Derivation of DSGE Model with Financial Frictions by Mohamad Hasni Shaari

Using the dissertation of Mohamad Hasni Shaari, written by me. Sometimes I almost copy word by word, cause the original text is concise and hard to simplify.

1 Definitions

Lets start with definitions. consumption basket:

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \gamma \frac{1}{\eta} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

γ measures openness, H and F denote home and foreign goods, η is the elasticity of substitution.
Both domestic and foreign goods are given by

$$C_{H,t} = \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

so we get demands

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}$$

Note that we express both P_H and P_F in domestic currency.
We also get demands

$$C_{H,t} = (1-\gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t$$

and price index

$$P_t = \left[(1-\gamma) P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad p_t = (1-\gamma) p_{H,t} + \gamma p_{F,t}.$$

Now about inflation:

$$\pi_t = (1-\gamma)\pi_{H,t} + \gamma \pi_{F,t}$$

Define terms of trade as

$$TOT_t = \frac{P_{F,t}}{P_{H,t}}$$

so that

$$p_t = (1-\gamma)p_{H,t} + \gamma p_{F,t} = p_{H,t} + \gamma tot_t, \quad \pi_t = \pi_{H,t} + \gamma \Delta tot_t.$$

We have S_t the nominal exchange rate, increase = depreciation, and $RER_t = S_t \frac{P_t^*}{P_t}$. We assume incomplete exchange rate pass-through, so that $LOPG_t = \frac{S_t P_t^*}{P_{F,t}}$. For estimation, LOPG is assumed to follow AR(1) process (strange...). But anyway, we can express the log of RER as

$$rer_t = s_t + p_t^* - p_t = s_t + p_t^* - p_{H,t} - \gamma tot_t = s_t + p_t^* - p_{F,t} + p_{F,t} - p_{H,t} - \gamma tot_t = lopg_t + (1-\gamma) tot_t$$

While law of one price does not hold for imports, it holds for exports, so that $P_{H,t}^* = P_{H,t}/S_t$.

2 Households

Households maximize discounted future utility given by

$$U(C_t, L_t) = \log(C_t - hC_{t-1}) - \frac{L_{H,t}^{1+\Psi}}{1+\Psi}$$

where C_t is consumption, $L_{H,t}$ is labor supply by household and h is the parameter of external habit. Ψ is inverse elasticity of labor supply.
Budge constraint

$$\widetilde{W}_{H,t} L_{H,t} + R_{t-1} D_{t-1} + R_{t-1}^* \Psi^B(Z_{t-1}, A_{t-1}^{UIP}) S_t B_{t-1} + \Pi_t + T_t = P_t C_t + D_t + S_t B_t$$

means that HH gets income from labor $L_{H,t}$ and nominal wage $\widetilde{W}_{H,t}$. HH gets profits from retailer Π_t and left-over equity from entrepreneurs who die and leave economy T_t. HH can buy two assets: domestic D_t from domestic intermediary and foreign (denominated in foreign currency) which gives risk-adjusted return $R_t^* \Psi^B(Z_t, A_t^{UIP})$. Risk-adjustment is there to stationarise the model, so we specify

$$\Psi^B(Z_t, A_t^{UIP}) = \exp\left[-\psi^B(Z_t + A_t^{UIP})\right].$$

Here $Z_t = \frac{S_t B_t}{Y_t P_t}$ is the net foreign asset position of the domestic economy and A_t^{UIP} is a shock assumed to follow AR(1) process.
HH chooses quadruple $\{C_t, L_{H,t}, D_t, B_t\}$ and we get following FOCs

- wrt to $L_{H,t}$:

$$\widetilde{W}_{H,t} = -\frac{L_{H,t}^{\Psi}}{\lambda_t}$$

- wrt to C_t:

$$\frac{1}{C_t - hC_{t-1}} = -\lambda_t P_t$$

- wrt to B_t:

$$\lambda_t S_t = \beta \lambda_{t+1} R_t^* \Psi^B(Z_t, A_t^{UIP}) S_{t+1}$$

- wrt to D_t:

$$\lambda_t = \beta \lambda_{t+1} R_t$$

Now combine first two to get labor supply:

$$\frac{\widetilde{W}_{H,t}}{P_t} = W_{H,t} = L_{H,t}^{\psi}(C_t - hC_{t-1}).$$

Now we combine the second and fourth (HH decides if to consume or to invest in domestic bonds)

$$\beta R_t = \frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \frac{P_{t+1}}{P_t}.$$

Similarly, HH decides if to consume or invest in foreign bonds:

$$R_t^* \Psi^B(Z_t, A_t^{UIP}) = \frac{1}{\beta} \frac{S_t}{S_{t+1}} \frac{(C_t - hC_{t-1})}{(C_{t+1} - hC_t)} \frac{P_t}{P_{t+1}}$$

