Little´s law

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Based on Factory Physics (Hopp and Spearman)

Little´s law - definition (formula)

- Fundamental relationships among :
  - WIP (Work In Process)
  - Cycle Time (CT)
  - Throughput (T or sometimes TH)
- Formula

$$WIP=TH \times CT$$

- Can be applied to :
  - Single machine station
  - Complex production line
  - Entire plant

Relationships among these variables will serve to se clearly precise (quantitative) description of behaviour of the single production line . It helps user to use a given scale to benchmark actual production systems

Definition of basic parameters

- **Throughput (Throughput rate, TH) :** production per unit time that is sold (see TOC definition)
- If **TH** is measured in cost dollars rather than in prices it is typically called :

Cost of good sold (COGS)

- **Upper limit** of TH in production process is capacity

- If you release more raw material above capacity of the line (machine), system become unstable –> WIP goes up !!

Definition of basic parameters

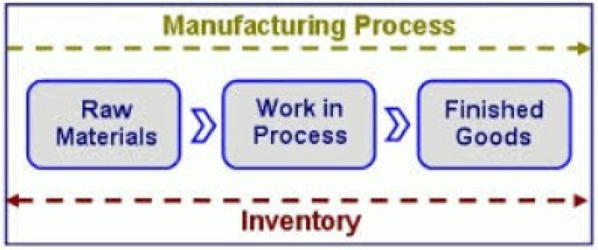
- **WIP (Work In Process)** : inventory between start and end points of the product routing

- **WIP** can be used as one parameter to calculate (measure) an **efficiency**

- **Efficiency** can be defined as **Turnover Ratio** = TH/FGI for warehouses or TH/(FGI+WIP) for production plants where **FGI**=Finished goods inventory

- **WIP** : inventory still in line

- **FGI** : inventory waiting for dispatch (shipping)



Definition of basic parameters

- **CT (Cycle Time, Flow Time or Throughput Time)** : average time from release of the job of the beginning of the routing until it reaches an inventory point at the end of the routing or time that part spends as a WIP.

- **LT (Lead Time)** : managerial constant used for planning of production

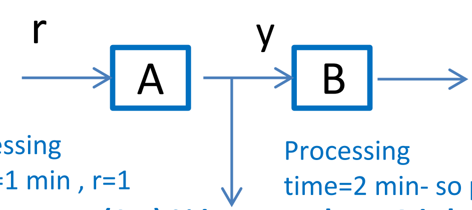
- **Service Level** **(especially for MTO lines, where plant have to satisfy orders with specific due dates)** :

Service level P{Cycle time=<Lead Time}

Definition of basic parameters

Utilization U(x) = Arrival rate / Effective production rate

- Where
  - **Arrival Rate =r** = amount of parts arriving to workstation per time unit
  - **Effective production time** is maximum average rate at which the workstation can process parts (considering effects of failures, setup times and so on)
- Bottleneck rate (see TOC)
  - **r**b = rate (parts per unit time or jobs per unit time) of workstation having the **highest long term utilization**
- Example : taking into consideration **only rate**, so bottleneck is B for sure



Processing time=1 min , r=1

Processing time=2 min- so processing r=0,5=1/2

(1-y) % is scraped, so A is heavily loaded

But having in mind some rejects on A output , so B have to process only **y**, where 1-**y** = quantity of rejects.

U(A)= r/1=1 and **U(B) =y*r/0,5 =2*y*r**

If **r<0,5**, that utilization of **A** is higher than U(B) and hence A workstation is bottleneck !

