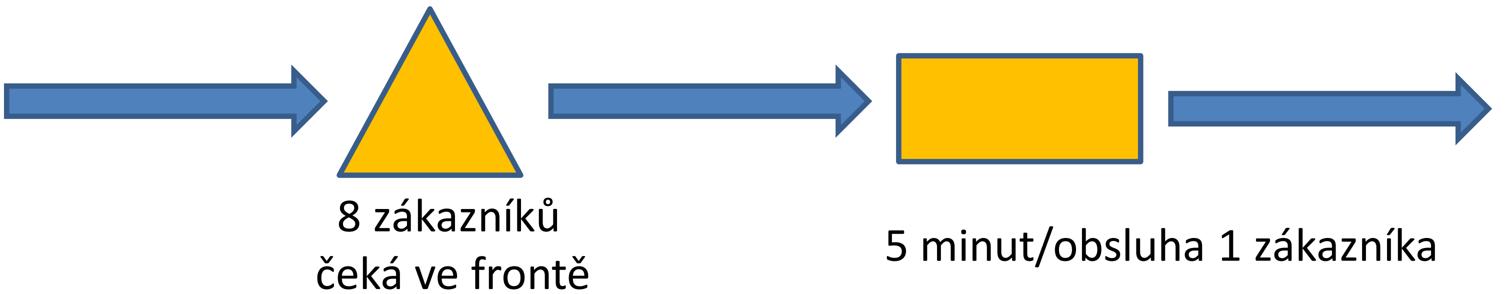
Littlův zákon – základ – 1.část

Běžná situace, kterou je potřeba řešit

- 30 zákazníků/hodina – (maximální kapacita provozovny)
- 8 zákazníků čeká ve frontě (jde v podstatě o nárazník)
- 5 minut trvá doba obsluhy jednoho zákazníka



- Potřeba odstranit všechny časy, které nepřinášejí hodnotu
- 1 obslužné místo=12 zákazníků za hodinu (60/5=12), takže pro 30 zákazníků/hodinu je potřeba kapacita 2,5 obslužného místa, což je 30/12=2,5
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Otázky

- Jak dlouho průměrně čeká zákazník ve frontě ?
- Kolik průměrně lidí může být naráz obslouženo ?
- Kolik zákazníků je v provozovně v jenom okamžiku (jak čekající, tak ti, které personál obsluhuje) ?
- Jaká je průměrná doba „průstupu“ (průtoku) zákazníka provozovnou (čekání i obsluha)
- **Zjednodušující podmínky**
  - „Vstupní tok“ (průměr) =„Výstupní tok“ (průměr)
  - Díky průměrování neuvažujeme fluktuace (viz hody mincí)

Klíčová měřítka (proměnné)

- Lead Time =LT (jak dlouho trvá proces)
- Inventory = **WIP**   (kolik jednotek je v procesu= nedokončená výroba-**W**ork **I**n **P**rogress)
- **Throughput Rate** =**T** (počet zákazníků nebo výrobků /jednotka času)
- Lead Time = čas/jednotka = počet minut /zákazník nebo počet hodin/výrobek apod.
-
- Tyto měřítka jsou propojena Littlovým zákonem : **WIP=T x LT**
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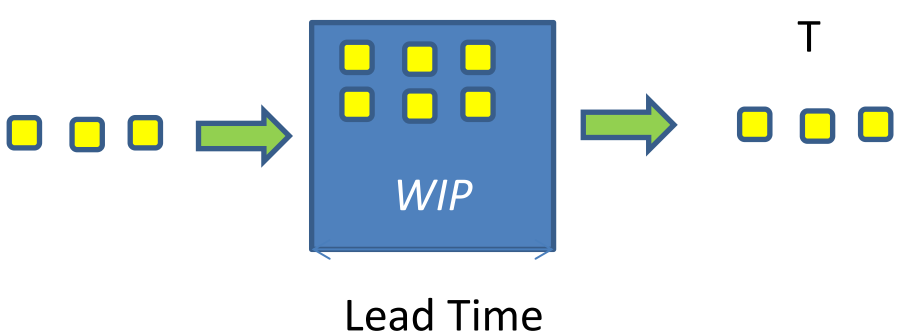
- **Příklad 1**   **: T= 30** výrobků/hod, výroba jednoho výrobku trvá **0,5** hodiny (0,5 hod/výrobek=**LT**).
- **Otázka :** kolik je **WIP** ?
- **Odpověď :** **WIP** =**30***0,5=**15** výrobků