The latter two equations define optimal choice between foreign and domestic bonds and therefore imply UIP. Combine to get:

$$\begin{aligned} R_t^* \Psi^B(Z_t, A_t^{UIP}) &= \frac{S_t}{S_{t+1}} R_t \\ R_t^* \exp\left[-\psi^B(Z_t + A_t^{UIP})\right] &= R_t \frac{RER_t P_t}{P_t^*} \frac{P_{t+1}^*}{P_{t+1} RER_{t+1}} \end{aligned}$$

When log linearized, these equations become:

$$\begin{aligned} l_{H,t} &= \frac{1}{\Psi}\left[w_{H,t} - \frac{1}{1-h}(c_t - hc_{t-1})\right] \\ (1-h)(r_t - E_t\pi_{t+1}) &= (c_{t+1} - hc_t) - (c_t - hc_{t-1}) \\ rer_{t+1} - rer_t &= (r_t - E_t\pi_{t+1}) - (r_t * - E_t\pi_{t+1}^*) + \psi^B z_t + A_t^{UIP} \end{aligned}$$

3 Entrepreneurs

We add capital and entrepreneurs. Entrepreneurs produce intermediate goods and capital goods. They also own all capital. We want them to face a financing constraint, so we need to prevent them from living infinitely and accumulating enough new worth so that the constraint wouldn't matter. We let a fraction of $(1-\xi)$ of entrepreneurs die every period.
Entrepreneurs produce intermediate (wholesale) goods $Y_{H,t}$ and seel them for price $P_{H,t}^{W}$. They use both HH and own labor

$$L_t = L_{H,t}^{\Omega} L_{E,t}^{1-\Omega},$$

but we normalize $L_{E,t}$ to one for simplicity. HH is paid $\widetilde{W}_{H,t}$ for labor unit, entrepreneur is paid $\widetilde{W}_{E,t}$. The gross nominal return on capital is $\widetilde{R}_{G,t}$.
They employ production function

$$Y_{H,t} = A_t^Y K_t^{\alpha} L_{H,t}^{(1-\alpha)\Omega},$$

where A_t^Y is productivity common for all firms assumed to follow AR(1) process. Log-linearize to get

$$y_{H,t} = \alpha k_t + (\Omega(1-\alpha)) l_{H,t} + A_t^Y.$$

Entrepreneurs every period minimize costs

$$\widetilde{R}_{G,t} K_t + L_{H,t} \widetilde{W}_{H,t} + \widetilde{W}_{E,t}$$

s.t. the production function. We get following FOCs:

$$\begin{aligned}
\widetilde{R}_{G,t} &= \lambda \alpha \frac{Y_{H,t}}{K_t} \\
\widetilde{W}_{H,t} &= \lambda \Omega (1-\alpha) \frac{Y_{H,t}}{L_{H,t}} \\
\widetilde{W}_{E,t} &= \lambda (1-\Omega)(1-\alpha) \frac{Y_{H,t}}{L_{E,t}}
\end{aligned}$$

Realize that λ is the nominal marginal cost of producing one more unit of output, which is equal to $P_{H,t}^{W}$ (no profit), and that $L_{E,t}$ is 1, and we get the demand schedules:

$$\begin{aligned}
\widetilde{R}_{G,t} &= P^W_{H,t}\alpha\frac{Y_{H,t}}{K_t} \\
\widetilde{W}_{H,t} &= P^W_{H,t}\Omega(1-\alpha)\frac{Y_{H,t}}{L_{H,t}} \\
\widetilde{W}_{E,t} &= P^W_{H,t}(1-\Omega)(1-\alpha)Y_{H,t}
\end{aligned}$$

We will manipulate a bit to show how the LOPG and RER influence this. Define $MC_{H,t} = \frac{P^W_{H,t}}{P_{H,t}}$ to be the real marginal costs expressed in terms of the domestic goods price level. Next, divide all by P_t to get real variables. We get (also log-lin)

$$\begin{aligned}
\frac{\widetilde{R}_{G,t}}{P_t} = R_{G,t} &= \alpha\frac{Y_{H,t}}{K_t}MC_{H,t}\frac{P_{H,t}}{P_t} \\
r_{G,t} &= y_{H,t} + mc_{H,t} - k_t - \left(\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right) \\
\frac{\widetilde{W}_{H,t}}{P_t} = W_{H,t} &= \Omega(1-\alpha)\frac{Y_{H,t}}{L_{H,t}}MC_{H,t}\frac{P_{H,t}}{P_t} \\
w_{H,t} &= y_{H,t} + mc_{H,t} - l_{H,t} - \left(\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right) \\
\frac{\widetilde{W}_{E,t}}{P_t} = W_{E,t} &= (1-\Omega)(1-\alpha)Y_{H,t}MC_{H,t}\frac{P_{H,t}}{P_t} \\
w_{E,t} &= y_{H,t} + mc_{H,t} - \left(\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right)
\end{aligned}$$

where we use the fact that

$$\begin{aligned}
p_{H,t} - p_t &= \gamma tot_t \\
rer_t &= (1-\gamma)tot_t + lopg_t \\
tot_t &= \frac{rer_t - lopg_t}{1-\gamma}
\end{aligned}$$

