rather than their true, but unobservable,

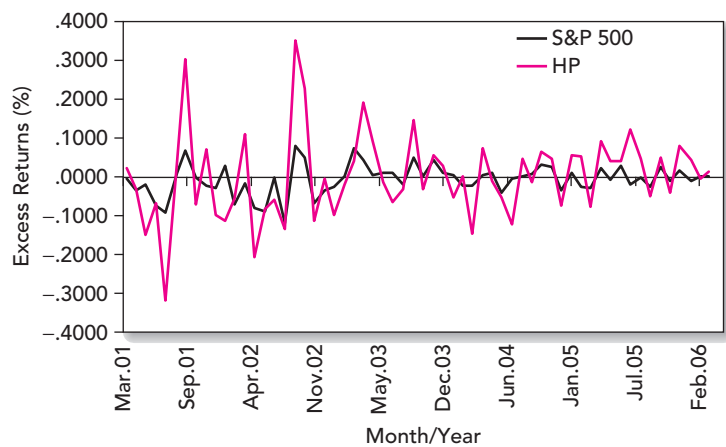


FIGURE 8.2 Excess returns on HP and S&P 500 for April 2001 to March 2006

⁶In general, the adjusted R -square (R_A^2) is derived from the unadjusted by $R_A^2 = 1 - (1 - R^2) \frac{n-1}{n-k-1}$, where k is the number of independent variables (here, $k = 1$). An additional degree of freedom is lost to the estimate of the intercept.

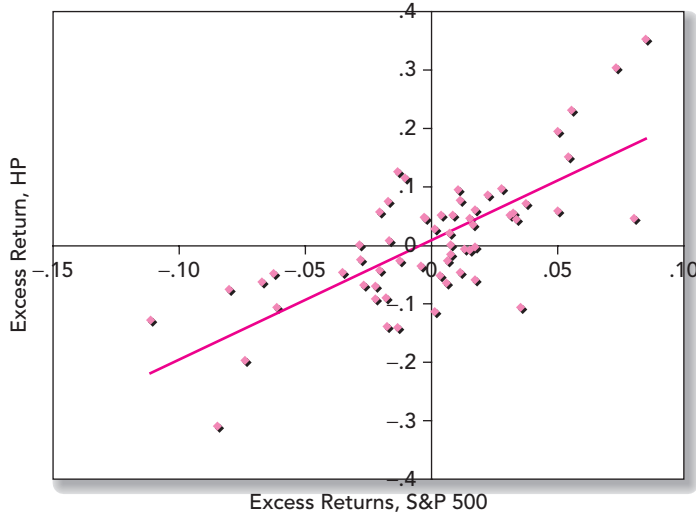


FIGURE 8.3 Scatter diagram of HP, the S&P 500, and the security characteristic line (SCL) for HP

values. With 60 observations, this bias is small. The standard error of the regression is the standard deviation of the residual, which we discuss in more detail shortly. This is a measure of the slippage in the average relationship between the stock and the index due to the impact of firm-specific factors, and is based on *in-sample* data. A more severe test is to look at returns from periods after the one covered by the regression sample and test the power of the independent variable (the S&P 500) to predict the dependent variable (the return on HP). Correlation between regression forecasts and realizations of *out-of-sample* data is almost always considerably lower than in-sample correlation.

Analysis of Variance

The next panel of Table 8.1 shows the analysis of variance (ANOVA) for the SCL. The sum of squares (SS) of the regression (.3752) is the portion of the vari-

ance of the dependent variable (HP’s return) that is explained by the independent variable (the S&P 500 return); it is equal to $\beta_{HP}^2 \sigma_{S\&P\ 500}^2$. The MS column for the residual (.0059) shows the variance of the *unexplained* portion of HP’s return, that is, the portion of return that is independent of the market index. The square root of this value is the standard error (SE) of the regression (.0767) reported in the first panel. If you divide the total SS of the regression (.7162) by 59, you will obtain the estimate of the variance of the dependent variable (HP), .012 per month, equivalent to a monthly standard deviation of 11%. When

TABLE 8.1

Excel output: Regression statistics for the SCL of Hewlett-Packard

Regression Statistics				
Multiple R		.7238		
R-square		.5239		
Adjusted R-square		.5157		
Standard error		.0767		
Observations		60		
ANOVA				
	df	SS	MS	
Regression	1	.3752	.3752	
Residual	58	.3410	.0059	
Total	59	.7162		
	Coefficients	Standard Error	t-Stat	p-Value
Intercept	0.0086	.0099	0.8719	.3868
S&P 500	2.0348	.2547	7.9888	.0000

annualized,⁷ we obtain an annualized standard deviation of 38.17%, as reported earlier. Notice that the R -square (the ratio of explained to total variance) equals the explained (regression) SS divided by the total SS.⁸

The Estimate of Alpha

Moving to the bottom panel, the intercept (.0086 = .86% per month) is the estimate of HP's alpha for the sample period. Although this is an economically large value (10.32% on an annual basis), it is statistically insignificant. This can be seen from the three statistics next to the estimated coefficient. The first is the standard error of the estimate (0.0099).⁹ This is a measure of the imprecision of the estimate. If the standard error is large, the range of likely estimation error is correspondingly large.

The t -statistic reported in the bottom panel is the ratio of the regression parameter to its standard error. This statistic equals the number of standard errors by which our estimate exceeds zero, and therefore can be used to assess the likelihood that the true but unobserved value might actually equal zero rather than the estimate derived from the data.¹⁰ The intuition is that if the true value were zero, we would be unlikely to observe estimated values far away (i.e., many standard errors) from zero. So large t -statistics imply low probabilities that the true value is zero.

In the case of alpha, we are interested in the average value of HP's return net of the impact of market movements. Suppose we define the nonmarket component of HP's return as its actual return minus the return attributable to market movements during any period. Call this HP's firm-specific return, which we abbreviate as R_{fs} .

$$R_{\text{firm-specific}} = R_{fs} = R_{HP} - \beta_{HP} R_{S\&P500}$$

If R_{fs} were normally distributed with a mean of zero, the ratio of its estimate to its standard error would have a t -distribution. From a table of the t -distribution (or using Excel's TINV function) we can find the probability that the true alpha is actually zero or even lower given the positive estimate of its value and the standard error of the estimate. This is called the *level of significance* or, as in Table 8.1, the probability or p -value. The conventional cut-off for statistical significance is a probability of less than 5%, which requires a t -statistic of about 2.0. The regression output shows the t -statistic for HP's alpha to

⁷When annualizing monthly data, average return and variance are multiplied by 12. However, because variance is multiplied by 12, standard deviation is multiplied by $\sqrt{12}$.

$$^8 R\text{-Square} = \frac{\beta_{HP}^2 \sigma_{S\&P500}^2}{\beta_{HP}^2 \sigma_{S\&P500}^2 + \sigma^2(e_{HP})} = \frac{.3752}{.7162} = .5239$$

Equivalently, R -square equals 1 minus the fraction of variance that is *not* explained by market returns, i.e., 1 minus the ratio of firm-specific risk to total risk. For HP, this is

$$1 - \frac{\sigma^2(e_{HP})}{\beta_{HP}^2 \sigma_{S\&P500}^2 + \sigma^2(e_{HP})} = 1 - \frac{.3410}{.7162} = .5239$$

⁹We can relate the standard error of the alpha estimate to the standard error of the residuals as follows:

$$SE(\alpha_{HP}) = \sigma(e_{HP}) \sqrt{\frac{1}{n} + \frac{(\text{AvgS\&P500})^2}{\text{Var}(S\&P500) \times (n-1)}}$$

¹⁰The t -statistic is based on the assumption that returns are normally distributed. In general, if we standardize the estimate of a normally distributed variable by computing its difference from a hypothesized value and dividing by the standard error of the estimate (to express the difference as a number of standard errors), the resulting variable will have a t -distribution. With a large number of observations, the bell-shaped t -distribution approaches the normal distribution.

be .8719, indicating that the estimate is not significantly different from zero. That is, we cannot reject the hypothesis that the true value of alpha equals zero with an acceptable level of confidence. The p -value for the alpha estimate (.3868) indicates that if the true alpha were zero, the probability of obtaining an estimate as high as .0086 (given the large standard error of .0099) would be .3868, which is not so unlikely. We conclude that the sample average of R_{j_s} is too low to reject the hypothesis that the true value of alpha is zero.

But even if the alpha value were both economically *and* statistically significant *within the sample*, we still would not use that alpha as a forecast for a future period. Overwhelming empirical evidence shows that 5-year alpha values do not persist over time, that is, there seems to be virtually no correlation between estimates from one sample period to the next. In other words, while the alpha estimated from the regression tells us the average return on the security when the market was flat during that estimation period, it does *not* forecast what the firm's performance will be in future periods. This is why security analysis is so hard. The past does not readily foretell the future. We elaborate on this issue in Chapter 11 on market efficiency.

The Estimate of Beta

The regression output in Table 8.1 shows the beta estimate for HP to be 2.0348, more than twice that of the S&P 500. Such high market sensitivity is not unusual for technology stocks. The standard error (SE) of the estimate is .2547.¹¹

The value of beta and its SE produce a large t -statistic (7.9888), and a p -value of practically zero. We can confidently reject the hypothesis that HP's true beta is zero. A more interesting t -statistic might test a null hypothesis that HP's beta is greater than the market-wide average beta of 1. This t -statistic would measure how many standard errors separate the estimated beta from a hypothesized value of 1. Here too, the difference is easily large enough to achieve statistical significance:

$$\frac{\text{Estimated value} - \text{Hypothesized value}}{\text{Standard error}} = \frac{2.03 - 1}{.2547} = 4.00$$

However, we should bear in mind that even here, precision is not what we might like it to be. For example, if we wanted to construct a confidence interval that includes the true but unobserved value of beta with 95% probability, we would take the estimated value as the center of the interval and then add and subtract about two standard errors. This produces a range between 1.43 and 2.53, which is quite wide.