- **Příklad 2 : T=**30 zákazníků/hod, obsluha jednoho je 5 minut, **WIP** =(30/60) *(5/1)=(1/2)*5 =**2,5**



Řešení |(home study)

Proces	**WIP**	T (Zák/hod)	LT
Buffer	8	30	
Obsluha		30	5
Celkem		30	

Proces	**WIP**	T (Zák/hod)	LT (min/Zák)	Time
Buffer	8	30	5	
Obsluha		30	5	5
Celkem		30	5	

WIP =T x LT

LT= WIP /T (třetí sloupec je kvůli jednotkám)

Proces	WIP	T (Zák/hod)	LT(min/Zák)	Time
Buffer	8,0	30	5	
Obsluha	2,5	30	5	5
Celkem	**10,5**	30	5	

Proces	WIP	T (Zák/hod)	LT (/min/Zák)	Time
Buffer	8	30	5	16
Obsluha	2,5	30	5	5
Celkem	**10,5**	30	5	**21**

Zadání (z předchozích snímků)

30 zákazníků/hodina – (max kapacita provozovny) = Throughput Rate = **T**

8 zákazníků čeká ve frontě (nárazník) = **WIP**

5 minut trvá doba obsluhy jednoho zákazníka = LT

WIP = TxLT=((30/60)*5)=(3*5)/6 = 2,5 , tedy kolik zákazníků naráz může být obslouženo a celkem jich je v provozovně 10,5=8,0+2,5 a dále pak :

LT= **WIP** /T=8/(3/6)= (8*6)/3=48/3=16 (**21**=5+16)tak dlouho čeká zákazník ve frontě a 5=(2,5/(3/6))=2,5*6/3=15/3 je doba obsluhy (zadáno)

Otázky

- Jak dlouho průměrně čeká zákazník ve frontě ?
- **Odpověď =16**
- Kolik průměrně lidí může být naráz obslouženo ?
- **Odpověď = 2,5**
- Kolik zákazníků je v provozovně v jenom okamžiku (jak čekající tak ty, které personál obsluhuje) ?
- **Odpověď = 10,5**
- Jaká je průměrná doba „průstupu“ zákazníka provozovnou (čekání i obsluha) ?
- **Odpověď = 21 minut**

Littlův zákon – základ – 2.část

Skorkovský ,KPH,ESF.MU

Based on resource : Factory Physics (Hopp and Spearman)

Little´s law - definition (formula)

- Fundamental relationships among :
  - WIP (Work In Process)
  - Cycle Time (CT) = Lead Time
  - Throughput (T or sometimes TH)
- Formula

