MACROECONOMICS I

Class 3. Explaining Economic Growth.

The Solow-Swan Model

March 7th, 2014

Announcement

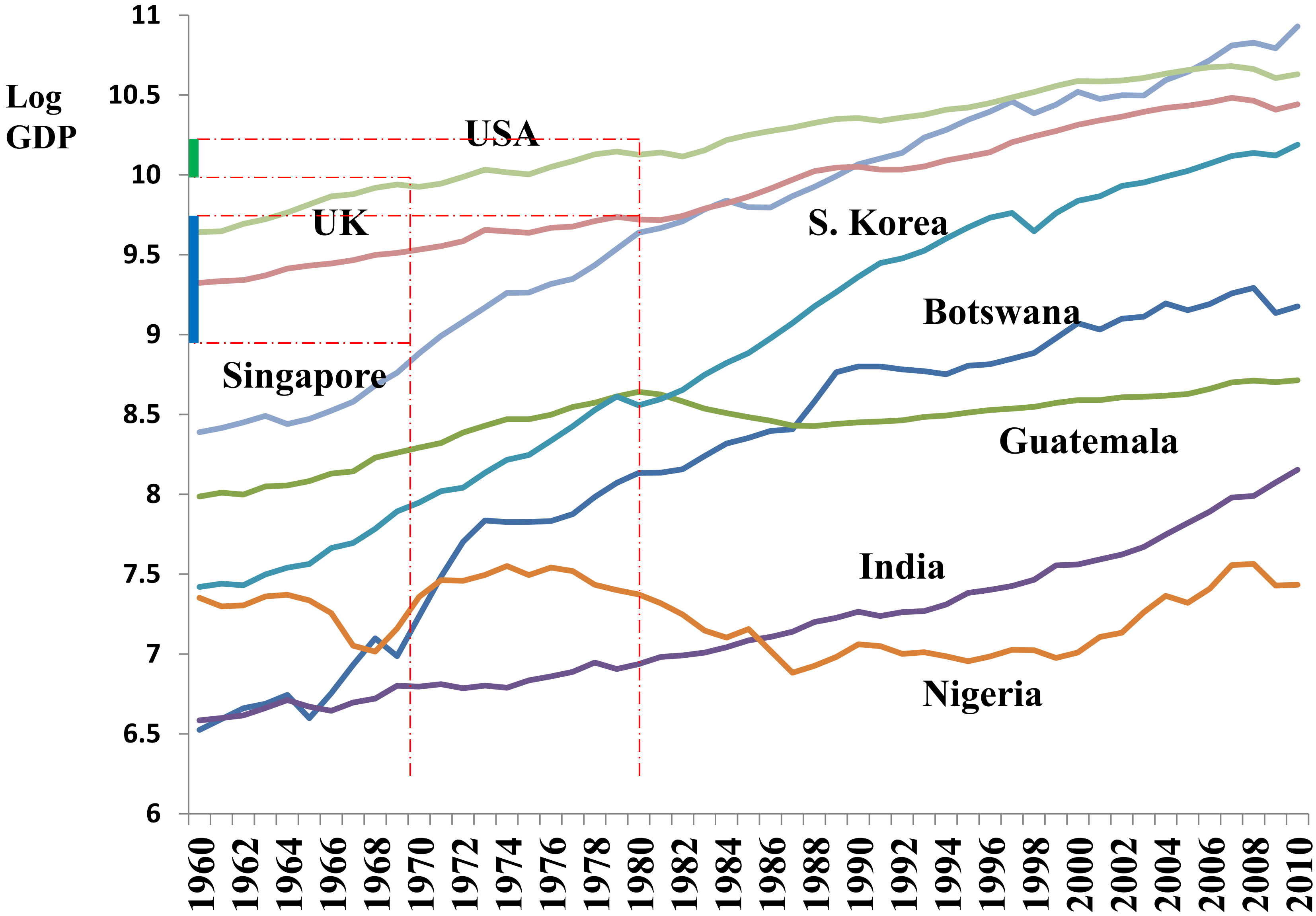
Homework assignment #1 is now posted on the web

Deadline: March 21st, before the class (12:00)

Submission: One hard copy of answers from a group

N!B! **NO** late submissions will be accepted

The evolution of GDP per capita, 1960-2010



Solow-Swan Model of Economic Growth(1956)

- What drives changes in GDP per capita **in the long run**?

- **Robert Solow (1956)**

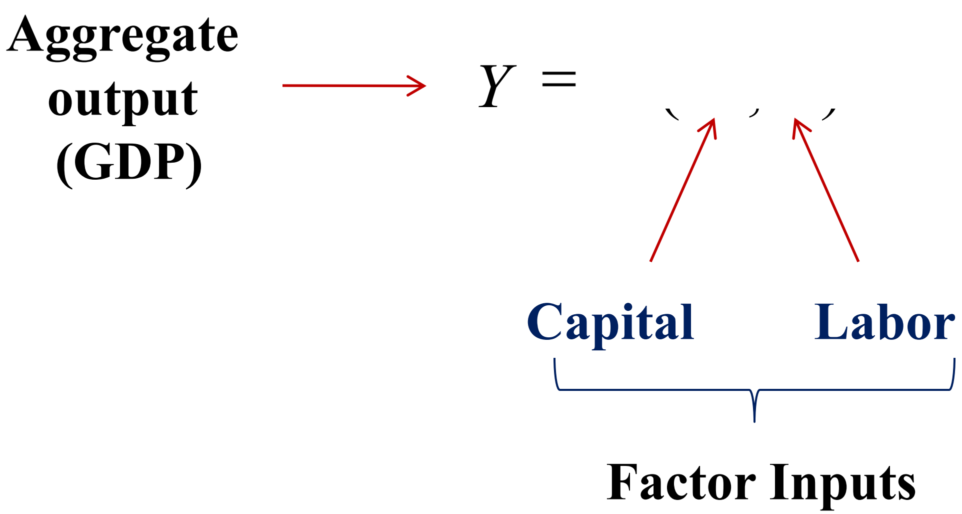
Economic environment (a set of assumptions)

- A single composite good
- **Two factors** of production: capital and labor
- **Two agents**: firms and households
- A **closed** economy

Solow-Swan Model: Supply Side

Production function (technology)

- Maximum output for given inputs



- Production of movies

Solow-Swan Model: Returns to Scale

- Output is a **positive** function of inputs $Y = \quad (+) \quad (+)$

What would happen to GDP is both inputs increase twice?