Firm-Specific Risk

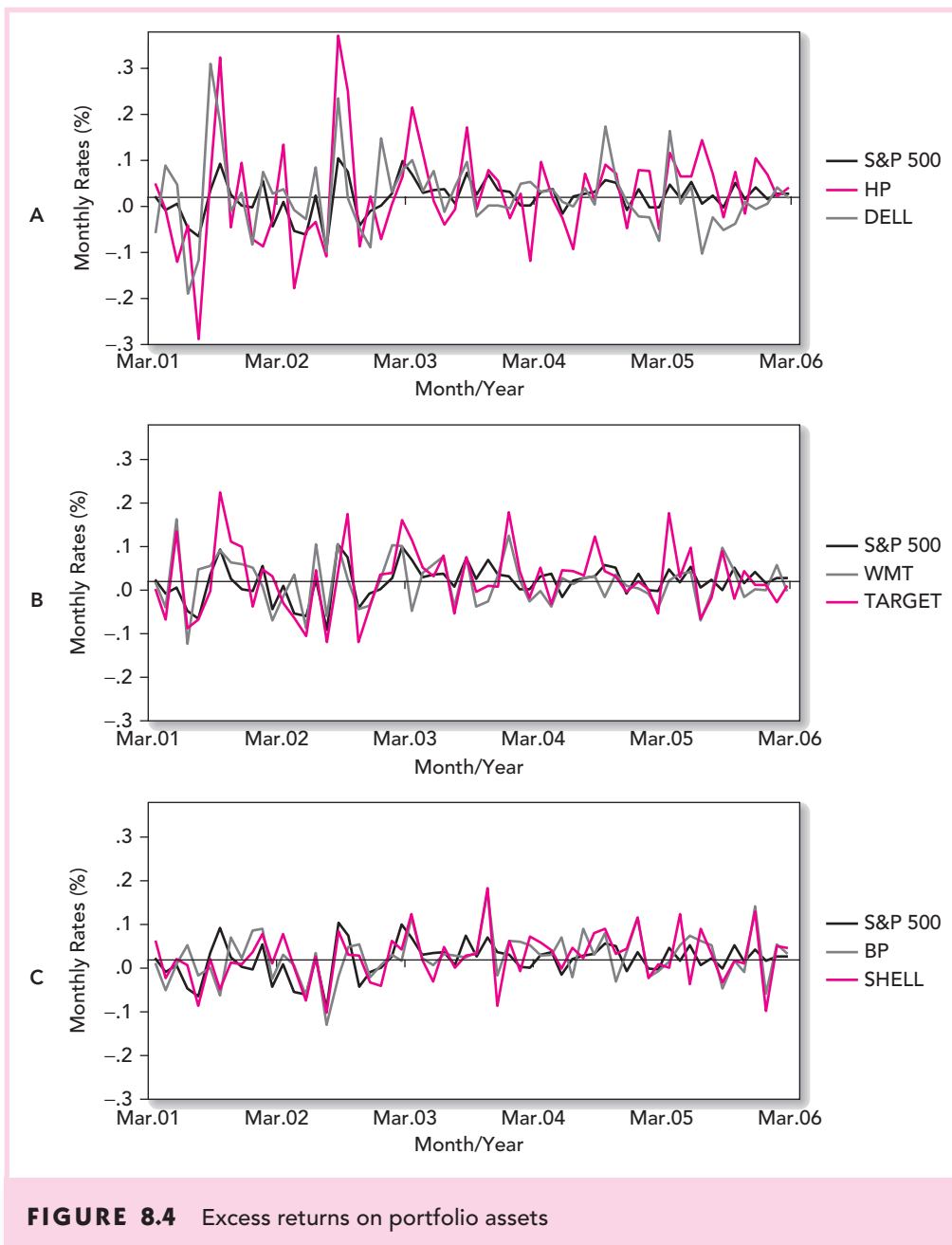
The monthly standard deviation of HP's residual is 7.67%, or 26.6% annually. This is quite large, on top of HP's high-level systematic risk. The standard deviation of systematic risk is $\beta \times \sigma(\text{S\&P 500}) = 2.03 \times 13.58 = 27.57\%$. Notice that HP's firm-specific risk is as large as its systematic risk, a common result for individual stocks.

Correlation and Covariance Matrix

Figure 8.4 graphs the excess returns of the pairs of securities from each of the three sectors with the S&P 500 index on the same scale. We see that the IT sector is the most variable, followed by the retail sector, and then the energy sector, which has the lowest volatility.

Panel 1 in Spreadsheet 8.1 shows the estimates of the risk parameters of the S&P 500 portfolio and the six analyzed securities. You can see from the high residual standard deviations (column E) how important diversification is. These securities have tremendous

¹¹ $SE(\beta) = \frac{\sigma(\epsilon_{HP})}{\sigma_{HP} \sqrt{n-1}}$



firm-specific risk. Portfolios concentrated in these (or other) securities would have unnecessarily high volatility and inferior Sharpe ratios.

Panel 2 shows the correlation matrix of the residuals from the regressions of excess returns on the S&P 500. The shaded cells show correlations of same-sector stocks, which are as high as .7 for the two oil stocks (BP and Shell). This is in contrast to the assumption of the index model that all residuals are uncorrelated. Of course, these correlations are, to a great extent, high by design, because we selected pairs of firms from the same industry. Cross-industry correlations are typically far smaller, and the empirical estimates

	A	B	C	D	E	F	G	H	I	J
1	Panel 1: Risk Parameters of the Investable Universe (annualized)									
2										
3		SD of Excess Return	Beta	SD of Systematic Component	SD of Residual	Correlation with the S&P 500				
4	S&P 500	0.1358	1.00	0.1358	0	1				
5	HP	0.3817	2.03	0.2762	0.2656	0.72				
6	DELL	0.2901	1.23	0.1672	0.2392	0.58				
7	WMT	0.1935	0.62	0.0841	0.1757	0.43				
8	TARGET	0.2611	1.27	0.1720	0.1981	0.66				
9	BP	0.1822	0.47	0.0634	0.1722	0.35				
10	SHELL	0.1988	0.67	0.0914	0.1780	0.46				
11										
12	Panel 2: Correlation of Residuals									
13										
14		HP	DELL	WMT	TARGET	BP				
15	HP	1								
16	DELL	0.08	1							
17	WMT	-0.34	0.17	1						
18	TARGET	-0.10	0.12	0.50	1					
19	BP	-0.20	-0.28	-0.19	-0.13	1				
20	SHELL	-0.06	-0.19	-0.24	-0.22	0.70				
21										
22	Panel 3: The Index Model Covariance Matrix									
23										
24			S&P 500	HP	DELL	WMT	TARGET	BP	SHELL	
25		Beta	1.00	2.03	1.23	0.62	1.27	0.47	0.67	
26	S&P 500	1.00	0.0184	0.0375	0.0227	0.0114	0.0234	0.0086	0.0124	
27	HP	2.03	0.0375	0.1457	0.0462	0.0232	0.0475	0.0175	0.0253	
28	DELL	1.23	0.0227	0.0462	0.0842	0.0141	0.0288	0.0106	0.0153	
29	WMT	0.62	0.0114	0.0232	0.0141	0.0374	0.0145	0.0053	0.0077	
30	TARGET	1.27	0.0234	0.0475	0.0288	0.0145	0.0682	0.0109	0.0157	
31	BP	0.47	0.0086	0.0175	0.0106	0.0053	0.0109	0.0332	0.0058	
32	SHELL	0.67	0.0124	0.0253	0.0153	0.0077	0.0157	0.0058	0.0395	
33										
34	Cells on the diagonal (shadowed) equal to variance									
35			formula in cell C26		= B4^2					
36	Off-diagonal cells equal to covariance									
37			formula in cell C27		= C\$25*\$B27*\$B\$4^2					
38	multiplies beta from row and column by index variance									
39										
40	Panel 4: Macro Forecast and Forecasts of Alpha Values									
41										
42										
43		S&P 500	HP	DELL	WMT	TARGET	BP	SHELL		
44	Alpha	0	0.0150	-0.0100	-0.0050	0.0075	0.012	0.0025		
45	Risk premium	0.0600	0.1371	0.0639	0.0322	0.0835	0.0400	0.0429		
46										
47	Panel 5: Computation of the Optimal Risky Portfolio									
48										
49		S&P 500	Active Pf A	HP	DELL	WMT	TARGET	BP	SHELL	Overall Pf
50	$\sigma^2(e)$			0.0705	0.0572	0.0309	0.0392	0.0297	0.0317	
51	$\alpha/\sigma^2(e)$		0.5505	0.2126	-0.1748	-0.1619	0.1911	0.4045	0.0789	
52	$w^0(i)$		1.0000	0.3863	-0.3176	-0.2941	0.3472	0.7349	0.1433	
53	$[w^0(i)]^2$			0.1492	0.1009	0.0865	0.1205	0.5400	0.0205	
54	α_A		0.0222							
55	$\sigma^2(e_A)$		0.0404							
56	w_A^0		0.1691							
57	w^1 (Risky portf)	0.8282	0.1718							
58	Beta	1	1.0922	2.0348	1.2315	0.6199	1.2672	0.4670	0.6736	1.0158
59	Risk premium	0.06	0.0878	0.1371	0.0639	0.0322	0.0835	0.0400	0.0429	0.0648
60	SD	0.1358	0.2497							0.1422
61	Sharpe ratio	0.44	0.35							0.46

SPREADSHEET 8.1

Implementing the index model



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of correlations of residuals for industry indexes (rather than individual stocks in the same industry) would be far more in accord with the model. In fact, a few of the stocks in this sample actually seem to have negatively correlated residuals. Of course, correlation also is subject to statistical sampling error, and this may be a fluke.