To get the expression for the real marginal costs for domestic prduced goods, just plug equations for $l_{H,t}$ and k_t into prioduction function to get:

$$\begin{aligned}
mc_{H,t} &= \frac{(1-\alpha)(1+\Omega)}{\alpha+(1-\alpha)\Omega}y_{H,t} + \frac{1}{\alpha+(1-\alpha)\Omega}\left[\alpha r_{G,t} + (1-\alpha)w_{H,t}\right] \\
&\quad + \frac{1}{\alpha+(1-\alpha)\Omega}\left[\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right] - \frac{1}{\alpha+(1-\alpha)\Omega}A^Y_t
\end{aligned}$$

Interestingly, depreciation of RER increases MC, while larger LOPG decreases it.

4 Investment

Entrepreneurs produce capital and sell it for nominal price $\widetilde{Q}_t$. Capital is produced using old capital and investment INV_t, which is produced exactly as consumption goods

$$INV_t = \left[(1-\gamma)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \gamma\frac{1}{\eta}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

so that demand functions of the entrepreneurs are exactly the same as HH's. Also, price of the investment is P_t and investment price index is equal to CPI. Capital accumulates according to

$$K_{t+1} = \Phi\left(\frac{INV_t}{K_t}\right)K_t + (1-\delta)K_t.$$

Φ is increasing concave, stands for adjustment costs:

$$\Phi\left(\frac{INV_t}{K_t}\right) = \frac{INV_t}{K_t} - \frac{\psi_I}{2}\left(\frac{INV_t}{K_t} - \delta\right)^2.$$

In steady state, we have $\Phi(SS) = \delta, \quad \Phi'(SS) = 1$. The first condition means that in steady state capital stock doesn't change, the second one ensures that the real price of capital is equal to one is SS. In log-lin:

$$\begin{aligned}
\bar{K}(1+k_{t+1}) &= \bar{K} + \Phi'\bar{K}\frac{\bar{I}}{\bar{K}}inv_t + (1-\delta)\bar{K}k_t + \Phi\bar{K}k_t - \Phi'\bar{K}\frac{\bar{I}}{\bar{K}^2}\bar{K}k_t \\
\bar{K}k_{t+1} &= \bar{K}\delta inv_t + (1-\delta)\bar{K}k_t + \delta\bar{K}k_t - \delta\bar{K}k_t \\
k_{t+1} &= \delta inv_t + (1-\delta)k_t
\end{aligned}$$

When deciding how much capital to produce, the entrepreneur solves

$$\max_{INV_t} \widetilde{Q}_t\Phi\left(\frac{INV_t}{K_t}\right)K_t - P_tINV_t$$

which gives FOC:

$$\begin{aligned}
\widetilde{Q}_t \Phi'\left(\frac{INV_t}{K_t}\right) &= P_t \\
Q_t &= \frac{1}{\Phi'\left(\frac{INV_t}{K_t}\right)}
\end{aligned}$$

Lets log-linearize, cause I don't find it straightforward at all:

$$\begin{aligned}
Q(1+q_t) &= \frac{1}{\Phi'\left(\frac{INV}{K}\right)} + (-1)\frac{1}{\Phi'\left(\frac{INV}{K}\right)^2}\Phi''\left(\frac{INV}{K}\right)\frac{INV}{K}inv_t + \\
&+ (-1)\frac{1}{\Phi'\left(\frac{INV}{K}\right)^2}\Phi''\left(\frac{INV}{K}\right)(-1)\frac{INV}{K^2}Kk_t \\
Qq_t &= \frac{1}{\Phi'\left(\frac{INV}{K}\right)^2}\Phi''\left(\frac{INV}{K}\right)\frac{INV}{K^2}Kk_t + \\
&(-1)\frac{1}{\Phi'\left(\frac{INV}{K}\right)^2}\Phi''\left(\frac{INV}{K}\right)\frac{INV}{K}inv_t \diagup : Q = \frac{1}{\Phi'\left(\frac{INV}{K}\right)} \\
q_t &= \frac{-\Phi''\left(\frac{INV}{K}\right)}{\Phi'\left(\frac{INV}{K}\right)}\frac{INV}{K}(inv_t - k_t)
\end{aligned}$$