Definition of basic parameters

- **T_0** =Row process time of the line is the sum of the long –term average process time of each workstation in the line (single job entering empty line from staring point to the ending one)

7, 8, 9, 7, 6, 10
7,83

5.3. 4. 5. 6. 8

9, 8, 4, 5, 5, 6
6,1

19,16=7,83+5,16+6,1

Definition of basic parameters

- **Critical WI**P (W_o) of the line is the WIP level for which a line with given values of $\mathbf{r}_b$ (bottleneck rate) and T_o achieves maximum throughput ($\mathbf{r}_b$) with minimum cycle time (which is in this case T_o)

$$W_o = r_b \times T_o$$

Use of defined parameters

- Simple production line that makes giant one-cent pieces
- It is as a model very with unrealistic assumption because process times are deterministic (no waiting times after every operation and no queue times before any other operation, 24 hours /day an unlimited market)→ balanced line
- Any machine can be regarded as the bottleneck (one- half part per hour)

Head stamping → Tail stamping → Rimming → Deburring

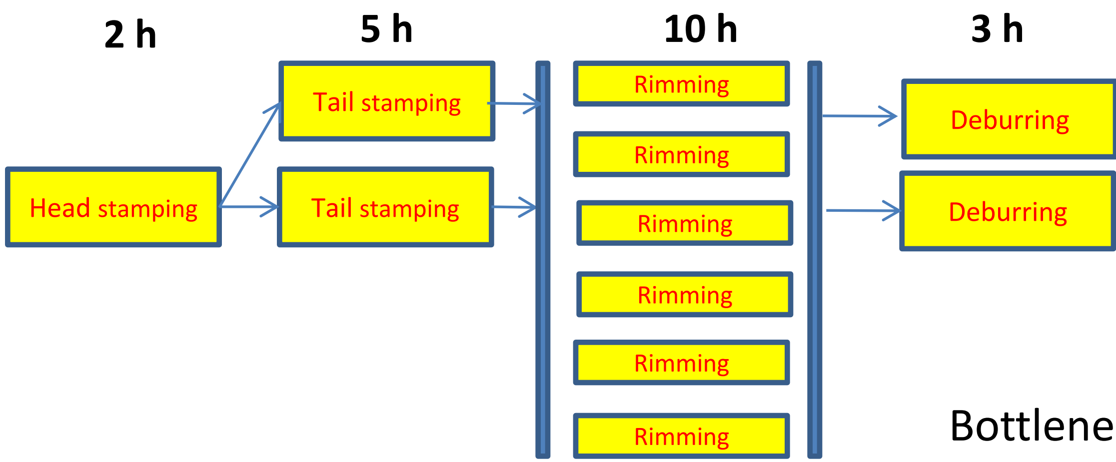
2 h | 2 h | 2 h | 2 h

rb = rate (parts per unit time or jobs per unit time) of workstation having the **highest long term utilization   =0.5 penny per hour , which means 24 hour x 0.5=12 pennies /day**

T_0 =Row process time of the line is the sum of the long –term average process time of each workstation in the line = 8 hours=2+2+2+2

Critical **WIP (W_0)** of the line is the WIP level for which a line with given values of **rb** and T_0 achieves maximum throughput (rb) with minimum cycle time (which is in this case T_0) = **rb x T_0 = 0.5 x 8 = 4 pennies**

Use of defined parameters



Bottleneck= **rb** =0,4

It is neither the station that contains the slowest machines nor the one with fewest machines !!!

Station number	Number of machines	Process time (hours)	Station capacity (jobs per hour)
1	1	2	0.5
2	2	5	0.4=(1/5)*2
3	6	10	0.6=(1/10)*6
4	2	3	0.67=(1/3)*2

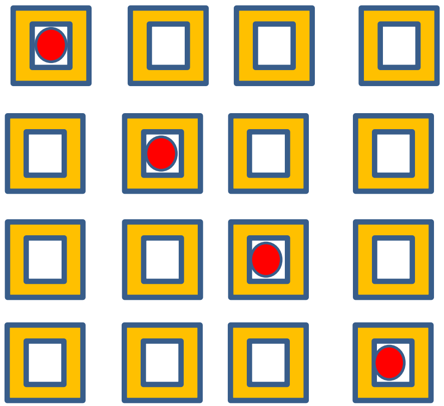
$\mathbf{T_0}$ =Row process time of the line =2+5+10+3 and