Panel 3 produces covariances derived from Equation 8.10 of the single-index model. Variances of the S&P 500 index and the individual covered stocks appear on the diagonal. The variance estimates for the individual stocks equal $\beta_i^2\sigma_M^2 + \sigma^2(e_i)$. The off-diagonal terms are covariance values and equal $\beta_i\beta_j\sigma_M^2$.

8.4

PORTFOLIO CONSTRUCTION AND
THE SINGLE-INDEX MODEL

In this section, we look at the implications of the index model for portfolio construction.¹² We will see that the model offers several advantages, not only in terms of parameter estimation, but also for the analytic simplification and organizational decentralization that it makes possible.

Alpha and Security Analysis

Perhaps the most important advantage of the single-index model is the framework it provides for macroeconomic and security analysis in the preparation of the input list that is so critical to the efficiency of the optimal portfolio. The Markowitz model requires estimates of risk premiums for each security. The estimate of expected return depends on both macroeconomic and individual-firm forecasts. But if many different analysts perform security analysis for a large organization such as a mutual fund company, a likely result is inconsistency in the macroeconomic forecasts that partly underlie expectations of returns across securities. Moreover, the underlying assumptions for market-index risk and return often are not explicit in the analysis of individual securities.

The single-index model creates a framework that separates these two quite different sources of return variation and makes it easier to ensure consistency across analysts. We can lay down a hierarchy of the preparation of the input list using the framework of the single-index model.

1. Macroeconomic analysis is used to estimate the risk premium and risk of the market index.
2. Statistical analysis is used to estimate the beta coefficients of all securities and their residual variances, $\sigma^2(e_i)$.
3. The portfolio manager uses the estimates for the market-index risk premium and the beta coefficient of a security to establish the expected return of that security *absent* any contribution from security analysis. The market-driven expected return is conditional on information common to all securities, not on information gleaned from security analysis of particular firms. This market-driven expected return can be used as a benchmark.
4. Security-specific expected return forecasts (specifically, security alphas) are derived from various security-valuation models (such as those discussed in Part Five). Thus, the alpha value distills the incremental risk premium attributable to private information developed from security analysis.

¹² The use of the index model to construct optimal risky portfolios was originally developed in Jack Treynor and Fischer Black, "How to Use Security Analysis to Improve Portfolio Selection," *Journal of Business*, January 1973.

In the context of Equation 8.9, the risk premium on a security not subject to security analysis would be $\beta_i E(R_M)$. In other words, the risk premium would derive solely from the security's tendency to follow the market index. Any expected return beyond this benchmark risk premium (the security alpha) would be due to some nonmarket factor that would be uncovered through security analysis.

The end result of security analysis is the list of alpha values. Statistical methods of estimating beta coefficients are widely known and standardized; hence, we would not expect this portion of the input list to differ greatly across portfolio managers. In contrast, macro and security analysis are far less of an exact science and therefore provide an arena for distinguished performance. Using the index model to disentangle the premiums due to market and nonmarket factors, a portfolio manager can be confident that macro analysts compiling estimates of the market-index risk premium and security analysts compiling alpha values are using consistent estimates for the overall market.

In the context of portfolio construction, alpha is more than just one of the components of expected return. It is the key variable that tells us whether a security is a good or a bad buy. Consider an individual stock for which we have a beta estimate from statistical considerations and an alpha value from security analysis. We easily can find many other securities with identical betas and therefore identical systematic components of their risk premiums. Therefore, what really makes a security attractive or unattractive to a portfolio manager is its alpha value. In fact, we've suggested that a security with a positive alpha is providing a premium over and above the premium it derives from its tendency to track the market index. This security is a bargain and therefore should be overweighted in the overall portfolio compared to the passive alternative of using the market-index portfolio as the risky vehicle. Conversely, a negative-alpha security is overpriced and, other things equal, its portfolio weight should be reduced. In more extreme cases, the desired portfolio weight might even be negative, that is, a short position (if permitted) would be desirable.

The Index Portfolio as an Investment Asset

The process of charting the efficient frontier using the single-index model can be pursued much like the procedure we used in Chapter 7, where we used the Markowitz model to find the optimal risky portfolio. Here, however, we can benefit from the simplification the index model offers for deriving the input list. Moreover, portfolio optimization highlights another advantage of the single-index model, namely, a simple and intuitively revealing representation of the optimal risky portfolio. Before we get into the mechanics of optimization in this setting, however, we start by considering the role of the index portfolio in the optimal portfolio.

Suppose the prospectus of an investment company limits the universe of investable assets to only stocks included in the S&P 500 portfolio. In this case, the S&P 500 index captures the impact of the economy on the large stocks the firm may include in its portfolio. Suppose that the resources of the company allow coverage of only a relatively small subset of this so-called *investable universe*. If these analyzed firms are the only ones allowed in the portfolio, the portfolio manager may well be worried about limited diversification.

A simple way to avoid inadequate diversification is to include the S&P 500 portfolio as one of the assets of the portfolio. Examination of Equations 8.8 and 8.9 reveals that if we treat the S&P 500 portfolio as the market index, it will have a beta of 1.0 (its sensitivity to itself), no firm-specific risk, and an alpha of zero—there is no nonmarket component in its expected return. Equation 8.10 shows that the covariance of any security, i , with the index is $\beta_i \sigma_M^2$. To distinguish the S&P 500 from the n securities covered by the firm, we will designate it the $(n + 1)$ th asset. We can think of the S&P 500 as a *passive portfolio* that the manager would select in the absence of security analysis. It gives broad market exposure without the need for expensive security analysis. However, if the manager is willing to engage in such research, she may devise an *active portfolio* that can be mixed with the index to provide an even better risk–return trade-off.

The Single-Index-Model Input List

If the portfolio manager plans to compile a portfolio from a list of n actively researched firms and a passive market index portfolio, the input list will include the following estimates:

1. Risk premium on the S&P 500 portfolio.
2. Estimate of the standard deviation of the S&P 500 portfolio.
3. n sets of estimates of (a) beta coefficients, (b) stock residual variances, and (c) alpha values. (The alpha values for each security, together with the risk premium of the S&P 500 and the beta of each security, will allow for determination of the expected return on each security.)

The Optimal Risky Portfolio of the Single-Index Model

The single-index model allows us to solve for the optimal risky portfolio directly and to gain insight into the nature of the solution. First we confirm that we easily can set up the optimization process to chart the efficient frontier in this framework along the lines of the Markowitz model.

With the estimates of the beta and alpha coefficients, plus the risk premium of the index portfolio, we can generate the $n + 1$ expected returns using Equation 8.9. With the estimates of the beta coefficients and residual variances, together with the variance of the index portfolio, we can construct the covariance matrix using Equation 8.10. Given a column of risk premiums and the covariance matrix, we can conduct the optimization program described in Chapter 7.

We can take the description of how diversification works in the single-index framework of Section 8.2 a step further. We showed earlier that the alpha, beta, and residual variance of an equally weighted portfolio are the simple averages of those parameters across component securities. Moreover, this result is not limited to equally weighted portfolios. It applies to any portfolio, where we need only replace “simple average” with “weighted average,” using the portfolio weights. Specifically,

$$\begin{aligned}\alpha_p &= \sum_{i=1}^{n+1} w_i \alpha_i && \text{for the index } \alpha_{n+1} = \alpha_M = 0 \\ \beta_p &= \sum_{i=1}^{n+1} w_i \beta_i && \text{for the index } \beta_{n+1} = \beta_M = 1 \\ \sigma^2(e_p) &= \sum_{i=1}^{n+1} w_i^2 \sigma^2(e_i) && \text{for the index } \sigma^2(e_{n+1}) = \sigma^2(e_M) = 0\end{aligned}\tag{8.18}$$

The objective is to maximize the Sharpe ratio of the portfolio by using portfolio weights, w_1, \dots, w_{n+1} . With this set of weights, the expected return, standard deviation, and Sharpe ratio of the portfolio are

$$\begin{aligned}E(R_p) &= \alpha_p + E(R_M)\beta_p = \sum_{i=1}^{n+1} w_i \alpha_i + E(R_M) \sum_{i=1}^{n+1} w_i \beta_i \\ \sigma_p &= [\beta_p^2 \sigma_M^2 + \sigma^2(e_p)]^{1/2} = \left[\sigma_M^2 \left(\sum_{i=1}^{n+1} w_i \beta_i \right)^2 + \sum_{i=1}^{n+1} w_i^2 \sigma^2(e_i) \right]^{1/2} \\ S_p &= \frac{E(R_p)}{\sigma_p}\end{aligned}\tag{8.19}$$

At this point, as in the standard Markowitz procedure, we could use Excel's optimization program to maximize the Sharpe ratio subject to the adding-up constraint that the portfolio weights sum to 1. However, this is not necessary because the optimal portfolio can be derived explicitly using the index model. Moreover, the solution for the optimal portfolio provides considerable insight into the efficient use of security analysis in portfolio construction. It is instructive to outline the logical thread of the solution. We will not show every algebraic step, but will instead present the major results and interpretation of the procedure.