- **Constant returns to scale (CRS)**

If the quantity of **both** inputs doubles, the output also doubles

$$2Y = \quad L)$$

- *Decreasing* vs. *Increasing* returns to scale

Solow-Swan Model: Returns to Factor Inputs

What would happen to GDP if **only one** input increases?

- **Diminishing returns** to factor inputs

For a **fixed L**, an increase in K would lead to smaller and smaller increase in Y

For a **fixed K**, an increase in L would lead to smaller and smaller increase in Y

- Increasing returns to factor inputs

Solow-Swan Model: GDP Per Capita

- Transforming the model to **per capital** terms

$$Y =$$

$$\frac{Y}{L} =$$

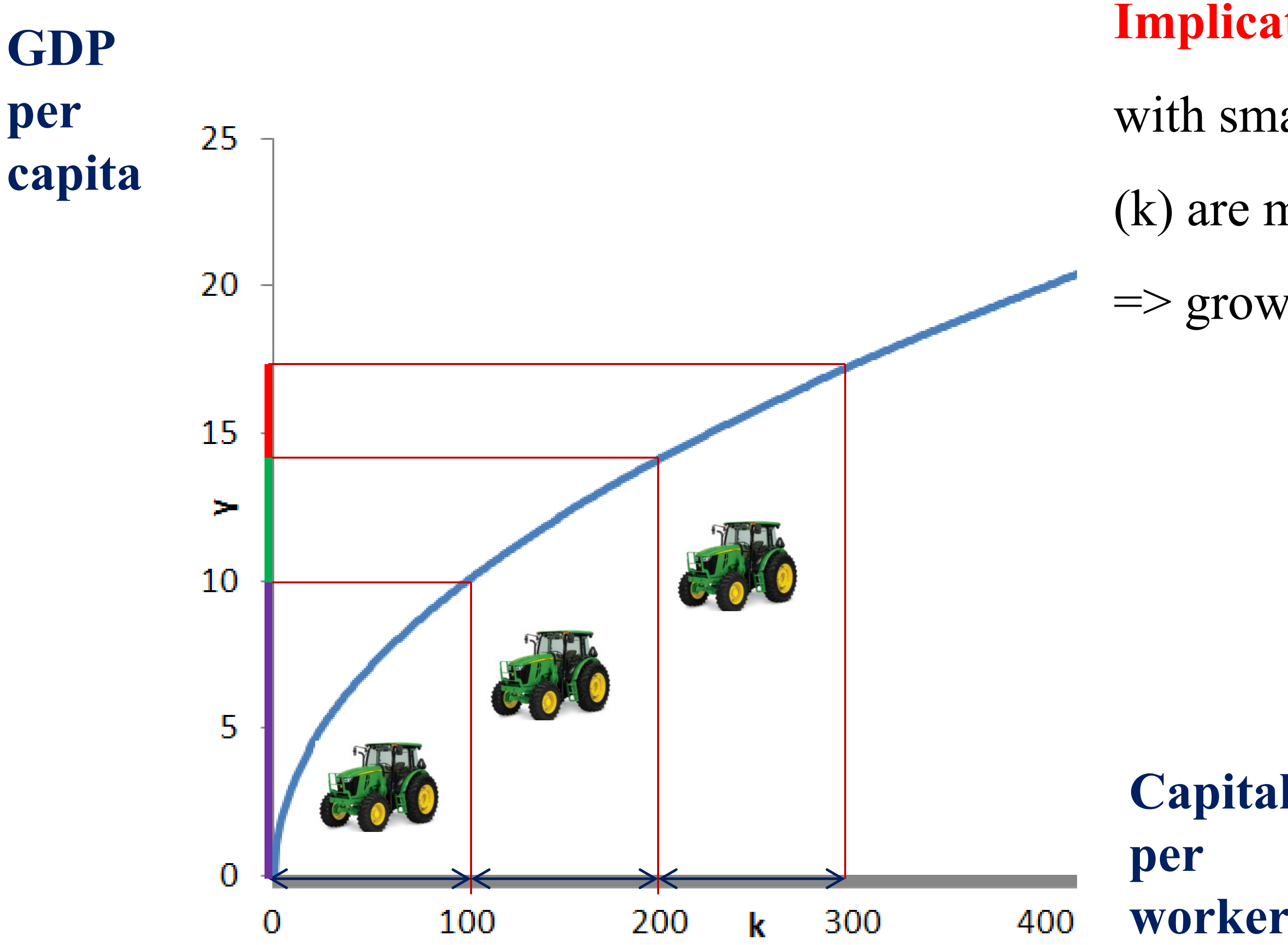
$$y =$$

GDP per capita **Capital per worker** **or Capital/Labor ratio**

N!B! The level of **capital per worker** determines the level of **output per worker**

Solow-Swan Model: Diminishing Returns

Production function: $y = \; , \; (\; , = \;$



Implication: Countries with small capital stock (k) are more productive => grow faster

Solow-Swan Model: Diminishing Returns (Cont.

Country	Average annual growth rate of GDP per capita	
	1950-1960	**1980-1990**
Germany	6.6 %	1.9 %
Japan	6.8 %	3.4 %
France	9.6 %	2.8%
USA	1.2 %	2.3 %

Source: Blanchard et al (2010)

Solow-Swan Model: Demand Side

$$Y = \quad + \quad$$

Consumption **Investment**

- A fixed fraction of HH income is saved

$$\boldsymbol{I = sY \;\; \& \;\; C = (1-s)Y}$$

- Constant savings rate (s): $s = 30\ \%$

Savings rate determines the allocation of income between consumption and investment

Solow-Swan Model: Demand Side (Cont.)