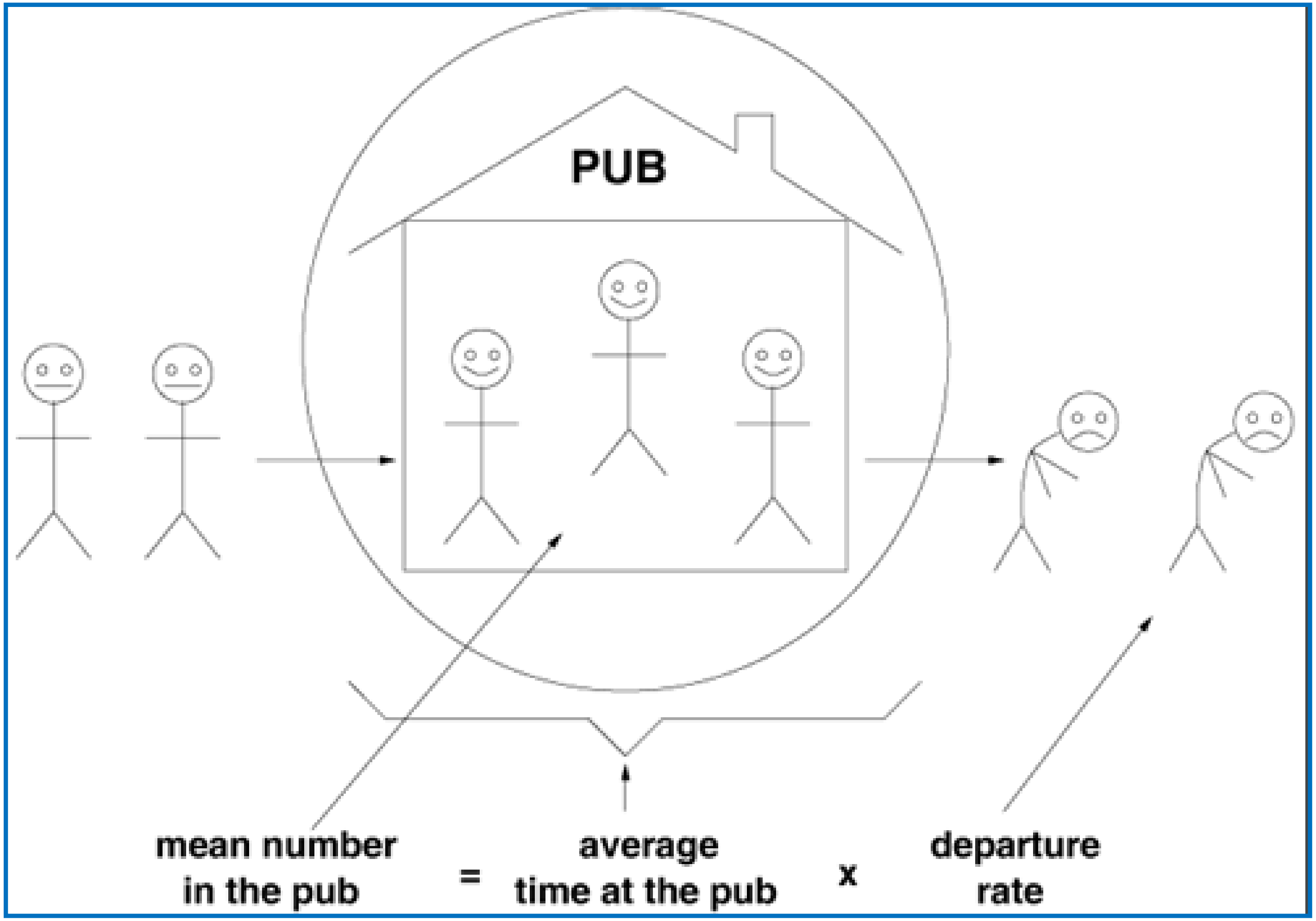
$$WIP = TH \times CT = T \times LT$$

- Can be applied to :
  - Single machine station
  - Complex production line
  - Entire plant

Relationships among these variables will serve to se clearly precise (quantitative) description of behaviour of the ***single production line*** *. It helps user to use a given scale to benchmark actual production systems*

I finally figured it out !!!!

Daily application of the law….



Definition of basic parameters

- **Throughput (Throughput rate, TH,T)** : production per unit time that is sold (see TOC definition !!!!)
- **If TH (T)** is measured in **Cost Dollars** rather than in prices, it is typically called :

Cost of good sold (COGS)

- **Upper limit** of TH in production process is Capacity !!!
- If you release more raw material above capacity of the line (machine), system become **unstable** –> **WIP goes up !!**

Definition of basic parameters

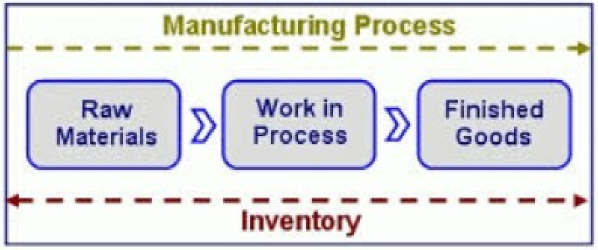
- **WIP (Work In Process)** : inventory between start and end points of the product routing

- **WIP** can be used as one parameter to calculate (measure) an **efficiency**

- **Efficiency** can be defined as **Turnover Ratio** = TH/FGI for warehouses or TH/(FGI+WIP) for production plants where **FGI**=Finished goods inventory

- **WIP** : **inventory still in production line**

- **FGI** : inventory waiting for dispatch (shipping)



Definition of basic parameters (home study)