Before delving into the results, let us first explain the basic trade-off the model reveals. If we were interested only in diversification, we would just hold the market index. Security analysis gives us the chance to uncover securities with a nonzero alpha and to take a differential position in those securities. The cost of that differential position is a departure from efficient diversification, in other words, the assumption of unnecessary firm-specific risk. The model shows us that the optimal risky portfolio trades off the search for alpha against the departure from efficient diversification.

The optimal risky portfolio turns out to be a combination of two component portfolios: (1) an *active portfolio*, denoted by A , comprised of the n analyzed securities (we call this the *active portfolio* because it follows from active security analysis), and (2) the market-index portfolio, the $(n + 1)$ th asset we include to aid in diversification, which we call the *passive portfolio* and denote by M .

Assume first that the active portfolio has a beta of 1. In that case, the optimal weight in the active portfolio would be proportional to the ratio $\alpha_A/\sigma^2(e_A)$. This ratio balances the contribution of the active portfolio (its alpha) against its contribution to the portfolio variance (residual variance). The analogous ratio for the index portfolio is $E(R_M)/\sigma_M^2$, and hence the initial position in the active portfolio (i.e., if its beta were 1) is

$$w_A^0 = \frac{\frac{\alpha_A}{\sigma_A^2}}{\frac{E(R_M)}{\sigma_M^2}} \quad (8.20)$$

Next, we amend this position to account for the actual beta of the active portfolio. For any level of σ_A^2 , the correlation between the active and passive portfolios is greater when the beta of the active portfolio is higher. This implies less diversification benefit from the passive portfolio and a lower position in it. Correspondingly, the position in the active portfolio increases. The precise modification for the position in the active portfolio is:¹³

$$w_A^* = \frac{w_A^0}{1 + (1 - \beta_A)w_A^0} \quad (8.21)$$

Notice that when $\beta_A = 1$, $w_A^* = w_A^0$.

The Information Ratio

Equations 8.20 and 8.21 yield the optimal position in the active portfolio once we know its alpha, beta, and residual variance. With w_A^* in the active portfolio and $1 - w_A^*$ invested in the index portfolio, we can compute the expected return, standard deviation, and Sharpe ratio of the optimal risky portfolio. The Sharpe ratio of an optimally constructed risky

¹³With a little algebraic manipulation, beta can be shown to equal the product of correlation between the index and the active portfolio and the ratio of SD(index)/SD(active portfolio). If $\beta_A > 1$, then correlation is higher than envisioned in Equation 8.20, so the diversification value of the index is lower. This requires the modification in Equation 8.21.

portfolio will exceed that of the index portfolio (the passive strategy). The exact relationship is

$$S_P^2 = S_M^2 + \left[\frac{\alpha_A}{\sigma(e_A)} \right]^2 \quad (8.22)$$

Equation 8.22 shows us that the contribution of the active portfolio (when held in its optimal weight, w_A^*) to the Sharpe ratio of the overall risky portfolio is determined by the ratio of its alpha to its residual standard deviation. This important ratio is called the **information ratio**. This ratio measures the extra return we can obtain from security analysis compared to the firm-specific risk we incur when we over- or underweight securities relative to the passive market index. Equation 8.22 therefore implies that to maximize the overall Sharpe ratio, we must maximize the information ratio of the active portfolio.

It turns out that the information ratio of the active portfolio will be maximized if we invest in each security in proportion to its ratio of $\alpha_i/\sigma^2(e_i)$. Scaling this ratio so that the total position in the active portfolio adds up to w_A^* , the weight in each security is

$$w_i^* = w_A^* \frac{\frac{\alpha_i}{\sigma^2(e_i)}}{\sum_{i=1}^n \frac{\alpha_i}{\sigma^2(e_i)}} \quad (8.23)$$

With this set of weights, we find that the contribution of each security to the information ratio of the active portfolio depends on its *own* information ratio, that is,

$$\left[\frac{\alpha_A}{\sigma(e_A)} \right]^2 = \sum_{i=1}^n \left[\frac{\alpha_i}{\sigma(e_i)} \right]^2 \quad (8.24)$$

The model thus reveals the central role of the information ratio in efficiently taking advantage of security analysis. The positive contribution of a security to the portfolio is made by its addition to the nonmarket risk premium (its alpha). Its negative impact is to increase the portfolio variance through its firm-specific risk (residual variance).

In contrast to alpha, notice that the market (systematic) component of the risk premium, $\beta_i E(R_M)$, is offset by the security's nondiversifiable (market) risk, $\beta_i^2 \sigma_M^2$, and both are driven by the same beta. This trade-off is not unique to any security, as any security with the same beta makes the same balanced contribution to both risk and return. Put differently, the beta of a security is neither vice nor virtue. It is a property that simultaneously affects the risk *and* risk premium of a security. Hence we are concerned only with the aggregate beta of the active portfolio, rather than the beta of each individual security.

We see from Equation 8.23 that if a security's alpha is negative, the security will assume a short position in the optimal risky portfolio. If short positions are prohibited, a negative-alpha security would simply be taken out of the optimization program and assigned a portfolio weight of zero. As the number of securities with nonzero alpha values (or the number with positive alphas if short positions are prohibited) increases, the active portfolio will itself be better diversified and its weight in the overall risky portfolio will increase at the expense of the passive index portfolio.

Finally, we note that the index portfolio is an efficient portfolio only if all alpha values are zero. This makes intuitive sense. Unless security analysis reveals that a security has a nonzero alpha, including it in the active portfolio would make the portfolio less attractive. In addition to the security's systematic risk, which is compensated for by the market risk premium (through beta), the security would add its firm-specific risk to portfolio variance.

With a zero alpha, however, the latter is not compensated by an addition to the nonmarket risk premium. Hence, if all securities have zero alphas, the optimal weight in the active portfolio will be zero, and the weight in the index portfolio will be 1. However, when security analysis uncovers securities with nonmarket risk premiums (nonzero alphas), the index portfolio is no longer efficient.

Summary of Optimization Procedure

To summarize, once security analysis is complete, and the index-model estimates of security and market index parameters are established, the optimal risky portfolio can be formed using these steps:

1. Compute the initial position of each security in the active portfolio as $w_i^0 = \alpha_i / \sigma^2(e_i)$.
2. Scale those initial positions to force portfolio weights to sum to 1 by dividing by their sum, that is, $w_i = \frac{w_i^0}{\sum_{i=1}^n w_i^0}$.
3. Compute the alpha of the active portfolio: $\alpha_A = \sum_{i=1}^n w_i \alpha_i$.
4. Compute the residual variance of the active portfolio: $\sigma^2(e_A) = \sum_{i=1}^n w_i^2 \sigma^2(e_i)$.

5. Compute the initial position in the active portfolio: $w_A^0 = \frac{\left[\frac{\alpha_A}{\sigma^2(e_A)} \right]}{\left[\frac{E(R_M)}{\sigma_M^2} \right]}$.

6. Compute the beta of the active portfolio: $\beta_A = \sum_{i=1}^n w_i \beta_i$.

7. Adjust the initial position in the active portfolio: $w_A^* = \frac{w_A^0}{1 + (1 - \beta_A)w_A^0}$.

8. Note: the optimal risky portfolio now has weights: $w_M^* = 1 - w_A^*$; $w_i^* = w_A^* w_i$.

9. Calculate the risk premium of the optimal risky portfolio from the risk premium of the index portfolio and the alpha of the active portfolio:
 $E(R_p) = (w_M^* + w_A^* \beta_A)E(R_M) + w_A^* \alpha_A$. Notice that the beta of the risky portfolio is $w_M^* + w_A^* \beta_A$ because the beta of the index portfolio is 1.

10. Compute the variance of the optimal risky portfolio from the variance of the index portfolio and the residual variance of the active portfolio:
 $\sigma_p^2 = (w_M^* + w_A^* \beta_A)^2 \sigma_M^2 + [w_A^* \sigma(e_A)]^2$.

An Example

We can illustrate the implementation of the index model by constructing an optimal portfolio from the S&P 500 index and the six stocks for which we analyzed risk parameters in Section 8.3.

This example entails only six analyzed stocks, but by virtue of selecting three *pairs* of firms from the same industry, it is designed to produce relatively high residual correlations. This should put the index model to a severe test, as the model ignores the correlation between residuals when producing estimates for the covariance matrix. Therefore, comparison

of results from the index model with the full-blown covariance (Markowitz) model should be instructive.

Risk Premium Forecasts Panel 4 of Spreadsheet 8.1 contains estimates of alpha and the risk premium for each stock. These alphas ordinarily would be the most important production of the investment company in a real-life procedure. Statistics plays a small role here; in this arena, macro/security analysis is king. In this example, we simply use illustrative values to demonstrate the portfolio construction process and possible results. You may wonder why we have chosen such small, forecast alpha values. The reason is that even when security analysis uncovers a large apparent mispricing, that is, large alpha values, these forecasts must be substantially trimmed to account for the fact that such forecasts are subject to large estimation error. We discuss the important procedure of adjusting actual forecasts in Chapter 27.

The Optimal Risky Portfolio Panel 5 of Spreadsheet 8.1 displays calculations for the optimal risky portfolio. They follow the summary procedure of Section 8.4 (you should try to replicate these calculations in your own spreadsheet). In this example we allow short sales. Notice that the weight of each security in the active portfolio (see row 52) has the same sign as the alpha value. Allowing short sales, the positions in the active portfolio are quite large (e.g., the position in BP is .7349); this is an aggressive portfolio. As a result, the alpha of the active portfolio (2.22%) is larger than that of any of the individual alpha forecasts. However, this aggressive stance also results in a large residual variance (.0404, which corresponds to a residual standard deviation of 20%). Therefore, the position in the active portfolio is scaled down (see Equation 8.20) and ends up quite modest (.1718; cell C57), reinforcing the notion that diversification considerations are paramount in the optimal risky portfolio.