- Transforming to per capita terms

$$I=sY \;\;\&\;\; C=(1-s)Y$$

$$i = sf(k) \quad \& \quad c = (1-s)f(k)$$

- $y = f(k)$ – GDP per capita
- $i = 0.3y$ – Investment per capita
- $c = (1-0.3)y$ – Consumption per capita

Solow-Swan Model: Capital Accumulation

•**No population growth**: L= const

• GDP per capital will increase only due to increase in capital stock

$$\frac{Y_t}{L} = \left(\quad \right)$$

• Households' savings are used as investment into **capital accumulation (**K)

- New capital

-Replacement of old capital

• Capital **depreciation:** every year a fraction of capital **δ** breaks down and becomes useless

$$K_{t+} = \quad + \quad - \quad t$$

Solow-Swan Model: Capital Accumulation (Cont

- Capital **accumulation**

$$k_{t+1} = sf(k) + (1 - \square\ )k_t$$

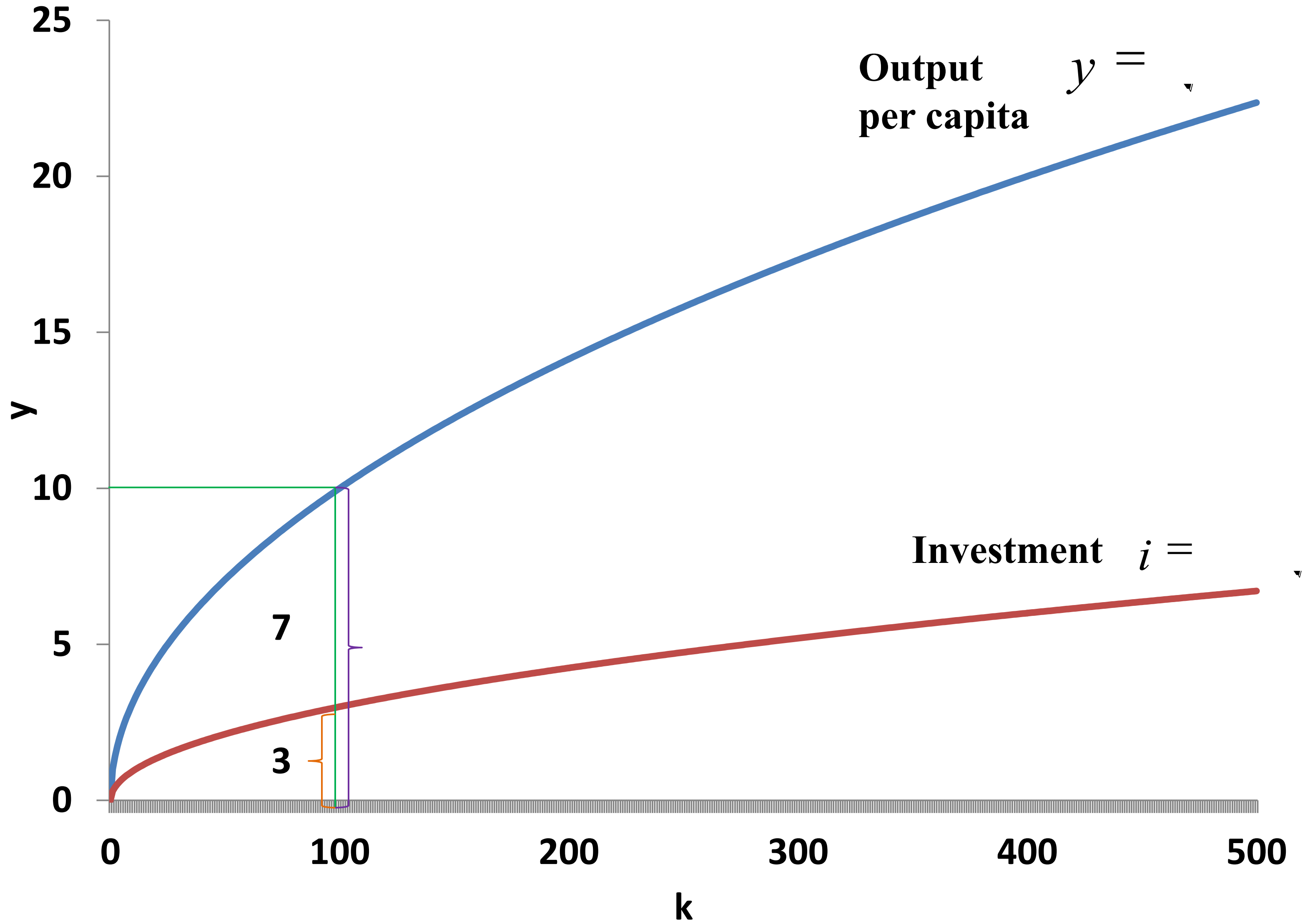
- Change in capital from year t to year t+1

$$\underbrace{k_{t+1} - k_t}_{} = s\,f(k) - \square\ k_t$$

$$\Delta k$$ – change in capital stock

- If $\Delta k > 0$ (capital stock **increases**) if $sf(k) > \square\ k_t$
- If $\Delta k < 0$ (capital stock **decreases**) if $sf(k) < \square\ k_t$

Solow-Swan Model: Graphical Representation



Solow Model: Steady-State (Cont.)

- **Steady-state**: the long-run equilibrium of the economy

The amount of savings per worker is just sufficient to cover the depreciation of the capital stock per worker

• Economy will **remain** in the steady state (unless additional channels of growth are introduced)