Now $R_{K,t} = \frac{\{R_{G,t}+(1-\delta)Q_t\}K_t}{Q_{t-1}K_t}$ is the real gross return on capital received by the entrepreneurs. In log-lin:

$$\begin{aligned}
R_K(1+r_{K,t}) &= \frac{[R_G+(1-\delta)Q]K}{QK} + \frac{R_G K}{QK}r_{G,t} + (1-\delta)\frac{QK}{QK}q_t - \frac{[R_G+(1-\delta)Q]K}{(QK)^2}QKq_{-1} \\
r_{K,t} + q_{t-1} &= \frac{R_G}{QR_K}r_{G,t} + \frac{1-\delta}{R_K}q_t = \frac{R_G}{R_G+(1-\delta)Q}r_{G,t} + \frac{1-\delta}{R_K}q_t = \\
&= (1 - \frac{(1-\delta)Q}{R_G+(1-\delta)Q})r_{G,t} + \frac{1-\delta}{R_K}q_t = \left(1 - \frac{1-\delta}{R_K}\right)r_{G,t} + \frac{1-\delta}{R_K}q_t \\
r_{K,t} + q_{t-1} &= \left(1 - \frac{1-\delta}{R_K}\right)r_{G,t} + \frac{1-\delta}{R_K}q_t
\end{aligned}$$

So two things determine the return on investment by entrepreneurs. First, capital is used by intermediate (wholesale firms), which pay rental rate $r_{G,t}$. Second, since the entrepreneurs own the capital and rent it, any change in the price of capital influences the return on investment. It also influences the entrepreneur's net worth.

4.1 Frictions and Net Worth

Entrepreneurs (E) finance their production operations and owning of capital using their net worth N_{t+1} and financing from intermediary F_{t+1}. So their budget constraint is

$$Q_t K_{t+1} = F_{t+1} + N_{t+1}.$$

When borrowing from financial intermediary, E pays not only the gross real interest rate $R_t \frac{P_t}{P_{t+1}}$, but also a premium dependent on the leverage ratio of the E. BGG explain this using principal-agent problem. The premium is given by

$$Premium = \left(\frac{N_{t+1}}{Q_t K_{t+1}}\right)^{-\chi},$$

where χ measures the elasticity of the premium. BGG provide detailed explanation for why the premium should be in such a relation with leverage ratio, but I find it natural.
E are risk-neutral and choose K_{t+1} to maximize profit. Chosen K_{t+1} implies necessary F_{t+1}. On the optimal margin, expected marginal return on investment is equal to marginal financing cost:

$$E_t(R_{K,t+1}) = E_t\left[\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right)^{-\chi} R_t \frac{P_t}{P_{t+1}}\right]$$

which again in log-lin is

$$E_t r_{K,t+1} = r_t - \pi_{t+1} - \chi(n_{t+1} - q_t - k_{t+1}).$$

Now we need to determine the evolution of E's net worth. The new net worth consists of entrepreneurial equity held by the fraction ξ of Es that survive this period and E labor income $W_{E,t}$:

$$N_{t+1} = \xi V_t + W_{E,t}.$$

The remaining Es who leave the economy transfer their wealth to HHs $T_t = (1-\xi)V_t$. We assume that labor income of Es is small, so that $(1-\Omega) = 0.01$. This mechanism only ensures that net worth is pinned down in steady state. Equity is given by

$$V_t = R_{K,t} Q_{t-1} K_t - \left(\frac{N_t}{Q_{t-1} K_t}\right)^{-\chi} R_{t-1} \frac{P_{t-1}}{P_t} F_t.$$

So the equity is the return minus the repayment of loans. Note that an increase in interest rate lowers E's net worth, which increases the premium and further lowers the net worth. Before log-linearizing, realize that

$$\begin{aligned}
R_K &= \left(\frac{N}{QK}\right)^{-\chi} R\frac{P}{P} \\
Q &= \frac{1}{\Phi'\left(\frac{INV}{K}\right)} = 1 \\
N_{t+1} &= \xi V_t + W_{E,t} \\
Vv_t &= \frac{Nn_{t+1} - W_E w_{E,t}}{\xi} \\
F_t &= Q_{t-1}K_t - N_t \\
f_t &= \frac{K(q_{t-1} + k_t) - Nn_t}{F}
\end{aligned}$$

and log-linearize (I wont bother with writing the first term of Taylor expansion, I mean the one equal to the equation in the steady state, since it immediately cancels out on both sides):

$$\begin{aligned}
\frac{Nn_{t+1} - W_E w_{E,t}}{\xi} &= R_K QK(r_{K,t} + q_{t-1} + k_t) + \chi\left(\frac{N}{QK}\right)^{-\chi-1}\frac{N}{QK}R(QK-N)n_t \\
&\left[-\chi\left(\frac{N}{QK}\right)^{-\chi-1}\frac{N}{(QK)^2}QKR(QK-N) - \left(\frac{N}{QK}\right)^{-\chi}RQK\right](q_{t-1}+k_t) \\
&-\left(\frac{N}{QK}\right)^{-\chi}R(QK-N)(r_t - \pi_t) + \left(\frac{N}{QK}\right)^{-\chi}RNn_t
\end{aligned}$$