Utilization U(x) = Arrival rate / Effective production rate

- Where
  - **Arrival Rate =r** = amount of parts arriving to workstation per time unit
  - **Effective production time** is maximum average rate at which the workstation can process parts (considering effects of failures, setup times and so on)

- Bottleneck rate (see TOC)
  - **r**b = rate (parts per unit time or jobs per unit time) of workstation having the **highest long term utilization**

Definition of basic parameters (home study)

- **T_0** =Row process time of the line is the sum of the long –term average process time of each workstation in the line (single job entering empty line from staring point to the ending one)

7, 8, 9, 7, 6, 10
7,83

5,3, 4, 5, 6, 8
5,16

9, 8, 4, 5, 5, 6
6,10

19,16=7,83+5,16+6,1

Definition of basic parameters (home study)

- **Critical WI**P (**W_0**) of the line is the WIP level for which a line with given values of **r_b** (bottleneck rate) and T_0 achieves **maximum throughput (r_b)** with minimum cycle time (which is in this case **T_0**)

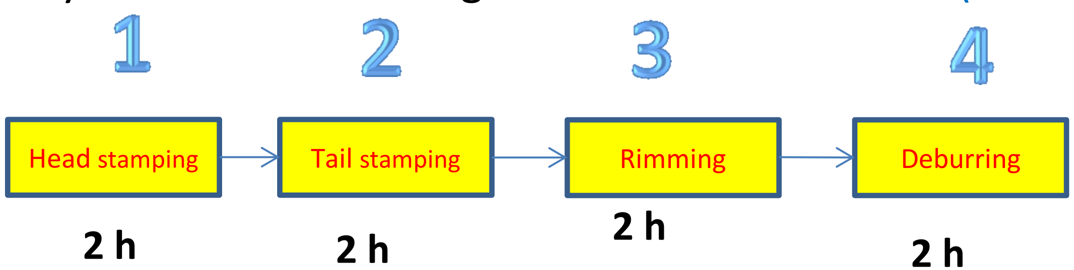
$$W_0 = r_b \times T_0$$

$$\mathbf{WIP = TH \times CT}$$

Poznámka vyučujícího : problém s reprezentací algoritmu je např. ten, že v různé literatuře se využívá jiné označení proměnných. Principy jsou samozřejmě identické.

Use of defined parameters (home study)

- Simple production line that makes giant one-cent pieces
- It is as a model very with unrealistic assumption because process times are deterministic (no waiting times after every operation and no queue times before any other operation, 24 hours /day an unlimited market)→ balanced line
- Any machine can be regarded as the bottleneck (one half part per hour=0,5)

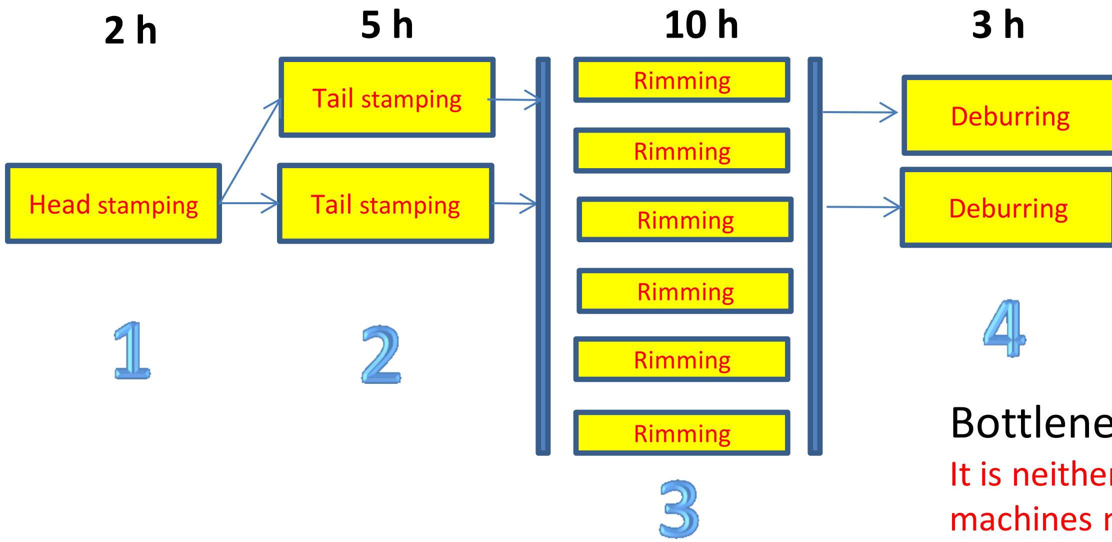


rb = bottleneck rate (parts per unit time or jobs per unit time) of workstation having the **highest long term utilization =0.5 penny per hour, which means 24 hour x 0.5=12 pennies /day**