$$\Delta \quad = \quad - \quad =$$

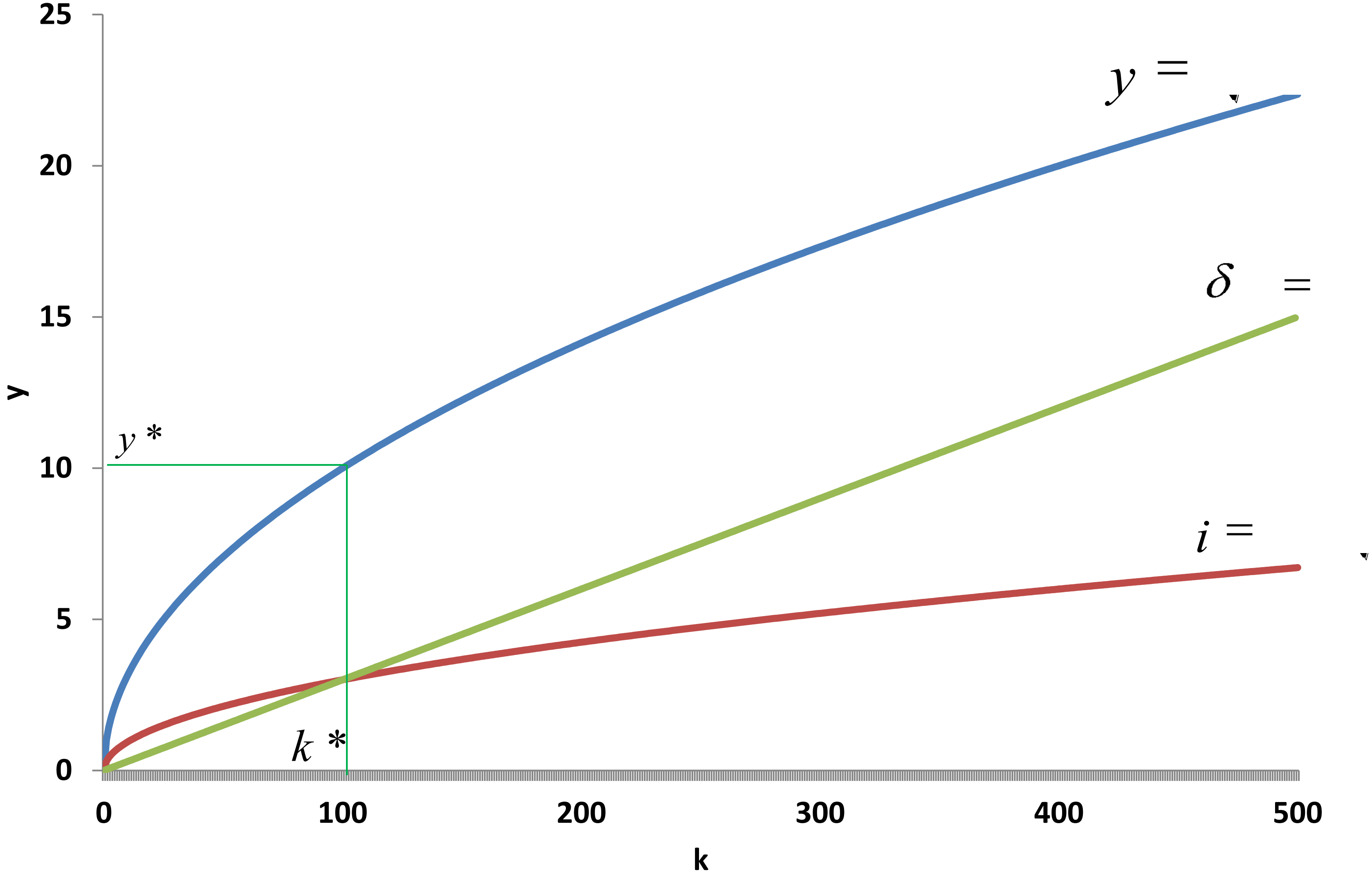
$$y^* = \quad \rightarrow \Delta \quad =$$

• Economy which is not in the steady state will go there => convergence to the constant level of output per worker over time

• Different economies have **different steady state** value of capital

Solow Model: Steady-State

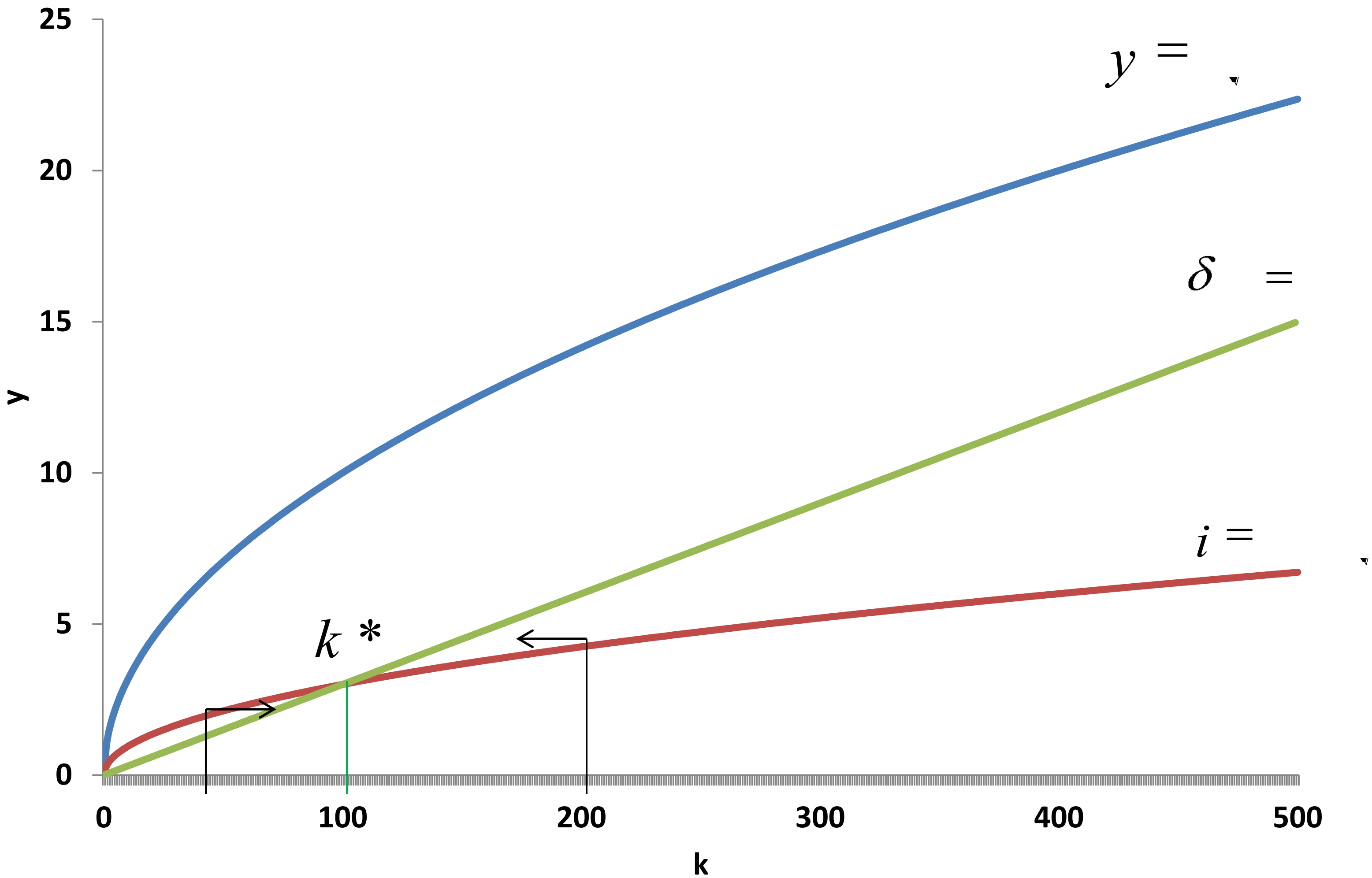
- **Steady-state**: investment and depreciation just **balance**