Now we make use of

$$R_K = \left(\frac{N}{QK}\right)^{-\chi} R$$

and $Q = 1$ to get

$$\begin{aligned}
Nn_{t+1} &= W_E w_{E,t} + \xi R_K K(r_{K,t} + q_{t-1} + k_t) + \xi\chi R_K(K-N)n_t - \\
&-\xi\left[\chi R_K(K-N) + R_K K\right](q_{t-1}+k_t) - \xi R_K(K-N)(r_{t-1} - \pi_t) + \xi R_K N n_t \\
Nn_{t+1} &= \xi R_K\left[K(r_{K,t} + q_{t-1} + k_t) + \chi(K-N)n_t - \chi(K-N)(q_{t-1}+k_t) - K(q_{t-1}+k_t)\right] + \\
&+\xi R_K\left[-(K-N)(r_{t-1} - \pi_t) + Nn_t\right] + W_E w_{E,t}
\end{aligned}$$

Denote $\Gamma_5 = \left(\frac{K}{N} - 1\right)$ to get:

$$\begin{aligned}
\frac{K-N}{N} &= \Gamma_5 \\
\frac{K}{N} &= \Gamma_5 + 1 \\
\frac{W_E}{N} &= \frac{W_E}{K}\frac{K}{N} = (\Gamma_5 + 1)\frac{W_E}{K}
\end{aligned}$$

Now collect terms and divide by N:

$$n_{t+1} = \xi R_K \left[(\Gamma_5 + 1) r_{K,t} - \chi\Gamma_5 (q_{t-1} + k_t) + (\chi\Gamma_5 + 1) n_t - \Gamma_5 (r_{t-1} - \pi_t)\right] + (\Gamma_5 + 1)\frac{W_E}{K} w_{E,t}$$

5 Retailers

We assume Calvo pricing on final goods market. There are importers and domestic firms, that sell on the domestic market and export to the foreign economy. Retailers buy the consumption good for price $P_{H,t}^{W}$. Each period, fraction $(1-\theta_H)$ resets its price to new optimal price $P_{H,t}^{NEW}$. The rest update their price according to

$$P_{H,t}(z) = P_{H,t-1}(z)(\pi_{t-1})^{\kappa},$$

where κ measures the degree of inflation indexation. This implies aggregate price level

$$P_{H,t} = \left[(1-\theta_H)\left(P_{H,t}^{NEW}\right)^{1-\varepsilon} + \theta_H \left(P_{H,t-1}\pi_{t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

Let $Y_{H,t}(z)$ be the composite good sold by a retailer z in period t. The aggregate good sold is given by

$$Y_{H,t} = \left(\int_0^1 Y_{H,t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Expected future demands by households are given by

$$Y_{H,t+k}(z) = \left(\frac{P_{H,t}^{NEW}}{P_{H,t+k}}(\pi_{t-1,t+k})^{\kappa}\right) Y_{H,t+k}$$

The representative firm maximizes

$$\max_{P_{H,t}^{NEW}} E_t \sum_{k=0}^{\infty} \beta^k \theta_H^k \left[Y_{H,t+k}(z) \left(P_{H,t}^{NEW} (\pi_{t-1,t+k})^{\kappa} - P_{H,t+k} \frac{P_{H,t+k}^W}{P_{H,t+k}} \right) \right]$$

where note that

$$\frac{P_{H,t+k}^W}{P_{H,t+k}} = MC_{H,t+k}.$$

FOC is

$$\sum_{k=0}^{\infty} (\beta\theta_H)^k E_t Y_{H,t+k} \left[P_{H,t}^{NEW} (\pi_{t-1,t+k})^{\kappa} - \frac{\varepsilon}{\varepsilon - 1} P_{H,t+k} MC_{H,t+k} \right] = 0$$

It is good to use this FOC to write the optimal price as

$$P_{H,t}^{NEW} = \mu \frac{\sum (\beta\theta_H)^k E_t Y_{H,t+k} \left[P_{H,t+k} MC_{t+k} \right]}{\sum (\beta\theta_H)^k E_t Y_{H,t+k} \left[(\pi_{t+k-1})^{\kappa} \right]}.$$

where $\mu = \frac{\varepsilon}{\varepsilon - 1}$ is the gross desired markup.
Log-linearize the equation above to get

$$p_{H,t}^{NEW} = (1 - \beta\theta_H)(p_{H,t} + mc_{H,t}) + (\beta\theta_H) \left[E_t \{ p_{H,t+1}^{NEW} - \kappa\pi_{t-1} \} \right]$$

where $mc_{H,t+k} = p_{H,t+k}^W - p_{H,t+k}$. Log-linearize the price index equation to get

$$\pi_{H,t} = (1 - \theta_H) \left[p_{H,t}^{NEW} - p_{H,t-1} \right] + \theta_H \kappa \pi_{t-1}$$