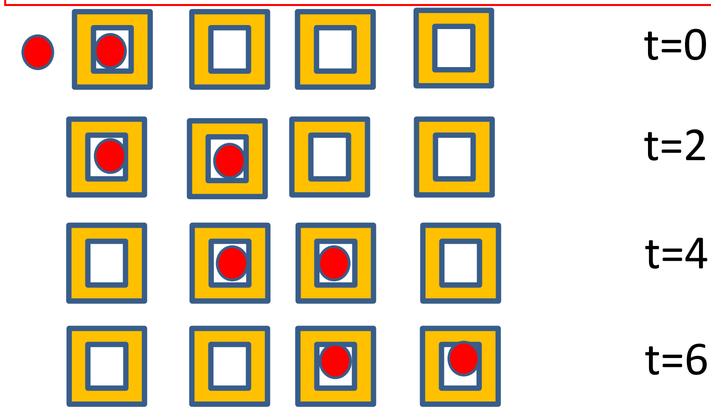
Critical **WIP($\mathbf{W_0}$)** =rb x T_0 = 0.4 x20 = 8 pennies <number of machines (11). This is because system is not balanced. Not balanced line is the line where some stations are not fully utilized

Best case performance I

WIP=1, Total cycle time= T_0=8,
Throughput=1/8=part/hour= **rb**/4=0,125,
Bottleneck rate = **rb**=0,5

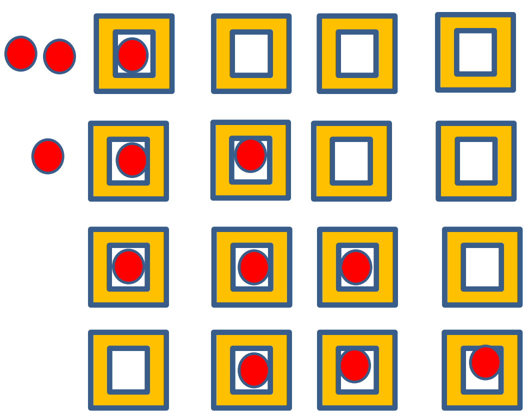
WIP=2, Total cycle t ime= T_0=8,
Throughput=WIP/CT=WIP/T_0= 2/8=part/hour= **rb**/2=0,250, což je 50 % of rb where bottleneck rate = **rb**=0,5