The optimal risky portfolio has a risk premium of 6.48%, standard deviation of 14.22%, and a Sharpe ratio of .46 (cells J58–J61). By comparison, the Sharpe ratio of the index portfolio is $.06/.1358 = .44$ (cell B61), which is quite close to that of the optimal risky portfolio. The small improvement is a result of the modest alpha forecasts that we used. In Chapter 11 on market efficiency and Chapter 24 on performance evaluation we demonstrate that such results are common in the mutual fund industry. Of course, some portfolio managers can and do produce portfolios with better performance.

The interesting question here is the extent to which the index model produces results that are inferior to that of the full-covariance (Markowitz) model. Figure 8.5 shows the efficient frontiers from the two models with the example data. We find that the difference is in fact negligible. Table 8.2 compares the compositions and expected performance of the global minimum variance (G) and the optimal risky portfolios derived from the two models. The significant difference between the two portfolios is limited to the minimum-variance portfolios that are driven only by considerations of variance. As we move up the efficient frontier, the required expected returns obviate the impact of the differences in covariance and the portfolios become similar in performance.

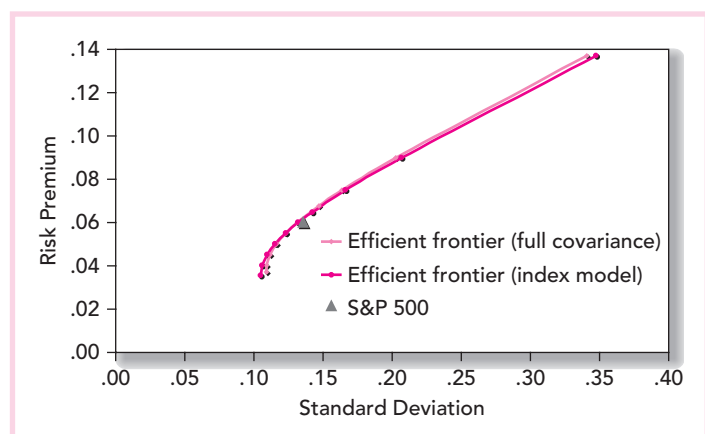


FIGURE 8.5 Efficient frontiers with the index model and full-covariance matrix

TABLE 8.2

Comparison of portfolios from the single-index and full-covariance models

	Global Minimum Variance Portfolio		Optimal Portfolio	
	Full-Covariance Model	Index Model	Full-Covariance Model	Index Model
Mean	.0371	.0354	.0677	.0649
SD	.1089	.1052	.1471	.1423
Sharpe ratio	.3409	.3370	.4605	.4558
Portfolio Weights				
S&P 500	.88	.83	.75	.83
HP	-.11	-.17	.10	.07
DELL	-.01	-.05	-.04	-.06
WMT	.23	.14	-.03	-.05
TARGET	-.18	-.08	.10	.06
BP	.22	.20	.25	.13
SHELL	-.02	.12	-.12	.03

8.5

PRACTICAL ASPECTS OF PORTFOLIO MANAGEMENT WITH THE INDEX MODEL

The tone of our discussions in this chapter indicates that the index model is the preferred one for practical portfolio management. Switching from the Markowitz to an index model is an important decision and hence the first question is whether the index model is really inferior to the Markowitz full-covariance model.

Is the Index Model Inferior to the Full-Covariance Model?

This question is partly related to a more general question of the value of parsimonious models. As an analogy, consider the question of adding additional explanatory variables in a regression equation. We know that adding explanatory variables will in most cases increase R -square, and in no case will R -square fall. But this does not necessarily imply a better regression equation.¹⁴ A better criterion is contribution to the predictive power of the regression. The appropriate question is whether inclusion of a variable that contributes to in-sample explanatory power is likely to contribute to out-of-sample forecast precision. Adding variables, even ones that may appear significant, sometimes can be hazardous to forecast precision. Put differently, a parsimonious model that is stingy about inclusion of independent variables is often superior. Predicting the value of the dependent variable depends on two factors, the precision of the coefficient estimates and the precision of the forecasts of the independent variables. When we add variables, we introduce errors on both counts.

This problem applies as well to replacing the single-index with the full-blown Markowitz model, or even a multi-index model of security returns. To add another index, we need both a forecast of the risk premium of the additional index portfolio and estimates of security betas with respect to that additional factor. The Markowitz model allows far more flexibility in our modeling of asset covariance structure compared to the single-index model. But that advantage may be illusory if we can't estimate those covariances with any degree

¹⁴In fact, the adjusted R -square may fall if the additional variable does not contribute enough explanatory power to compensate for the extra degree of freedom it uses.

of confidence. Using the full-covariance matrix invokes estimation risk of thousands of terms. Even if the full Markowitz model would be better *in principle*, it is very possible that cumulative effect of so many estimation errors will result in a portfolio that is actually inferior to that derived from the single-index model.

Against the potential superiority of the full-covariance model, we have the clear practical advantage of the single-index framework. Its aid in decentralizing macro and security analysis is another decisive advantage.

The Industry Version of the Index Model

Not surprisingly, the index model has attracted the attention of practitioners. To the extent that it is approximately valid, it provides a convenient benchmark for security analysis.

A portfolio manager who has no special information about a security nor insight that is unavailable to the general public will take the security's alpha value as zero, and, according to Equation 8.9, will forecast a risk premium for the security equal to $\beta_i R_M$. If we restate this forecast in terms of total returns, one would expect

$$E(r_{HP}) = r_f + \beta_{HP}[E(r_M) - r_f] \quad (8.25)$$

A portfolio manager who has a forecast for the market index, $E(r_M)$, and observes the risk-free T-bill rate, r_f , can use the model to determine the benchmark expected return for any stock. The beta coefficient, the market risk, σ_M^2 , and the firm-specific risk, $\sigma^2(e)$, can be estimated from historical SCLs, that is, from regressions of security excess returns on market index excess returns.

There are many sources for such regression results. One widely used source is Research Computer Services Department of Merrill Lynch, which publishes a monthly *Security Risk Evaluation* book, commonly called the "beta book." The Web sites for this chapter at the Online Learning Center (www.mhhe.com/bkm) also provide security betas.

Security Risk Evaluation uses the S&P 500 as the proxy for the market portfolio. It relies on the 60 most recent monthly observations to calculate regression parameters. Merrill Lynch and most services¹⁵ use total returns, rather than excess returns (deviations from T-bill rates), in the regressions. In this way they estimate a variant of our index model, which is

$$r = a + br_M + e^* \quad (8.26)$$

instead of

$$r - r_f = \alpha + \beta(r_M - r_f) + e \quad (8.27)$$

To see the effect of this departure, we can rewrite Equation 8.27 as

$$r = r_f + \alpha + \beta r_M - \beta r_f + e = \alpha + r_f(1 - \beta) + \beta r_M + e \quad (8.28)$$

Comparing Equations 8.26 and 8.28, you can see that if r_f is constant over the sample period, both equations have the same independent variable, r_M , and residual, e . Therefore, the slope coefficient will be the same in the two regressions.¹⁶

However, the intercept that Merrill Lynch calls alpha is really an estimate of $\alpha + r_f(1 - \beta)$. The apparent justification for this procedure is that, on a monthly basis, $r_f(1 - \beta)$ is small and is apt to be swamped by the volatility of actual stock returns. But it is worth noting that for $\beta \neq 1$, the regression intercept in Equation 8.26 will not equal the index model alpha as it does when excess returns are used as in Equation 8.27.

¹⁵ Value Line is another common source of security betas. Value Line uses weekly rather than monthly data and uses the New York Stock Exchange index instead of the S&P 500 as the market proxy.

¹⁶ Actually, r_f does vary over time and so should not be grouped casually with the constant term in the regression. However, variations in r_f are tiny compared with the swings in the market return. The actual volatility in the T-bill rate has only a small impact on the estimated value of β .

Another way the Merrill Lynch procedure departs from the index model is in its use of percentage changes in price instead of total rates of return. This means that the index model variant of Merrill Lynch ignores the dividend component of stock returns.

Table 8.3 illustrates a page from the beta book which includes estimates for Hewlett-Packard. The third column, Close Price, shows the stock price at the end of the sample period. The next two columns show the beta and alpha coefficients. Remember that Merrill Lynch's alpha is actually an estimate of $\alpha + r_f(1 - \beta)$.

Much of the output that Merrill Lynch reports is similar to the Excel output (Table 8.1) that we discussed when estimating the index model for Hewlett-Packard. The R -square statistic is the ratio of systematic variance to total variance, the fraction of total volatility attributable to market movements. Merrill Lynch actually reports adjusted R -squares (see footnote 6), which accounts for the instances of negative values. For most firms, R -square is substantially below .5, indicating that stocks have far more firm-specific than systematic risk. This highlights the practical importance of diversification.

The *Resid Std Dev-n* column is the standard deviation of the monthly regression residuals, also sometimes called the standard error of the regression. Like Excel, Merrill Lynch also reports the standard errors of the alpha and beta estimates so we can evaluate the precision of the estimates. Notice that the estimates of beta are far more precise than those of alpha.