Substitute and rearrange to get

$$\pi_{H,t} = \frac{1}{1 + \beta\kappa} \left[\beta E_t \{ \pi_{H,t+1} \} + \kappa\pi_{t-1} + \Lambda^H mc_{H,t}) \right], \quad \Lambda^H = \frac{(1 - \beta\theta_H)(1 - \theta_H)}{\theta_H}.$$

6 Importers

Importers buy for price $P_{F,t}^W = S_t P_t^*$ (in local currency). They sell at price $P_{F,t}$. We assume $P_{F,t} \neq S_t P_t^*$. Again, importers price a la Calvo, so the price index for imported goods is

$$P_{F,t} = \left[(1 - \theta_F)(P_{F,t}^{NEW})^{1-\varepsilon} + \theta_F \left(P_{F,t-1} (\pi_{t-1})^{\kappa} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Again importers solve the problem

$$\max_{P_{F,t}^{NEW}} \sum_{k=0}^{\infty} (\beta\theta_F)^k E_t \left(Y_{F,t+k}(z) \left[P_{F,t}^{NEW}(\pi_{t+k-1}^{\kappa}) - P_{F,t+k}\frac{P_{F,t}^{W}}{P_{F,t+k}} \right] \right)$$

which results in

$$P_{F,t}^{NEW} = \mu \frac{\sum_{k=0}^{\infty}(\beta\theta_F)^k E_t(Y_{F,t+k}[P_{F,t+k}MC_{F,t+k}])}{\sum_{k=0}^{\infty}(\beta\theta_F)^k E_t(Y_{F,t+k}(\pi_{t+k-1})^{\kappa}}$$

which in log-lin becomes

$$p_{F,t}^{NEW} = (1-\beta\theta_F)[p_{F,t} - mc_{F,t}] + (\beta\theta_F)\left[E_t\{p_{F,t+1}^{NEW}\} - \kappa\pi_{t-1}\right]$$

where $mc_{F,t} = p_{F,t}^{W} - p_{F,t}$. By definition $p_{F,t}^{W} = s_t + p_t^*$, so

$$mc_{F,t} = s_t + p_t^* - p_{F,t} = lopg_t.$$

Log-linearize the price index for imported goods and combine to get

$$\pi_{F,t} = \frac{1}{1+\beta\kappa}\left[\beta E_t\{\pi_{F,t+1}\} + \kappa\pi_{t-1} + \Lambda^F lopg_t\right], \quad \Lambda^F = \frac{(1-\beta\theta_F)(1-\theta_F)}{\theta_F}$$

Finally, realize that overall CPI inflation

$$\pi_t = (1-\gamma)\pi_{H,t} + \gamma\pi_{F,t}$$

whi after plugging gives

$$\pi_t = \frac{1}{1+\beta\kappa}\left[\beta\{\pi_{t+1}\} + \kappa\pi_{t-1} + (1-\gamma)\Lambda^H mc_{H,t} + \gamma\Lambda^F lopg_t\right]$$

7 Monetary policy

Interest rate is given by

$$r_t = (1-\rho)\left[\beta_\pi \pi_{t+1} + \Theta_y y_{t+1}\right] + \rho r_{t-1} + \varepsilon_t^{MP}.$$

8 Market clearing and equilibrium

Foreing agents are assumed to have identical preferences to the domestic agents, so the foreign demand for domestic exports is given by

$$C^*_{H,t} = \gamma \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\eta} Y^*_t$$

We assume that law of one price holds for exports, so $P^*_{H,t} = \frac{P_{H,t}}{S_t}$, we know that $RER_t = S_t \frac{P^*_t}{P_t}$ and we have

$$C^*_{H,t} = \gamma \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left(\frac{1}{RER_t} \right)^{-\eta} Y^*_t.$$

In every period, final good $Y_{H,t}$ is either consumed, exported or invested, so we get aggregate resource constraint

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1-\gamma)(C_t + INV_t) + \gamma \left(\frac{1}{RER_t} \right)^{-\eta} Y^*_t \right].$$

In log-lin

$$y_{H,t} = \frac{C}{Y_H}(1-\gamma)c_t + \frac{INV}{Y_H}(1-\gamma)inv_t + \gamma y^*_t + \eta\gamma \left(\frac{2-\gamma}{1-\gamma} \right) rer_t - \frac{\eta\gamma}{1-\gamma} lopg_t.$$