T_0 =Row process time of the line is the sum of the long –term average process time of each workstation in the line = 8 hours=2+2+2+2=8

Critical **WIP (W_0)** of the line is the WIP level for which a line with given values of **rb** and **T_0** achieves maximum throughput (rb) with minimum cycle time (which is in this case T_0) = **rb x T_0 = 0.5 x 8 = 4 pennies**

Use of defined parameters (home study)



Bottleneck= **rb** =**0,4**

It is neither the station that contains the slowest machines nor the one with fewest machines !!!

Station number	Number of machines	Process time (hours)	Station capacity (jobs per hour)
1	1	2	0.5=(1/2)*1
2	2	5	0.4=(1/5)*2
3	6	10	0.6=(1/10)*6
4	2	3	0.67=(1/3)*2

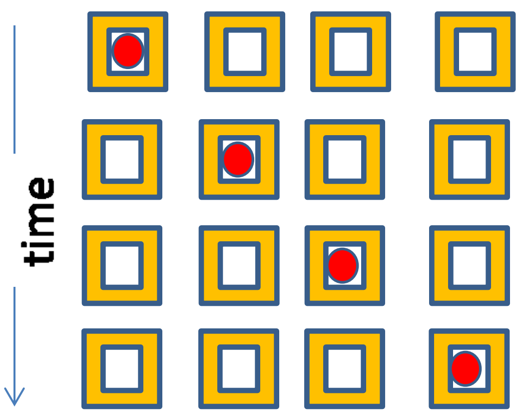
T_0 =Row process time of the line =2+5+10+3 =**20** and

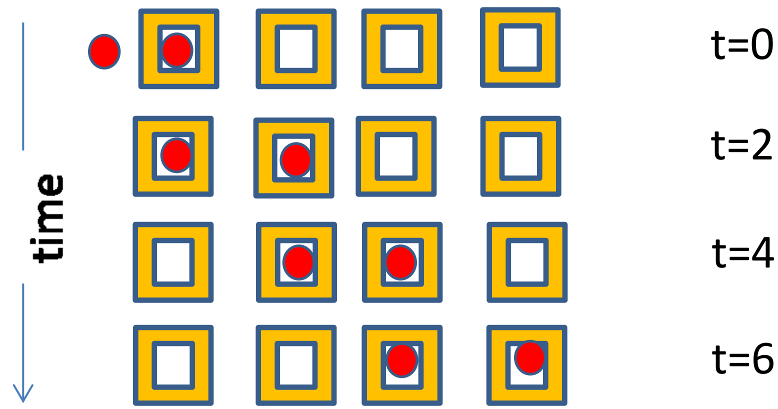
Critical **WIP(W_0)** =rb x T_0 = **0.4** x**20** = 8 pennies <number of machines (11). This is because system is not balanced. Not balanced line is the line where some stations are not fully utilized !!!

Best case performance I (home study)

WIP=1, Total cycle time= T_0=8=CT,
Throughput=1/8=part/hour= **rb**/4=0,125,
Bottleneck rate = **rb**=0,5=4*0,125

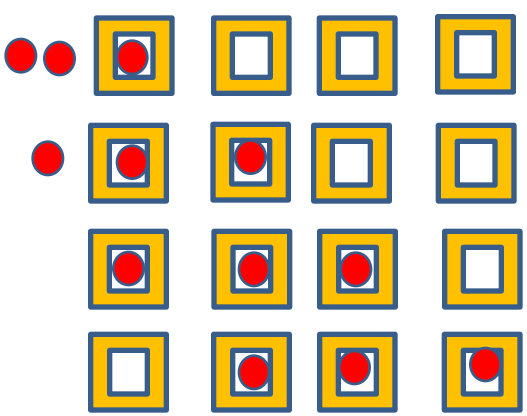
WIP=2, Total cycle t ime= T_0=8=CT
Throughput=WIP/CT=WIP/T_0= 2/8=part/hour= **rb**/2=0,250, což je 50 % of rb where bottleneck rate = **rb**=0,5