$I = \quad \rightarrow \Delta \quad = \quad \rightarrow \Delta \quad =$

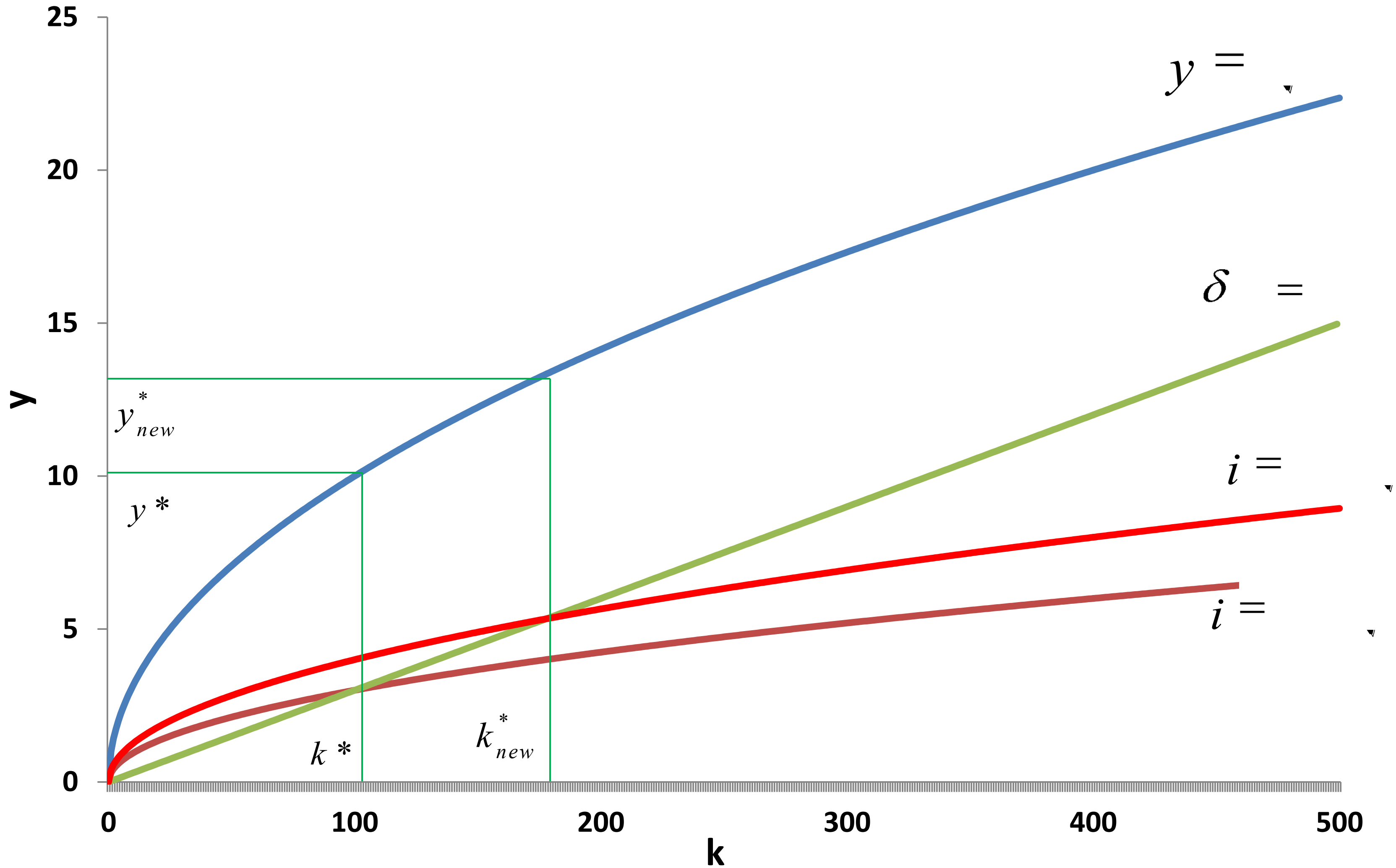
Solow Model: Steady-State Level of Capital per V

- **Convergence** to steady state



Solow Model: Increase in Savings Rate

- Savings rate increases from 30 % to 40 %



- Economy moves to a **new steady state** => Higher capital and output per capita

Solow Model: Steady-State (Cont.)

Implications

- Savings rate (s) has **no effect** on the long-run **growth rate of GDP** per capita