The next-to-last column is called Adjusted Beta. The motivation for adjusting beta estimates is that, on average, the beta coefficients of stocks seem to move toward 1 over time. One explanation for this phenomenon is intuitive. A business enterprise usually is established to produce a specific product or service, and a new firm may be more unconventional than an older one in many ways, from technology to management style. As it grows, however, a firm often diversifies, first expanding to similar products and later to more diverse operations. As the firm becomes more conventional, it starts to resemble the rest of the economy even more. Thus its beta coefficient will tend to change in the direction of 1.

Another explanation for this phenomenon is statistical. We know that the average beta over all securities is 1. Thus, before estimating the beta of a security, our best forecast of the beta would be that it is 1. When we estimate this beta coefficient over a particular sample period, we sustain some unknown sampling error of the estimated beta. The greater the difference between our beta estimate and 1, the greater is the chance that we incurred a large estimation error and that beta in a subsequent sample period will be closer to 1.

The sample estimate of the beta coefficient is the best guess for that sample period. Given that beta has a tendency to evolve toward 1, however, a forecast of the future beta coefficient should adjust the sample estimate in that direction.

Merrill Lynch adjusts beta estimates in a simple way.¹⁷ It takes the sample estimate of beta and averages it with 1, using weights of two-thirds and one-third:

$$\text{Adjusted beta} = \frac{2}{3} \text{ sample beta} + \frac{1}{3} (1) \quad (8.29)$$

CONCEPT CHECK

4

What was HP's index-model alpha per month during the period covered by the Merrill Lynch regression if during this period the average monthly rate of return on T-bills was .4%?

Always remember that these alpha estimates are ex post (after the fact) measures. They do not mean that anyone could have forecast these alpha values ex ante (before the fact). In fact, the name of the game in security analysis is to forecast alpha values

¹⁷A more sophisticated method is described in Oldrich A. Vasicek, "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas," *Journal of Finance* 28 (1973), pp. 1233–39.

Ticker Symbol	Security Nam	2004/12 Close Price	Beta	Alpha	R-Sqr	Resid Std		Std Error		Adjusted Beta	Number of Observ
						Dev-n	Alpha	Beta	Alpha		
HTBK	HERITAGE COMM CORP	19.020	0.23	0.72	0.01	6.86	0.19	0.89	0.49	60	
HPC	HERCULES INC	14.850	0.78	-0.09	0.07	12.13	0.34	1.57	0.85	60	
HFWA	HERITAGE FINL CORP WASH	22.120	0.09	1.69	-0.01	4.27	0.12	0.55	0.40	60	
HRLY	HERLEY INDS INC	20.340	-0.04	1.66	-0.02	10.37	0.29	1.34	0.31	60	
HT	HERSHA HOSPITALITY TR	11.450	0.46	1.67	0.12	5.62	0.16	0.73	0.64	60	
HSY	HERSHEY FOODS CORP	55.540	-0.21	1.66	0.00	7.72	0.21	1.00	0.20	60	
HSKA	HESKA CORP	1.169	1.87	3.88	0.06	31.26	0.86	4.04	1.58	60	
HPQ	HEWLETT PACKARD CO	20.970	1.76	-0.45	0.40	10.05	0.28	1.30	1.50	60	
HXL	HEXCEL CORP NEW	14.500	0.85	4.08	0.02	21.63	0.60	2.80	0.90	60	
HIFN	HI/FN INC	9.220	2.33	0.88	0.21	20.55	0.57	2.66	1.88	60	
HIBB	HIBBETT SPORTING GOODS	26.610	1.03	4.05	0.11	13.03	0.36	1.68	1.02	60	
HIB	HIBERNIA CORP	29.510	0.59	2.08	0.14	6.53	0.18	0.84	0.73	60	
HICK A	HICKOK INC	7.500	0.29	2.35	-0.01	19.21	0.53	2.48	0.53	60	
HTCO	HICKORY TECH CORP	10.690	0.13	-0.02	-0.01	10.74	0.30	1.39	0.42	60	
HSVL Y	HIGHVELD STL & VANADIUM ADR	8.200	0.34	2.64	0.00	14.42	0.40	1.86	0.56	60	
HIW	HIGHWOODS PROPERTIES IN	27.700	0.10	0.45	-0.01	5.70	0.16	0.74	0.40	60	

TABLE 8.3

Merrill Lynch, Pierce, Fenner & Smith, Inc.: Market sensitivity statistics*

*Based on S&P 500 index using straight regression.

EXAMPLE 8.1 Adjusted Beta

For the 60 months used in Table 8.3, HP's beta was estimated at 1.76. Therefore, its adjusted beta is $\frac{2}{3} \times 1.76 + \frac{1}{3} = 1.51$, taking it a third of the way toward 1.

In the absence of special information concerning HP, if our forecast for the market index is 11% and T-bills pay 5%, we learn from the Merrill Lynch beta book that the forecast for the rate of return on HP stock is

$$\begin{aligned} E(r_{\text{HP}}) &= r_f + \text{adjusted beta} \times [E(r_M) - r_f] \\ &= 5 + 1.51(11 - 5) = 14.06\% \end{aligned}$$

The sample period regression alpha is $-.45\%$. Because HP's beta is greater than 1, we know that this means that the index-model alpha estimate is somewhat larger. As in Equation 8.28, we have to subtract $(1 - \beta)r_f$ from the regression alpha to obtain the index model alpha. In any event, the standard error of the alpha estimate is 1.30%. The estimate of alpha is far less than twice its standard error. Consequently, we cannot reject the hypothesis that the true alpha is zero.

ahead of time. A well-constructed portfolio that includes long positions in future positive-alpha stocks and short positions in future negative-alpha stocks will outperform the market index. The key term here is "well constructed," meaning that the portfolio has to balance concentration on high-alpha stocks with the need for risk-reducing diversification as discussed earlier in the chapter.

Note that HP's RESID STD DEV- N is 10.05% per month and its R^2 is .40. This tells us that $\sigma_{\text{HP}}^2(e) = 10.05^2 = 101.0$ and, because $R^2 = 1 - \sigma^2(e)/\sigma^2$, we can solve for the estimate of HP's total standard deviation by rearranging as follows:

$$\sigma_{\text{HP}} = \left[\frac{\sigma_{\text{HP}}^2(e)}{1 - R^2} \right]^{1/2} = \left(\frac{101}{.60} \right)^{1/2} = 12.97\% \text{ per month}$$

This is HP's monthly standard deviation for the sample period. Therefore, the annualized standard deviation for that period was $12.97\sqrt{12} = 44.93\%$.

Finally, the last column shows the number of observations, which is 60 months, unless the stock is newly listed and fewer observations are available.

Predicting Betas

Merrill Lynch's adjusted betas are a simple way to recognize that betas estimated from past data may not be the best estimates of future betas: Betas seem to drift toward 1 over time. This suggests that we might want a forecasting model for beta.

One simple approach would be to collect data on beta in different periods and then estimate a regression equation:

$$\text{Current beta} = a + b (\text{Past beta}) \quad (8.30)$$

Given estimates of a and b , we would then forecast future betas using the rule

$$\text{Forecast beta} = a + b (\text{Current beta}) \quad (8.31)$$

There is no reason, however, to limit ourselves to such simple forecasting rules. Why not also investigate the predictive power of other financial variables in forecasting beta? For

example, if we believe that firm size and debt ratios are two determinants of beta, we might specify an expanded version of Equation 8.30 and estimate

$$\text{Current beta} = a + b_1(\text{Past beta}) + b_2(\text{Firm size}) + b_3(\text{Debt ratio})$$

Now we would use estimates of a and b_1 through b_3 to forecast future betas.

Such an approach was followed by Rosenberg and Guy¹⁸ who found the following variables to help predict betas:

1. Variance of earnings.
2. Variance of cash flow.
3. Growth in earnings per share.
4. Market capitalization (firm size).
5. Dividend yield.
6. Debt-to-asset ratio.

Rosenberg and Guy also found that even after controlling for a firm's financial characteristics, industry group helps to predict beta. For example, they found that the beta values of gold mining companies are on average .827 lower than would be predicted based on financial characteristics alone. This should not be surprising; the $-.827$ "adjustment factor" for the gold industry reflects the fact that gold values are inversely related to market returns.

Table 8.4 presents beta estimates and adjustment factors for a subset of firms in the Rosenberg and Guy study.

CONCEPT CHECK
5

Compare the first five and last four industries in Table 8.4. What characteristic seems to determine whether the adjustment factor is positive or negative?

Index Models and Tracking Portfolios

Suppose a portfolio manager believes she has identified an underpriced portfolio. Her security analysis team estimates the index model equation for this portfolio (using the S&P 500 index) in excess return form and obtains the following estimates:

$$R_p = .04 + 1.4R_{S\&P500} + e_p \quad (8.32)$$

Therefore, P has an alpha value of 4% and a beta of 1.4. The manager is confident in the quality of her security analysis but is wary about the performance of the broad market in the

Industry	Beta	Adjustment Factor
Agriculture	0.99	-.140
Drugs and medicine	1.14	-.099
Telephone	0.75	-.288
Energy utilities	0.60	-.237
Gold	0.36	-.827
Construction	1.27	.062
Air transport	1.80	.348
Trucking	1.31	.098
Consumer durables	1.44	.132

TABLE 8.4

Industry betas and adjustment factors

¹⁸Barr Rosenberg and J. Guy, "Prediction of Beta from Investment Fundamentals, Parts 1 and 2," *Financial Analysts Journal*, May–June and July–August 1976.

ALPHA BETTING

IT HAS never been easier to pay less to invest. No fewer than 136 exchange-traded funds (ETFs) were launched in the first half of 2006, more than in the whole of 2005.

For those who believe in efficient markets, this represents a triumph. ETFs are quoted securities that track a particular index, for a fee that is normally just a fraction of a percentage point. They enable investors to assemble a low-cost portfolio covering a wide range of assets from international equities, through government and corporate bonds, to commodities.