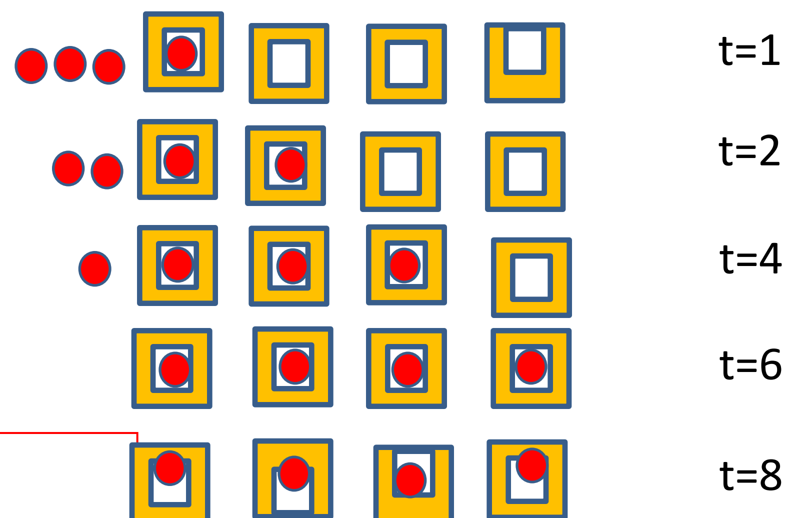




WIP=3

WIP=4





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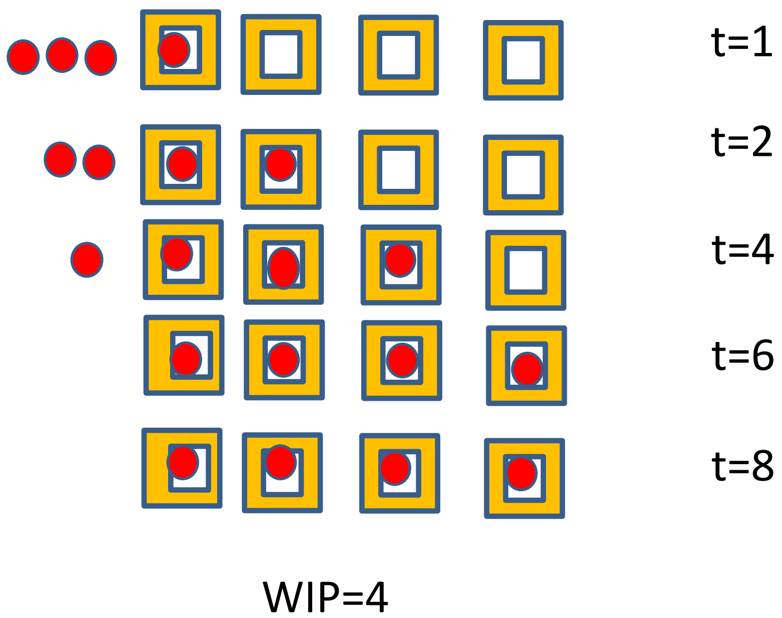
WIP=3, Total cycle time= T_0=8,
Throughput=WIP/CT=WIP/T_0= 3/8=part/hour
= 0,375, což je 75 % of rb where bottleneck rate = **rb**=0,5

Best case performance II



T_o =Row process time of the line is the sum of the long –term average process time of each workstation in the line (single job entering empty line from staring point to the ending one)

All stations stay busy all the time. Because no waiting so T_o=8 h., rb=0,5 .
So minimum value of T_o an maximum value of rb (throughput) is achieved only if WIP is set to critical value !

Critical **WIP (W_o)** of the line is the WIP level for which a line with given values of **rb** and **T_o** achieves maximum throughput (rb) with minimum cycle time (which is in this case T_o) = **rb x T_o = 0.5 x 8 = 4 pennies**

Best case performance III

Table 1.

Parameters :	100 % rb=0,5			
WIP	$CT=T_o$	% T_o	TH=Throughput	% rb
1	8	100	0,125	25
2	8	100	0,25	50
3	8	100	0,375	75
4	8	100	0,5	100
5	10	125	0,5	100
6	12	150	0,5	100
7	14	175	0,5	100
8	16	200	0,5	100
9	18	225	0,5	100
10	20	250	0,5	100

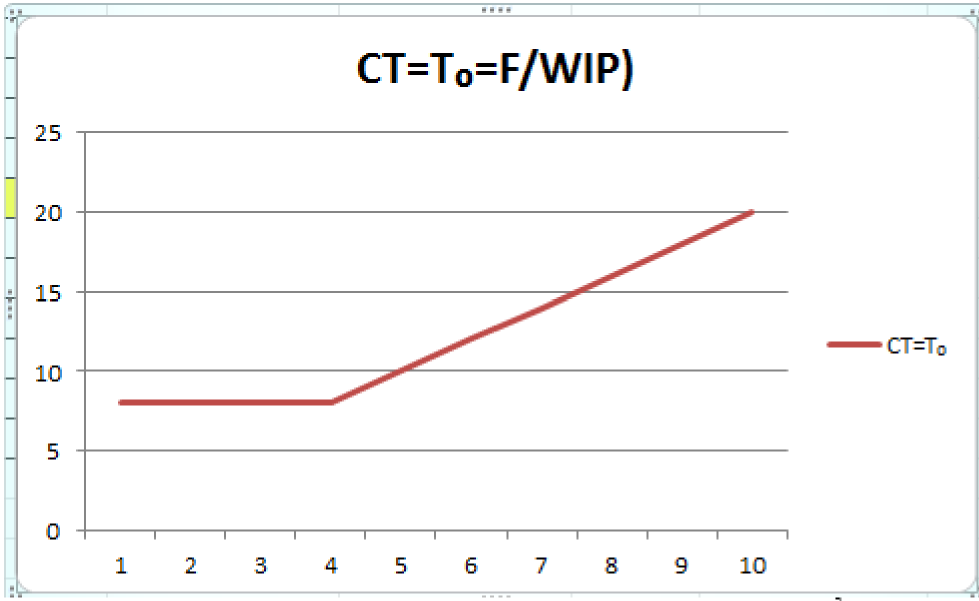
20 hours = 12 hours waiting before line and 8 hours processing