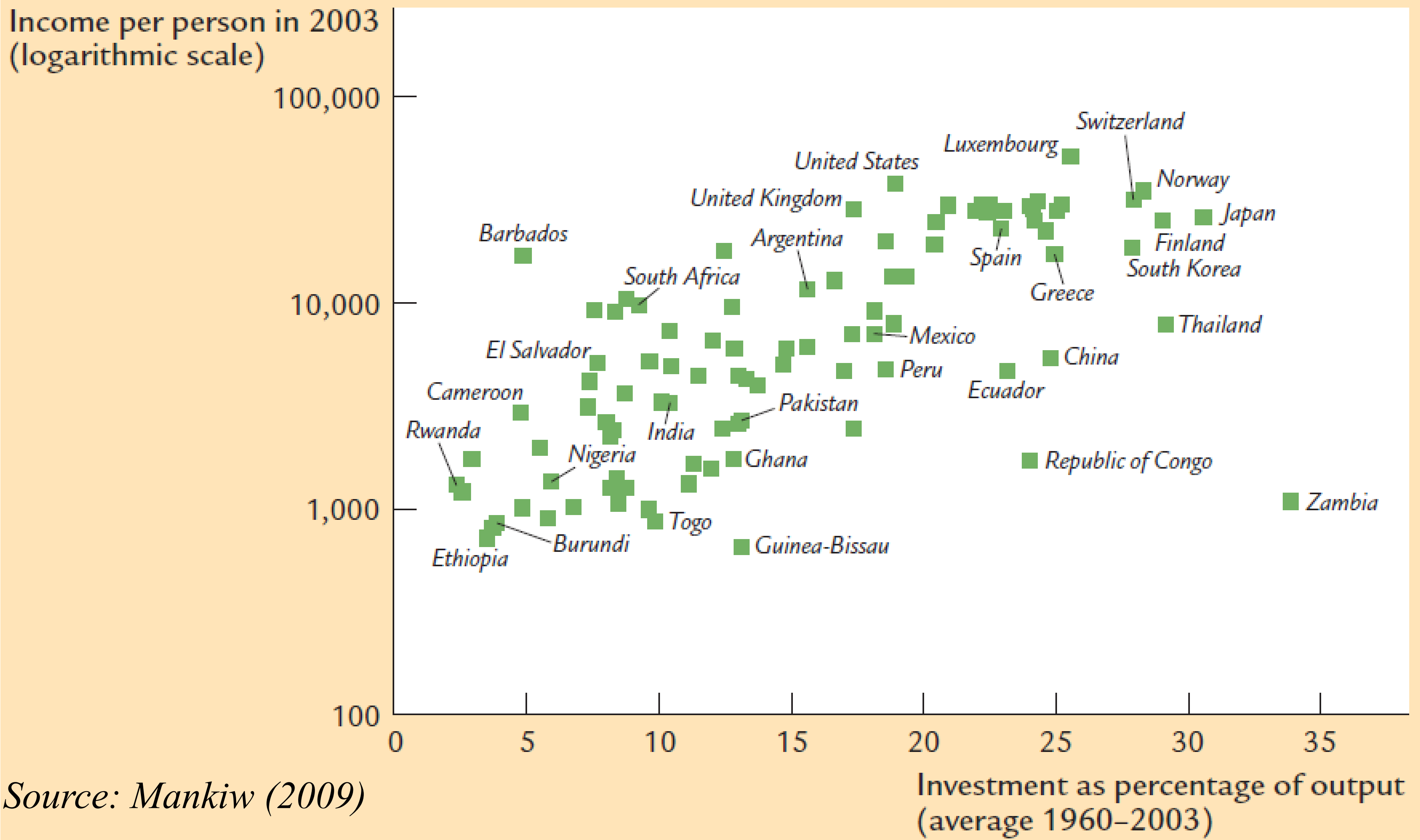
➢ Increase in savings rate will lead to higher growth of output per capita only *for some time*, but not forever.

➢ Saving rate is bounded by interval [0, 1]

- Savings rate determines the **level of GDP** per capita in a long run

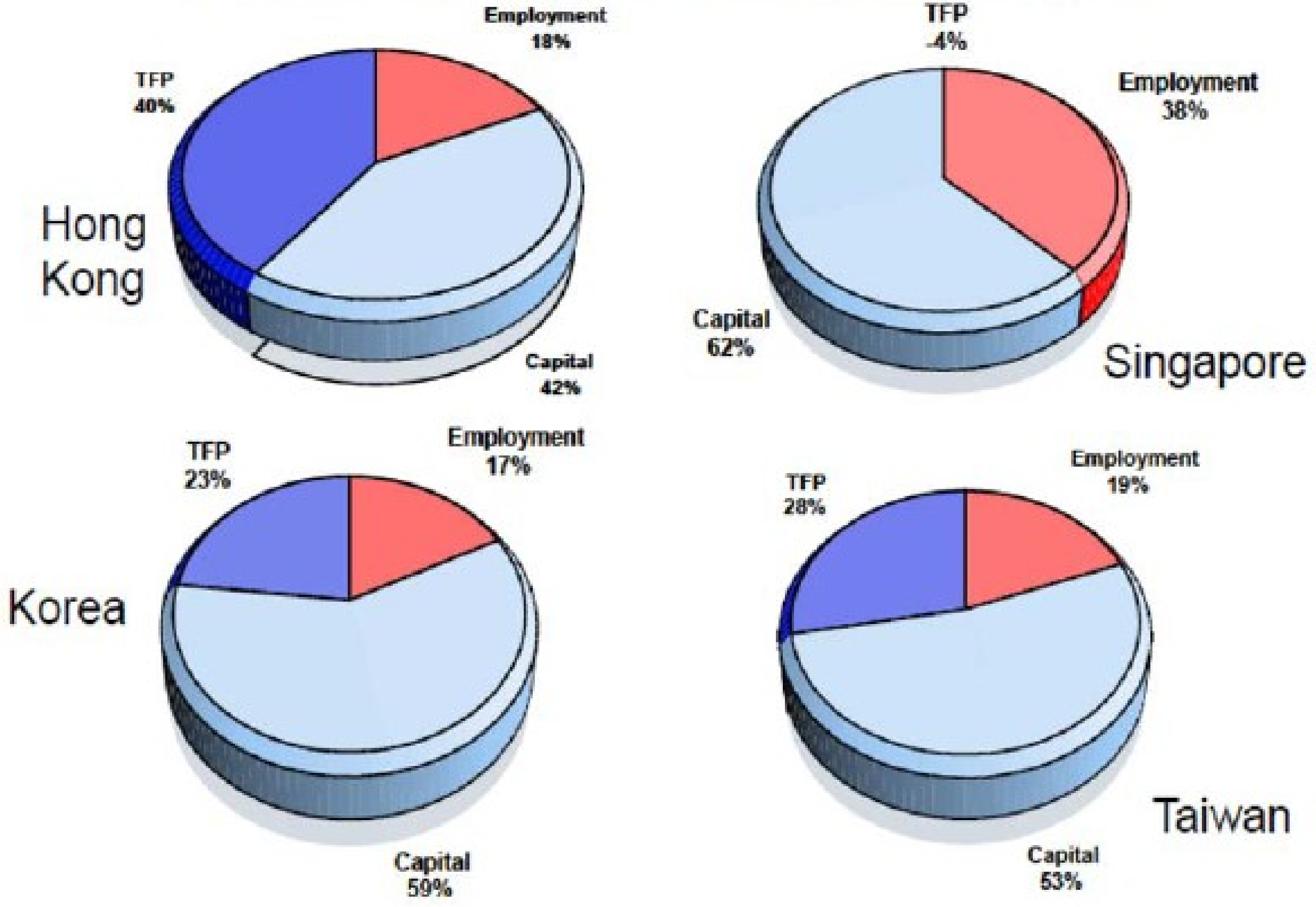
Solow Model: The Role of Savings

- A nation that devotes a large fraction of its income to savings will have a higher steady-state capital stock and a high level of income



Source: Mankiw (2009)

Growth Breakdown 1966-90 for Asian Dragons



The Solow-Swan Model: Steady State

- **Steady state**: the **long-run** equilibrium of the economy
  - Savings are just sufficient to cover the depreciation of the capital stock

❖ In the long run, capital per worker reaches its steady state for an **exogenous s**

❖ Increase in **s** leads to higher capital per worker and higher output per capita

❖ Output grows only during the transition to a new steady state (**not sustainable**)

❖ Economy will **remain** in the steady state (no further growth)

❖ Economy which is not in the steady state will go there => **Convergence**

Government policy response?