But as fast as the assets of ETFs and index-tracking mutual funds are growing, another section of the industry seems to be flourishing even faster. Watson Wyatt, a firm of actuaries, estimates that "alternative asset investment" (ranging from hedge funds through private equity to property) grew by around 20% in 2005, to \$1.26 trillion. Investors who take this route pay much higher fees in the hope of better performance. One of the fastest-growing assets, funds of hedge funds, charge some of the highest fees of all.

Why are people paying up? In part, because investors have learned to distinguish between the market return, dubbed beta, and managers' outperformance, known as alpha. "Why wouldn't you buy beta and alpha separately?" asks Arno Kitts of Henderson Global Investors, a fund-management firm. "Beta is a commodity and alpha is about skill."

Clients have become convinced that no one firm can produce good performance in every asset class. That has led to a "core and satellite" model, in which part of the portfolio is invested in index trackers with the rest in the hands of specialists. But this creates its own problems. Relations with a single balanced manager are simple. It is much harder to research and monitor the performance of specialists. That has encouraged the middlemen—managers of managers (in the traditional institutional business) and funds-of-funds (in the hedge-fund world), which are usually even more expensive.

That their fees endure might suggest investors can identify outperforming fund managers in advance. However, studies suggest this is extremely hard. And even where you can spot talent, much of the extra performance may be siphoned off into higher fees. "A disproportionate amount of the benefits of alpha go to the manager, not the client," says Alan Brown at Schroders, an asset manager.

In any event, investors will probably keep pursuing alpha, even though the cheaper alternatives of ETFs and tracking funds are available. Craig Baker of Watson Wyatt, says that, although above-market returns may not be available to all, clients who can identify them have a "first mover" advantage. As long as that belief exists, managers can charge high fees.

Source: *The Economist*, September 14, 2006. Copyright © 2007 The Economist Newspaper and The Economist Group. All rights reserved.

near term. If she buys the portfolio, and the market as a whole turns down, she still could lose money on her investment (which has a large positive beta) even if her team is correct that the portfolio is underpriced on a relative basis. She would like a position that takes advantage of her team's analysis but is independent of the performance of the overall market.

To this end, a **tracking portfolio** (T) can be constructed. A tracking portfolio for portfolio P is a portfolio designed to match the systematic component of P 's return. The idea is for the portfolio to "track" the market-sensitive component of P 's return. This means the tracking portfolio must have the same beta on the index portfolio as P and as little nonsystematic risk as possible. This procedure is also called *beta capture*.

A tracking portfolio for P will have a levered position in the S&P 500 to achieve a beta of 1.4. Therefore, T includes positions of 1.4 in the S&P 500 and -0.4 in T-bills. Because T is constructed from the index and bills, it has an alpha value of zero.

Now consider buying portfolio P but at the same time offsetting systematic risk by assuming a short position in the tracking portfolio. The short position in T cancels out the systematic exposure of the long position in P : the overall combined position is thus *market neutral*. Therefore, even if the market does poorly, the combined position should not be affected. But the alpha on portfolio P will remain intact. The combined portfolio, C , provides an excess return per dollar of

$$R_C = R_P - R_T = (.04 + 1.4R_{S\&P500} + e_P) - 1.4R_{S\&P500} = .04 + e_P \quad (8.33)$$

While this portfolio is still risky (due to the residual risk, e_p), the systematic risk has been eliminated, and if P is reasonably well-diversified, the remaining nonsystematic risk will be small. Thus the objective is achieved: the manager can take advantage of the 4% alpha without inadvertently taking on market exposure. The process of separating the search for alpha from the choice of market exposure is called *alpha transport*.

This “long-short strategy” is characteristic of the activity of many *hedge funds*. Hedge fund managers identify an underpriced security and then try to attain a “pure play” on the perceived underpricing. They hedge out all extraneous risk, focusing the bet only on the perceived “alpha” (see the box on p. 272). Tracking funds are the vehicle used to hedge the exposures to which they do *not* want exposure. Hedge fund managers use index regressions such as those discussed here, as well as more-sophisticated variations, to create the tracking portfolios at the heart of their hedging strategies.

1. A single-factor model of the economy classifies sources of uncertainty as systematic (macroeconomic) factors or firm-specific (microeconomic) factors. The index model assumes that the macro factor can be represented by a broad index of stock returns.
2. The single-index model drastically reduces the necessary inputs in the Markowitz portfolio selection procedure. It also aids in specialization of labor in security analysis.
3. According to the index model specification, the systematic risk of a portfolio or asset equals $\beta^2\sigma_M^2$ and the covariance between two assets equals $\beta_i\beta_j\sigma_M^2$.
4. The index model is estimated by applying regression analysis to excess rates of return. The slope of the regression curve is the beta of an asset, whereas the intercept is the asset's alpha during the sample period. The regression line is also called the *security characteristic line*.
5. Optimal active portfolios constructed from the index model include analyzed securities in proportion to their information ratios. The full risky portfolio is a mixture of the active portfolio and the passive market index portfolio. The index portfolio is used to enhance the diversification of the overall risky position.
6. Practitioners routinely estimate the index model using total rather than excess rates of return. This makes their estimate of alpha equal to $\alpha + r_f(1 - \beta)$.
7. Betas show a tendency to evolve toward 1 over time. Beta forecasting rules attempt to predict this drift. Moreover, other financial variables can be used to help forecast betas.

SUMMARY

Related Web sites for this chapter are available at www.mhhe.com/bkm

single-factor model
single-index model
regression equation

residuals
security characteristic line
scatter diagram

information ratio
tracking portfolio

KEY TERMS

1. What are the advantages of the index model compared to the Markowitz procedure for obtaining an efficiently diversified portfolio? What are its disadvantages?
2. What is the basic trade-off when departing from pure indexing in favor of an actively managed portfolio?
3. How does the magnitude of firm-specific risk affect the extent to which an active investor will be willing to depart from an indexed portfolio?
4. Why do we call alpha a “nonmarket” return premium? Why are high-alpha stocks desirable investments for active portfolio managers? With all other parameters held fixed, what would happen to a portfolio's Sharpe ratio as the alpha of its component securities increased?

PROBLEM SETS

Quiz

Problems

5. A portfolio management organization analyzes 60 stocks and constructs a mean-variance efficient portfolio using only these 60 securities.
- How many estimates of expected returns, variances, and covariances are needed to optimize this portfolio?
 - If one could safely assume that stock market returns closely resemble a single-index structure, how many estimates would be needed?
6. The following are estimates for two stocks.

Stock	Expected Return	Beta	Firm-Specific Standard Deviation
A	13%	0.8	30%
B	18	1.2	40

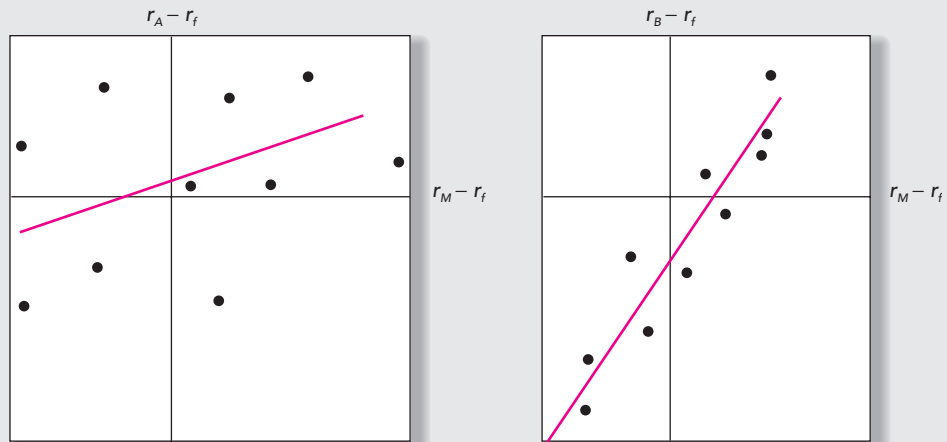
The market index has a standard deviations of 22% and the risk-free rate is 8%.

- What are the standard deviations of stocks A and B?
- Suppose that we were to construct a portfolio with proportions:

Stock A:	.30
Stock B:	.45
T-bills:	.25

Compute the expected return, standard deviation, beta, and nonsystematic standard deviation of the portfolio.

7. Consider the following two regression lines for stocks A and B in the following figure.



- Which stock has higher firm-specific risk?
 - Which stock has greater systematic (market) risk?
 - Which stock has higher R^2 ?
 - Which stock has higher alpha?
 - Which stock has higher correlation with the market?
8. Consider the two (excess return) index model regression results for A and B:
- $$R_A = 1\% + 1.2R_M$$
- $$R\text{-square} = .576$$
- $$\text{Residual standard deviation} = 10.3\%$$
- $$R_B = -2\% + .8R_M$$
- $$R\text{-square} = .436$$
- $$\text{Residual standard deviation} = 9.1\%$$

- Which stock has more firm-specific risk?
- Which has greater market risk?
- For which stock does market movement explain a greater fraction of return variability?
- If r_f were constant at 6% and the regression had been run using total rather than excess returns, what would have been the regression intercept for stock A?

