Seminar in macroeconomics – Economic growth II, 9th week

1. ☺ In the last fifty years, South Korea's economy grew at a rate of 6%, the US economy at a rate of 3%, and the Philippines grew at a rate of 1.7%. Does it contradict the predictions of the Solow model about the economic convergence? Explain.

2. ! An economy described by the Solow growth model has the following production function: $Y = F(K,L) = 10(K)^{1/4}(EL)^{3/4}$. The capital lifetime is 10 years, so that 10% of capital is depreciated every year. Suppose that the population growth rate is 4%, the rate of technological progress is 2 % and the saving rate is s = 0,128.

a) Express the production function in the form $y = Y/EL = f(k)$, where k is the capital per efficient worker.
b) Calculate the steady-state values of the following variables: capital per efficient worker, output per efficient worker, consumption per efficient worker, savings and investment per efficient worker and depreciation per efficient worker.
c) What is the steady-state growth rate of the following variables: capital *per worker*, output *per worker* and consumption *per worker*?
d) Calculate the steady-state growth rate of *total* capital, *total* output and *total* consumption.

3. ! In the United States, the capital share of GDP is about 30 percent, the average growth in output is about 3 percent per year, the depreciation rate is about 4 percent per year, and the capital-output ratio is about 2.5. Suppose that the production function is Cobb-Douglas, so that the capital share in output is constant, and that the United States has been in a steady state. Suppose Solow model with the population growth and technological progress.

a) What must the saving rate be in the initial steady state?
b) What is the marginal product of capital in the initial steady state?
c) Suppose that public policy raises the saving rate so that the economy reaches the Golden Rule level of capital. What will the marginal product of capital be at the Golden Rule steady state? Compare the marginal product at the Golden Rule steady state to the marginal product in the initial steady state. What should be the change in savings? Explain.
d) What will the capital-output ration be at the Golden Rule steady state? What is the Golden Rule saving rate?

4. ☺ The economy has a ratio of capital to output 2:1, a rate depreciation of 8 percent, asaving rate of 30 percent and a population growth rate of 2 percent. Income from equity constitutes 35 percent of the total product. Suppose that the economy is characterized by Cobb-Douglas production function and is in a steady state.

a) Calculate the rate of technological progress in this economy
b) What is the marginal product of capital in this economy?
c) Is the economy in the Golden Rule steady state? If not, should it increase or decrease the saving rate to achieve the Golden Rule level of capital? How does the change affect the marginal product of capital? How does it influence the ratio of capital to output?

5. ! Suppose that a country, currently in the steady state, is able to double the rate of technological progress.

a) What happens to the capital level per efficient worker?

b) What is the growth rate of capital and output per efficient worker in the original and in the new steady state? What can we say about the growth rates of these variables in adapting to a new steady state?
c) If we compare the initial and final steady state, what happens with the growth rate of capital, output and consumption *per worker*?

6. ☺ How can endogenous growth theory explain the long-term economic growth without the assumption of exogenous technological progress? How does the endogenous growth theory differ from Solow model in this point?

7. ☺ Medical research funding is a widely discussed public policy.
a) Use the concept of externalities, social and private returns to determine why the government should or should not subsidize medical research. How can the government encourage private businesses to increase spending on research?
b) If the government thinks that more resources should be used to finance medical research should it fund this research directly, or should it create incentives for private entities to finance this research instead? How would the final price of medicines be affected by the approaches?

8. ! Two countries, Richland and Poorland, are described by the Solow growth model. They have the same Cobb-Douglas production function, $F(K,L) = K^{\alpha}(EL)^{1-\alpha}$, but with different quantities of capital and labor. Richland saves 32 percent of its income, while Poorland saves 10 percent. Richland has population growth of 1 percent per year, while Poorland has population growth of 3 percent. (The numbers in this problem are chosen to be approximately realistic descriptions of rich and poor nations.) Both nations have technological progress at a rate of 2 percent per year and depreciation at a rate of 5 percent per year.
a) What is the production function $y=f(k)$, where $k = K/EL$?
b) Derive the steady-state capital per effective worker (as a function of parameters) and the steady-state output per worker (as a function of parameters).
c) Solve for the ratio of Richland's steady-state income per worker to Poorland's.
d) If the Cobb-Douglas parameter α takes the conventional value of about 1/3, how much higher should income per worker be in Richland compared to Poorland?
e) Income per worker in Richland is actually 16 times income per worker in Poorland. Can you explain this fact by changing the value of the parameter α? What must it be? Can you think of any way of justifying such a value for this parameter? How else might you explain the large difference in income between Richland and Poorland?

9. ☺ The amount of education the typical person receives varies substantially among countries. Suppose you were to compare a country with a highly educated labor force and a country with a less educated labor force. Assume that education affects only the level of the efficiency of labor. Also assume that the countries are otherwise the same: they have the same saving rate, the same depreciation rate, the same population growth rate, and the same rate of technological progress. Both countries are described by the Solow model and are in their steady states. What would you predict for the following variables?
a) The rate of growth of total income.
b) The level of income per worker.
c) The real rental price of capital.
d) The real wage.

10. ☺ The economy is described by the production function for firms, the production function for universities, and the capital-accumulation equation:

Production function in manufacturing firms: $Y = F[K,(1-u)LE]$

Production function in research universities: $\Delta E = g(u)E$

Capital-accumulation equation: $\Delta K = sY - \delta K$

where u is the fraction of the labor force in universities (and 1-u is the fraction in manufacturing), E is the stock of knowledge (which in turn determines the efficiency of labor), $g(u)$ is an increasing function that shows how the growth in knowledge depends on the fraction of the labor force in universities.

a) Rewrite the production function in manufacturing firms in output per effective worker and capital per effective worker.
b) What is the value of the investments needed to maintain a constant value of the capital per effective worker.
c) Write the equation describing the change in the capital stock per effective worker k, which shows ΔK depending on savings minus investment (from point b). Based on this equation draw a graph (similar to the basic Solow model) and explain the convergence to the steady state.
d) What is the steady-state growth in output per worker (Y/L) in this economy? As is the steady-state growth affected by a change in the saving rate and a change in the fraction of the labor force working in the education sector u.
e) To draw a graph with the impact of a rise in u. (Hint: This change affects both curves.) Describe the immediate impact and the long-run impact (steady state).
f) Does the rise in u have a clearly positive impact on the economy? Explain.

11. ! Production function has constant returns to scale and decreasing marginal product of capital. The share of capital income on domestic product is 30%. Capital grows at a rate of 6% and labor grows at a rate of 2% per year. Product grows at a rate of 4% per year. What is the growth contribution of the total factor productivity?

12. ! In the economy of Solovia, the owners of capital get two-thirds of national income, and the workers receive one-third. In year 1, the capital stock was 6, the labor input was 3, and output was 12. In year 2, the capital stock was 7, the labor input was 4, and output was 14. What happened to total factor productivity between the two years?

13. ☺ Suppose an economy described by the Solow model is in a steady state with population growth n of 1.8 percent per year and technological progress g of 1.8 percent per year. Total output and total capital grow at 3.6 percent per year. Suppose further that the capital share of output is 1/3. If you used the growth-accounting equation to divide output growth into three sources—capital, labor, and total factor productivity—how much would you attribute to each source? Compare your results to the figures we found for the United States in Table 8–3.