N!B! Savings rate is a fraction of wage, thus is bounded by the interval **[0, 1]**

The Solow-Swan Model: Numerical Example

Production function $Y =$ $=$

Production function in **per capita terms** $\frac{Y}{L} = \frac{\;^{0.5}\;^{0.5}}{L} = \frac{\;^{0.5}}{L^{0.5}}$

$k = \frac{\;}{L} = \frac{\;}{L}$

GDP per capita: $y =$

Savings rate: $s =$

Depreciation rate: $\delta =$

Initial stock of capital per worker: $k_0 =$

The Solow-Swan Model: Numerical Example (C

Year	k	y	i	c	δk	Δk
1	**4**					
2						
…						

Consumption: C = (1-s)Y

Consumption **per capita C/Y = c**

Steady state capital/labor ration:

$$s\sqrt{k} = \qquad \rightarrow \qquad = \frac{}{\delta}^{2}$$

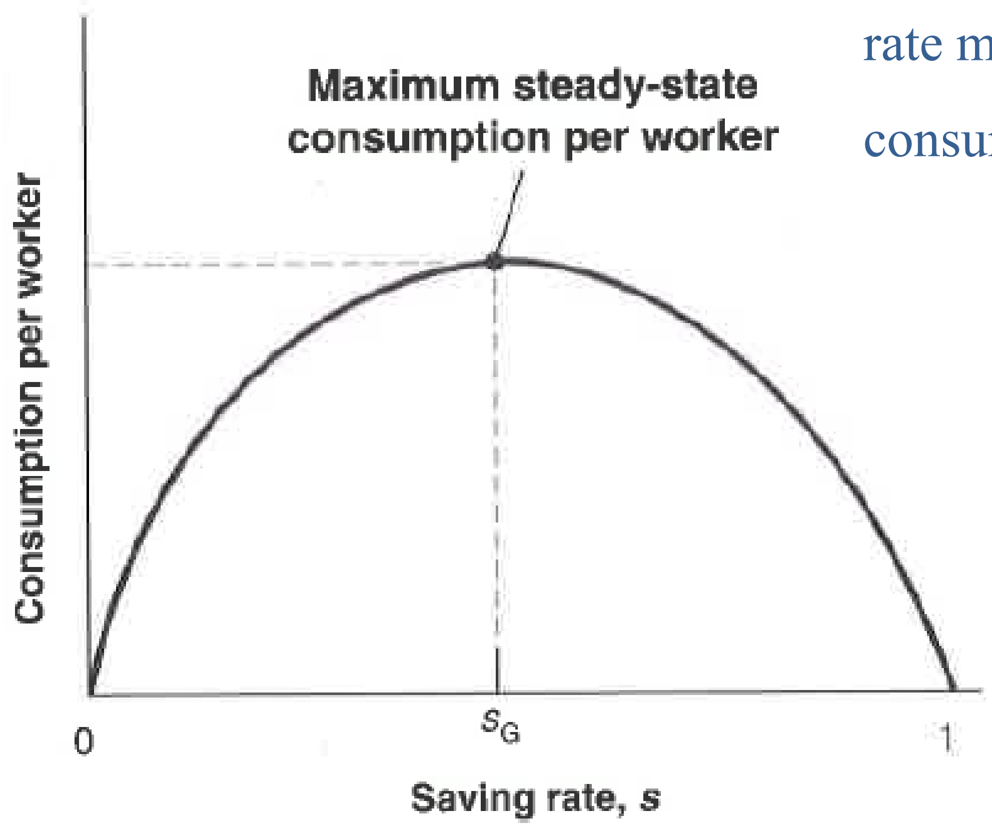
Year	k	y	c	i	δk	Δk
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
.						
.						
.						
10	5.602	2.367	1.657	0.710	0.560	0.150
.						
.						
.						
25	7.321	2.706	1.894	0.812	0.732	0.080
.						
.						
.						
100	8.962	2.994	2.096	0.898	0.896	0.002
.						
.						
.						
∞	9.000	3.000	2.100	0.900	0.900	0.000

The Golden Rule Level of Capital

- Increasing savings rate means less present consumption

What is the optimal savings rate?

