Seminar in macroeconomics – Economic growth I, 8th week

1. ! Consider the Solow model with population growth. What is the steady state condition in this model? What is the growth rate of capital per capita, output per capita and total capital and total output in the steady state? What is the steady-state growth rate of per capita consumption and total consumption?

2. ! Suppose that an economy in the steady state enters a war. During the war a large part of the capital stock is destroyed, but the number of employees does not change. What happens to the total output and the output per worker? Is the growth rate of output per worker after the war smaller or greater than normal? By contrast, suppose that the war does not directly affect the capital stock, but that casualties reduce the labor force? Which of these two scenarios can be used to analyze the post-war development in Germany?

3. ! Suppose a production function of the form $Y = 10(K)^{1/4}(L)^{3/4}$, capital lifetime is 50 years, the population growth rate is 0, and the saving rate is 0.128.

a) Does the production function exhibit constant returns to scale? Explain.
b) Derive the per-worker production function.
c) Calculate the steady-state value of these variables: capital per worker, output per worker, worker consumption, savings and investment per worker and amortization per worker.

4. ! Consider an economy described by the production function $Y = F(K, L) = K^{0,3}L^{0,7}$. Assume that the rate of capital depreciation is δ, and the population growth is 0. Derive the per-worker production function. Draw the following into a graph:

a) Saving rate that maximizes the output per worker.
b) Saving rate that maximizes the consumption per worker. Calculate the Golden Rule saving rate.

5. ☺ "Devoting a larger share of national output to investment would help to restore rapid productivity growth and rising living standards (measured by consumption per worker)." Do you agree with this claim? Explain.

6. ! Suppose that the current steady-state level of capital per worker is greater than the value required by the Golden Rule.

a) What change in the saving rate is required to achieve the Golden Rule?
b) If the change from (a) is implemented, what will be the short-run and long-run change of the consumption per worker?
c) What policies should be implemented to achieve the desired changes in the saving rate?

7. ☺ Suppose German firms invent capital that does not depreciate. How does it affect the steady state of world economies? Can the economies achieve unlimited growth? Consider the Solow model with and without the population growth.

8. ☺ At the end of the 19th century the US population grew rapidly due to strong immigration from Europe. Suppose that before beginning the immigration wave the US were in the steady state.

a) What is the impact of immigration on the steady-state level of capital per worker?

b) Describe the transition to the new steady state. What is the rate of growth of the capital per worker and the output per worker? What is the rate of growth in total product?
c) Suppose that the initial steady state was below the Golden Rule level of capital. Why should the policy-makers be reluctant to change the saving rate to achieve the Golden-Rule level of capital? Could an increase in the rate of population growth provide a cheaper way to reach the Golden Rule? Explain.

9. ☺ In the Solow model, population growth leads to steady-state growth in total output, but not in output per worker. Do you think this would still be true if the production function exhibited increasing or decreasing returns to scale? Explain.

10. ! Suppose that two countries have identical production function with constant returns to scale and start off with an equal amount of capital per worker. Both countries have a population growth rate of 2%, and the rate of depreciation of 10%. Assume further that country A saves 10% of output each year and country B saves 20% of output each year.
a) Assume that the initial capital stock per worker is below the steady-state level of both countries. Which country grows faster in the first years? Explain.
b) Now assume that the rate of population growth in country A increases to 5%. Which country will have a higher steady-state level of capital per worker? (For formal illustration use the production function $y = k^{\alpha}$)

11. ☺ Consider how unemployment would affect the Solow growth model. Suppose that output is produced according to the production function $Y = K^{\alpha}[(1-u)L]^{1-\alpha}$, where K is capital, L is labor, and u is the natural rate of unemployment. The national saving rate is s, the labor force grows at rate n, and capital depreciates at rate δ.
a) Express output per worker ($y = Y/L$) as a function of capital per worker ($k = K/L$) and the natural rate of unemployment. Describe the steady state of this economy.
b) Suppose that some change in government policy reduces the natural rate of unemployment. Describe how this change affects output both immediately and over time. Is the steady-state effect on output larger or smaller than the immediate effect? Explain.

12. ☺ Explore the data of two economies in the following table

	GDP (PPP) per capita in $	Population growth rate	Investment/GDP (in %)	Illiteracy rate (% of population above 15 years)
USA	31,900	1.5	18	0
Pakistan	1,890	3	10	54

Which of these factors is most responsible for the observed income difference? Is the Solow model useful for explaining the differences? (For formal illustration use the production function $y = k^{1/2}$, and the same rate of depreciation $\delta = 0.05$ for both countries.)