Financial intermediary lends to entrepreneurs. To finance itself, it collects funds from households at cost R_t. We assume zero-profit banking and borrowing only from domestic HHs. So in equilibrium

$$F_t = D_t.$$

9 Net foreign assets

Eveolution of net foreign assets is

$$Z_t = R^*_{t-1} \Psi^B (Z_{t-1}, A^{UIP}_{t-1}) Z_{t-1} + (Y_{H,t} - C_t - INV_t)$$

where $Z_t = \frac{S_t B_t}{Y_t P_t}$ is the economy share of net foreign assets on NGDP. The term $(Y_{H,t} - C_t - INV_t)$ stands for current account balance. In log-lin, we get

$$z_t = \frac{1}{\beta} z_{t-1} + (y_{H,t} - c_t - inv_t) - \frac{\gamma}{1-\gamma}(rer_t - \gamma lopg_t)$$

10 Log-linearized equations

The model consists of following set of equations:

$$l_{H,t} = \frac{1}{\Psi}\left[w_{H,t} - \frac{1}{1-h}(c_t - hc_{t-1})\right] \tag{1}$$

$$(1-h)(r_t - E_t\pi_{t+1}) = (c_{t+1} - hc_t) - (c_t - hc_{t-1}) \tag{2}$$

$$rer_{t+1} - rer_t = (r_t - E_t\pi_{t+1}) - (r_t * - E_t\pi^*_{t+1}) + \psi^B z_t + A_t^{UIP} \tag{3}$$

$$r_{G,t} = y_{H,t} + mc_{H,t} - k_t - \left(\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right) \tag{4}$$

$$w_{H,t} = y_{H,t} + mc_{H,t} - l_{H,t} - \left(\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right) \tag{5}$$

$$w_{E,t} = y_{H,t} + mc_{H,t} - \left(\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right) \tag{6}$$

$$y_{H,t} = \alpha k_t + (\Omega(1-\alpha))l_{H,t} + A_t^Y \tag{7}$$

$$k_{t+1} = \delta inv_t + (1-\delta)k_t \tag{8}$$

$$q_t = \frac{-\Phi''\left(\frac{INV}{K}\right)}{\Phi'\left(\frac{INV}{K}\right)}\frac{INV}{K}(inv_t - k_t) \tag{9}$$

$$r_{K,t} + q_{t-1} = \left(1 - \frac{1-\delta}{R_K}\right) r_{G,t} + \frac{1-\delta}{R_K} q_t \tag{10}$$

$$E_t r_{K,t+1} = r_t - \pi_{t+1} - \chi(n_{t+1} - q_t - k_{t+1}) \tag{11}$$

$$n_{t+1} = \xi R_K \left[(\Gamma_5 + 1) r_{K,t} - \chi\Gamma_5(q_{t-1} + k_t)\right] + \tag{12}$$

$$+\xi R_K\left[(\chi\Gamma_5 + 1)n_t - \Gamma_5(r_{t-1} - \pi_t)\right] + (\Gamma_5 + 1)\frac{W_E}{K} w_{E,t} \tag{13}$$

$$\pi_t = \frac{1}{1+\beta\kappa}\left[\beta\{\pi_{t+1}\} + \kappa\pi_{t-1} + (1-\gamma)\Lambda^H mc_{H,t} + \gamma\Lambda^F lopg_t\right] \tag{14}$$

$$r_t = (1-\rho)\left[\beta_\pi \pi_{t+1} + \Theta_y y_{t+1}\right] + \rho r_{t-1} + \varepsilon_t^{MP} \tag{15}$$

$$y_{H,t} = \frac{C}{Y_H}(1-\gamma)c_t + \frac{INV}{Y_H}(1-\gamma)inv_t + \gamma y_t^* + \eta\gamma\left(\frac{2-\gamma}{1-\gamma}\right) rer_t - \frac{\eta\gamma}{1-\gamma} lopg_t \tag{16}$$

$$z_t = \frac{1}{\beta} z_{t-1} + (y_{H,t} - c_t - inv_t) - \frac{\gamma}{1-\gamma}(rer_t - \gamma lopg_t) \tag{17}$$

$$lopg_t = \rho_{lopg} * lopg_{t-1} + \varepsilon_t^{lopg} \tag{18}$$

11 Foreign economy

We can either introduce foreign economy as VAR(1) process or try to do it structurally. The equations describing the foreign block are

$$l^*_{H,t} = \frac{1}{\Psi}\left[w^*_{H,t} - \frac{1}{1-h}(c^*_t - hc^*_{t-1})\right] \tag{19}$$

$$(1-h)(r^*_t - E_t\pi^*_{t+1}) = (c^*_{t+1} - hc^*_t) - (c^*_t - hc^*_{t-1}) \tag{20}$$