N!B! Optimal savings rate maximizes consumption per capita



$c^* = \quad - \quad \rightarrow$

The Solow-Swan Model: Convergence to Steady

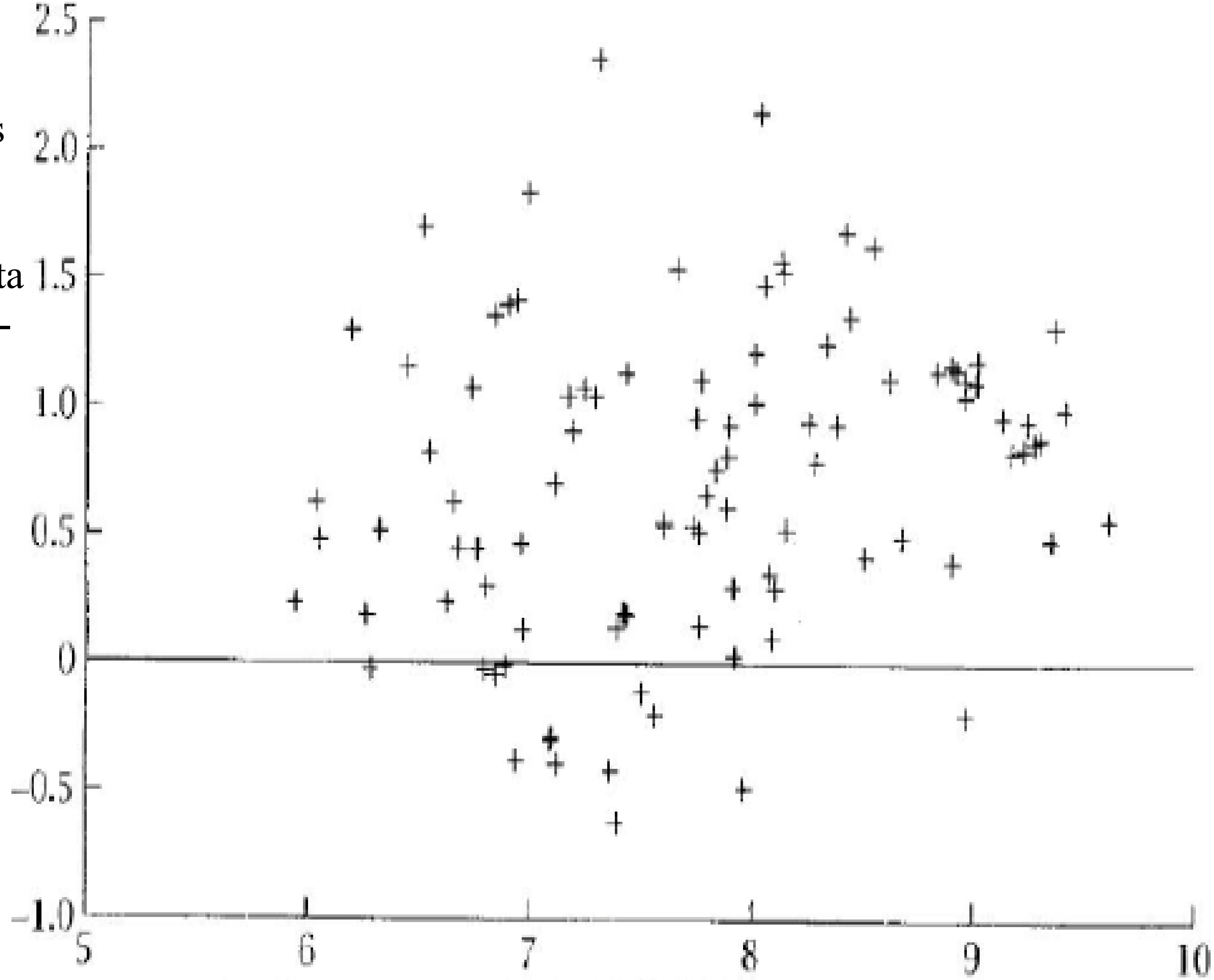
N!B! Regardless of k_0, if two economies have the same **s, δ, L**, they will reach the **same** steady state

- The property of catching-up is known as **convergence**
- If countries have the same steady state, poorest countries grow faster
- Not much convergence worldwide

  Different countries have different **institutions and policies**

- **Conditional convergence:** comparison of countries with similar savings rates

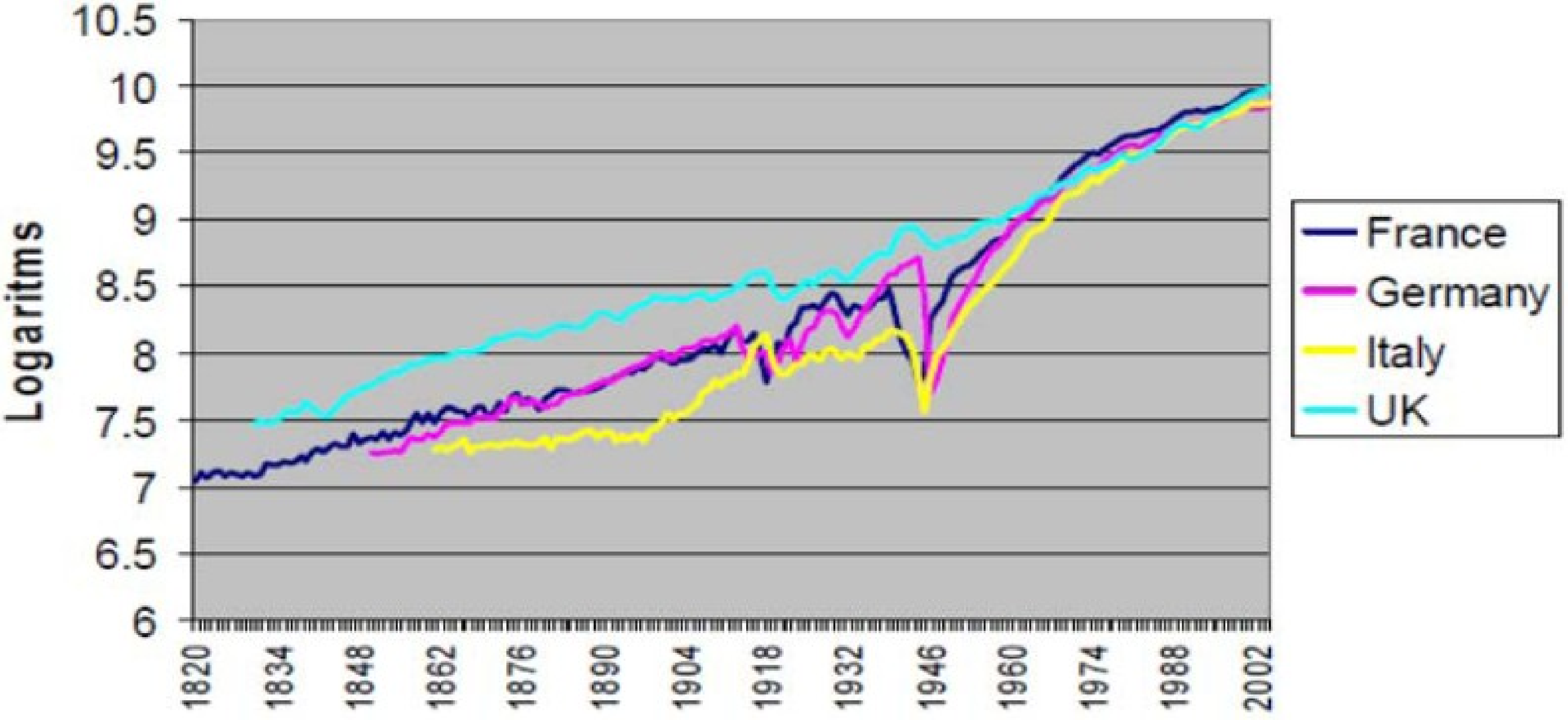
World Wide Convergence



Changes in Log **income** per capita in 1960 - 1990

Log income per capita in 1960 (100=1996)

Catch up amongst Europe's big 4



Output per head (US $ 1990)

Source : Maddison and GGDC

Next class: Solow-Swan Growth Model (Cont.)

N!B! Reading Assignment: Handout “Theories that don’t work”