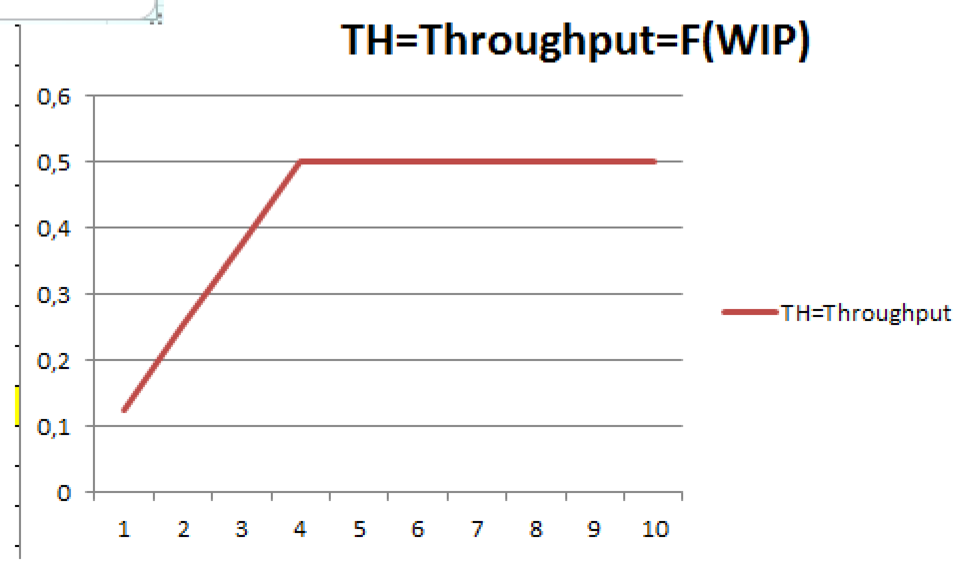
All machines remains busy so rb=0,5 (2 hours per processing one penny)

% T_o =(CT/ T_o) *100, so e.g. 250=(20/8)*100

% rb =((WIP/ T_o)/0,5)*100, where rb=0,5

Best case performance IV





Conclusion

- Close examination of Table 1 reveals fundamental relationship among WIP, CT and TH (throughput)
- At every WIP level WIP is equal to the product of TH and CT (cycle time)
- This relation is known as Little´s law.
- **WIP=TH * CT**
- *Source : Factory Physiscs, Wallace J Hopp and Mark L. Spearman ; ISBN 13: 978-1-57766-739-1   or   ISBN 10 :1-57766-739-5*
- http://www.factoryphysics.com/principle/littleslaw.htm

Example 1

- **Estimating Waiting Times:** If are in a grocery queue behind 10 persons and estimate that the clerk is taking around 5 minutes/per customer, we can calculate that it will take us 50 minutes (10 persons x 5 minutes/person) to start service.
- This is essentially **Little's law.** We take the number of persons in the queue (10) as the "inventory".
- The inverse of the average time per customer (1/5 customers/minute) provides us the rate of service or the throughput.
- Finally, we obtain the waiting time as equal to number of persons in the queue divided by the processing rate 10/(1/5) = 50 minutes).

Example 2

- **Planned Inventory Time:** Suppose a product is scheduled so that we expect it to wait for 2 days in finished goods inventory before shipping to the customer. This two days is called planned inventory time and is sometimes used as protection against system variability to ensure high delivery service. Using Little's law the total amount of inventory in finished goods can be computed as :
- **FGI = throughput × planned inventory time**

Youtube examples (6 minutes)

- http://www.youtube.com/watch?v=VU8TUSnQ-vw
- http://www.youtube.com/watch?v=rtGihR-bm-U