$$r^*_{G,t} = y^*_t + mc^*_t - k^*_t \tag{21}$$

$$w^*_{H,t} = y^*_t + mc^*_t - l^*_{H,t} \tag{22}$$

$$w^*_{E,t} = y^*_t + mc^*_t \tag{23}$$

$$y^*_{H,t} = \alpha k^*_t + (\Omega(1-\alpha))l^*_{H,t} + A^{Y*}_t \tag{24}$$

$$k^*_{t+1} = \delta inv^*_t + (1-\delta)k^*_t \tag{25}$$

$$q^*_t = \frac{-\Phi''\left(\frac{INV^*}{K^*}\right)}{\Phi'\left(\frac{INV^*}{K^*}\right)}\frac{INV^*}{K^*}(inv^*_t - k^*_t) \tag{26}$$

$$r^*_{K,t} + q^*_{t-1} = \left(1 - \frac{1-\delta}{R^*_K}\right)r^*_{G,t} + \frac{1-\delta}{R^*_K}q^*_t \tag{27}$$

$$E_t r^*_{K,t+1} = r^*_t - \pi^*_{t+1} - \chi(n^*_{t+1} - q^*_t - k^*_{t+1}) \tag{28}$$

$$n^*_{t+1} = \xi R^*_K\left[(\Gamma_5+1)r^*_{K,t} - \chi\Gamma_5(q^*_{t-1} + k^*_t)\right] + \tag{29}$$

$$+\xi R^*_K\left[(\chi\Gamma_5+1)n^*_t - \Gamma_5(r^*_{t-1} - \pi^*_t)\right] + (\Gamma_5+1)\frac{W^*_E}{K^*}w^*_{E,t} \tag{30}$$

$$\pi^*_t = \frac{1}{1+\beta\kappa}\left[\beta\{\pi^*_{t+1}\} + \kappa\pi^*_{t-1} + \Lambda^* mc^*_t\right] \tag{31}$$

$$r^*_t = (1-\rho)\left[\beta_{\pi^*}\pi^*_{t+1} + \Theta_y y^*_{t+1}\right] + \rho r^*_{t-1} + \varepsilon^{MP*}_t \tag{32}$$

$$y^*_t = \frac{C^*}{Y^*}c^*_t + \frac{INV^*}{Y^*}inv^*_t \tag{33}$$

12 Model without financial frictions

Lets now remove financial frictions. We will assume that entrepreneurs have enough new worth to be able to cover the cost of their operations

$$N_t = Q_{t-1}K_t.$$

Therefore, E don't need to borrow from banks. We can also let them live indefinitely and we can remove the entreprenurial labor from the model. Therefore: $n_t, w_{E,t}$ disappear.

$$l_{H,t} = \frac{1}{\Psi}\left[w_{H,t} - \frac{1}{1-h}(c_t - hc_{t-1})\right] \tag{34}$$

$$(1-h)(r_t - E_t\pi_{t+1}) = (c_{t+1} - hc_t) - (c_t - hc_{t-1}) \tag{35}$$

$$rer_{t+1} - rer_t = (r_t - E_t\pi_{t+1}) - (r_t * - E_t\pi^*_{t+1}) + \psi^B z_t + A_t^{UIP} \tag{36}$$

$$r_{G,t} = y_{H,t} + mc_{H,t} - k_t - \left(\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right) \tag{37}$$

$$w_{H,t} = y_{H,t} + mc_{H,t} - l_{H,t} - \left(\frac{\gamma}{1-\gamma}(rer_t - lopg_t)\right) \tag{38}$$

$$y_{H,t} = \alpha k_t + (\Omega(1-\alpha))l_{H,t} + A_t^Y \tag{39}$$

$$k_{t+1} = \delta inv_t + (1-\delta)k_t \tag{40}$$

$$q_t = \frac{-\Phi''\left(\frac{INV}{K}\right)}{\Phi'\left(\frac{INV}{K}\right)}\frac{INV}{K}(inv_t - k_t) \tag{41}$$

$$r_{K,t} + q_{t-1} = \left(1 - \frac{1-\delta}{R_K}\right) r_{G,t} + \frac{1-\delta}{R_K} q_t \tag{42}$$

$$E_t r_{K,t+1} = r_t - \pi_{t+1} \tag{43}$$

$$r_t = (1-\rho)\left[\beta_\pi \pi_{t+1} + \Theta_y y_{t+1}\right] + \rho r_{t-1} + \varepsilon_t^{MP} \tag{44}$$

$$y_{H,t} = \frac{C}{Y_H}(1-\gamma)c_t + \frac{INV}{Y_H}(1-\gamma)inv_t + \gamma y_t^* + \eta\gamma\left(\frac{2-\gamma}{1-\gamma}\right) rer_t - \frac{\eta\gamma}{1-\gamma} lopg_t \tag{45}$$

$$z_t = \frac{1}{\beta} z_{t-1} + (y_{H,t} - c_t - inv_t) - \frac{\gamma}{1-\gamma}(rer_t - \gamma lopg_t) \tag{46}$$

$$lopg_t = \rho_{lopg} * lopg_{t-1} + \varepsilon_t^{lopg} \tag{47}$$