Use the following data for Problems 9 through 14. Suppose that the index model for stocks A and B is estimated from excess returns with the following results:

$$R_A = 3\% + .7R_M + e_A$$

$$R_B = -2\% + 1.2R_M + e_B$$

$$\sigma_M = 20\%; R\text{-square}_A = .20; R\text{-square}_B = .12$$

- What is the standard deviation of each stock?
- Break down the variance of each stock to the systematic and firm-specific components.
- What are the covariance and correlation coefficient between the two stocks?
- What is the covariance between each stock and the market index?
- For portfolio P with investment proportions of .60 in A and .40 in B, rework Problems 9, 10, and 12.
- Rework Problem 13 for portfolio Q with investment proportions of .50 in P , .30 in the market index, and .20 in T-bills.
- A stock recently has been estimated to have a beta of 1.24:
 - What will Merrill Lynch compute as the “adjusted beta” of this stock?
 - Suppose that you estimate the following regression describing the evolution of beta over time:

$$\beta_t = .3 + .7\beta_{t-1}$$

What would be your predicted beta for next year?

- Based on current dividend yields and expected growth rates, the expected rates of return on stocks A and B are 11% and 14%, respectively. The beta of stock A is .8, while that of stock B is 1.5. The T-bill rate is currently 6%, while the expected rate of return on the S&P 500 index is 12%. The standard deviation of stock A is 10% annually, while that of stock B is 11%. If you currently hold a passive index portfolio, would you choose to add either of these stocks to your holdings?
- A portfolio manager summarizes the input from the macro and micro forecasters in the following table:

Micro Forecasts

Asset	Expected Return (%)	Beta	Residual Standard Deviation (%)
Stock A	20	1.3	58
Stock B	18	1.8	71
Stock C	17	0.7	60
Stock D	12	1.0	55

Macro Forecasts

Asset	Expected Return (%)	Standard Deviation (%)
T-bills	8	0
Passive equity portfolio	16	23

- Calculate expected excess returns, alpha values, and residual variances for these stocks.
- Construct the optimal risky portfolio.
- What is Sharpe’s measure for the optimal portfolio and how much of it is contributed by the active portfolio?
- What should be the exact makeup of the complete portfolio for an investor with a coefficient of risk aversion of 2.8?

18. Recalculate Problem 17 for a portfolio manager who is not allowed to short sell securities.
 - a. What is the cost of the restriction in terms of Sharpe's measure?
 - b. What is the utility loss to the investor ($A = 2.8$) given his new complete portfolio?
19. Suppose that based on the analyst's past record, you estimate that the relationship between forecast and actual alpha is:

$$\text{Actual abnormal return} = .3 \times \text{Forecast of alpha}$$

Use the alphas from Problem 17. How much is expected performance affected by recognizing the imprecision of alpha forecasts?

20. Suppose that the alpha forecasts in row 44 of Spreadsheet 8.1 are doubled. All the other data remain the same. Recalculate the optimal risky portfolio. Before you do any calculations, however, use the Summary of Optimization Procedure to estimate a back-of-the-envelope calculation of the information ratio and Sharpe ratio of the newly optimized portfolio. Then recalculate the entire spreadsheet example and verify your back-of-the-envelope calculation.

Challenge Problem



1. When the annualized monthly percentage rates of return for a stock market index were regressed against the returns for ABC and XYZ stocks over a 5-year period ending in 2008, using an ordinary least squares regression, the following results were obtained:

Statistic	ABC	XYZ
Alpha	-3.20%	7.3%
Beta	0.60	0.97
R^2	0.35	0.17
Residual standard deviation	13.02%	21.45%

Explain what these regression results tell the analyst about risk–return relationships for each stock over the sample period. Comment on their implications for future risk–return relationships, assuming both stocks were included in a diversified common stock portfolio, especially in view of the following additional data obtained from two brokerage houses, which are based on 2 years of weekly data ending in December 2008.

Brokerage House	Beta of ABC	Beta of XYZ
A	.62	1.45
B	.71	1.25

2. Assume the correlation coefficient between Baker Fund and the S&P 500 Stock Index is .70. What percentage of Baker Fund's total risk is specific (i.e., nonsystematic)?
3. The correlation between the Charlottesville International Fund and the EAFE Market Index is 1.0. The expected return on the EAFE Index is 11%, the expected return on Charlottesville International Fund is 9%, and the risk-free return in EAFE countries is 3%. Based on this analysis, what is the implied beta of Charlottesville International?
4. The concept of *beta* is most closely associated with:
 - a. Correlation coefficients.
 - b. Mean-variance analysis.
 - c. Nonsystematic risk.
 - d. Systematic risk.
5. Beta and standard deviation differ as risk measures in that beta measures:
 - a. Only unsystematic risk, while standard deviation measures total risk.
 - b. Only systematic risk, while standard deviation measures total risk.

- c. Both systematic and unsystematic risk, while standard deviation measures only unsystematic risk.
- d. Both systematic and unsystematic risk, while standard deviation measures only systematic risk.

Go to www.mhhe.com/edumarketinsight and click on the *Company* link. Enter the ticker symbol for the stock of your choice and click on the *Go* button. In the *Excel Analytics* section go to the *Market Data* section and get the *Monthly Adjusted Prices* data for the past 4 years. The page will also show monthly returns for your stock and for the S&P 500. Copy the data into an *Excel* worksheet and then do a regression to generate the characteristic line for the stock. (Use the menus for *Tools, Data Analysis, Regression*, input the X range and the Y range, select *New Worksheet Ply* under *Output Options*, and click on *OK*.) Based on the regression results, what is the beta coefficient for your stock?

Next use *Excel* to plot an X-Y Scatter graph of the stock's returns versus the S&P 500's returns. Once the graph is constructed, select one of the data points and right click on it. Choose the *Add Trendline* option and select the *Linear* type. On the *Options* tab, select *Display Equation on Chart*. How does the equation compare with your regression results?

Go back to the main page for your stock's information and select *S&P Stock Reports* from the menu. Choose *Stock Report* from the submenu and when the stock report opens, find the beta coefficient for the firm. How does this beta compare to your results? What are possible reasons for any differences?



E-Investments

Beta Estimates

Go to <http://finance.yahoo.com> and click on *Stocks* link under the *Investing* tab. Look for the *Stock Screener* link under *Research Tools*. The *Java Yahoo! Finance Screener* lets you create your own screens. In the *Click to Add Criteria* box, find *Trading and Volume* on the menu and choose *Beta*. In the *Conditions* box, choose \leq and in the *Values* box, enter 1. Hit the *Enter* key and then request the top 200 matches in the *Return Top_Matches* box. Click on the *Run Screen* button.

Select the *View Table* tab and sort the results to show the lowest betas at the top of the list by clicking on the *Beta* column header. Which firms have the lowest betas? In which industries do they operate?

Select the *View Histogram* tab and when the histogram appears, look at the bottom of the screen to see the *Show Histogram for* box. Use the menu that comes up when you click on the down arrow to select *beta*. What pattern(s), if any, do you see in the distributions of betas for firms that have betas less than 1?

SOLUTIONS TO CONCEPT CHECKS

1. a. Total market capitalization is $3,000 + 1,940 + 1,360 = 6,300$. Therefore, the mean excess return of the index portfolio is

$$\frac{3,000}{6,300} \times 10 + \frac{1,940}{6,300} \times 2 + \frac{1,360}{6,300} \times 17 = 9.05\% = .0905$$

b. The covariance between stocks A and B equals

$$\text{Cov}(R_A, R_B) = \beta_A \beta_B \sigma_M^2 = 1 \times .2 \times .25^2 = .0125$$

c. The covariance between stock B and the index portfolio equals

$$\text{Cov}(R_B, R_M) = \beta_B \sigma_M^2 = .2 \times .25^2 = .0125$$

d. The total variance of B equals

$$\sigma_B^2 = \text{Var}(\beta_B R_M + e_B) = \beta_B^2 \sigma_M^2 + \sigma^2(e_B)$$

$$\text{Systematic risk equals } \beta_B^2 \sigma_M^2 = .2^2 \times .25^2 = .0025.$$

Thus the firm-specific variance of B equals

$$\sigma^2(e_B) = \sigma_B^2 - \beta_B^2 \sigma_M^2 = .30^2 - .2^2 \times .25^2 = .0875$$

2. The variance of each stock is $\beta^2 \sigma_M^2 + \sigma^2(e)$.

For stock A , we obtain

$$\sigma_A^2 = .9^2(20)^2 + 30^2 = 1,224$$

$$\sigma_A = 35\%$$

For stock B ,

$$\sigma_B^2 = 1.1^2(20)^2 + 10^2 = 584$$

$$\sigma_B = 24\%$$

The covariance is

$$\beta_A \beta_B \sigma_M^2 = .9 \times 1.1 \times 20^2 = 396$$

3. $\sigma^2(e_p) = (\frac{1}{2})^2[\sigma^2(e_A) + \sigma^2(e_B)]$
 $= \frac{1}{4}(.30^2 + .10^2)$
 $= .0250$

Therefore $\sigma(e_p) = .158 = 15.8\%$

4. Merrill Lynch's alpha is related to the index-model alpha by

$$\alpha_{\text{Merrill}} = \alpha_{\text{index model}} + (1 - \beta)r_f$$

For HP, $\alpha_{\text{Merrill}} = -.45\%$, $\beta = 1.76$, and we are told that r_f was $.4\%$. Thus

$$\alpha_{\text{index model}} = -.45\% - (1 - 1.76).4\% = -.146\%.$$

HP's return was somewhat disappointing even after correcting Merrill Lynch's alpha. It underperformed its "benchmark" return by an average of $.146\%$ per month.

5. The industries with positive adjustment factors are most sensitive to the economy. Their betas would be expected to be higher because the business risk of the firms is higher. In contrast, the industries with negative adjustment factors are in business fields with a lower sensitivity to the economy. Therefore, for any given financial profile, their betas are lower.