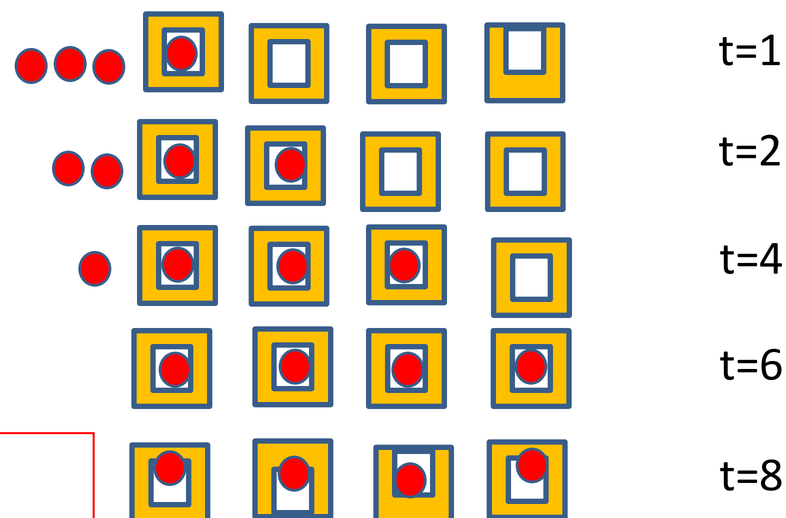




WIP=3

WIP=4

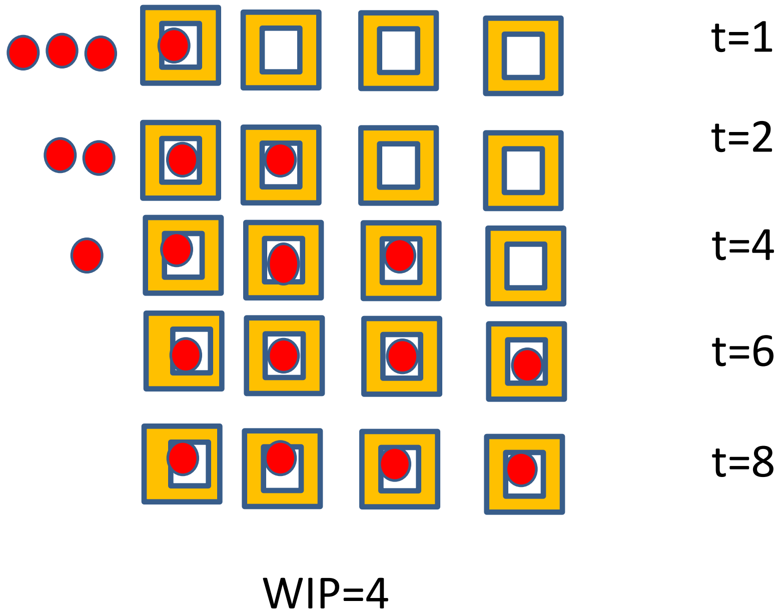




Text on the next slide

WIP=3, Total cycle time= T_0=8=CT,
Throughput=WIP/CT=WIP/T_0= 3/8=part/hour
= 0,375, což je 75 % of rb where bottleneck rate = **rb**=0,5

Best case performance II (home study)



T_o =Row process time of the line is the sum of the long –term average process time of each workstation in the line (single job entering empty line from staring point to the ending one)

All stations stay busy all the time. Because no waiting so T_o=8 h., rb=0,5 .
So minimum value of T_o an maximum value of rb (throughput) is achieved only if WIP is set to critical value !

Critical **WIP (W_o)** of the line is the WIP level, for which a line with given values of **rb** and **T_o** achieves maximum throughput (**rb**) with minimum cycle time (which is in this case T_o), so **WIP (W_o)** = **rb x T_o = 0.5 x 8 = 4 pennies-** on the next slide se red line

Best case performance III (home study)

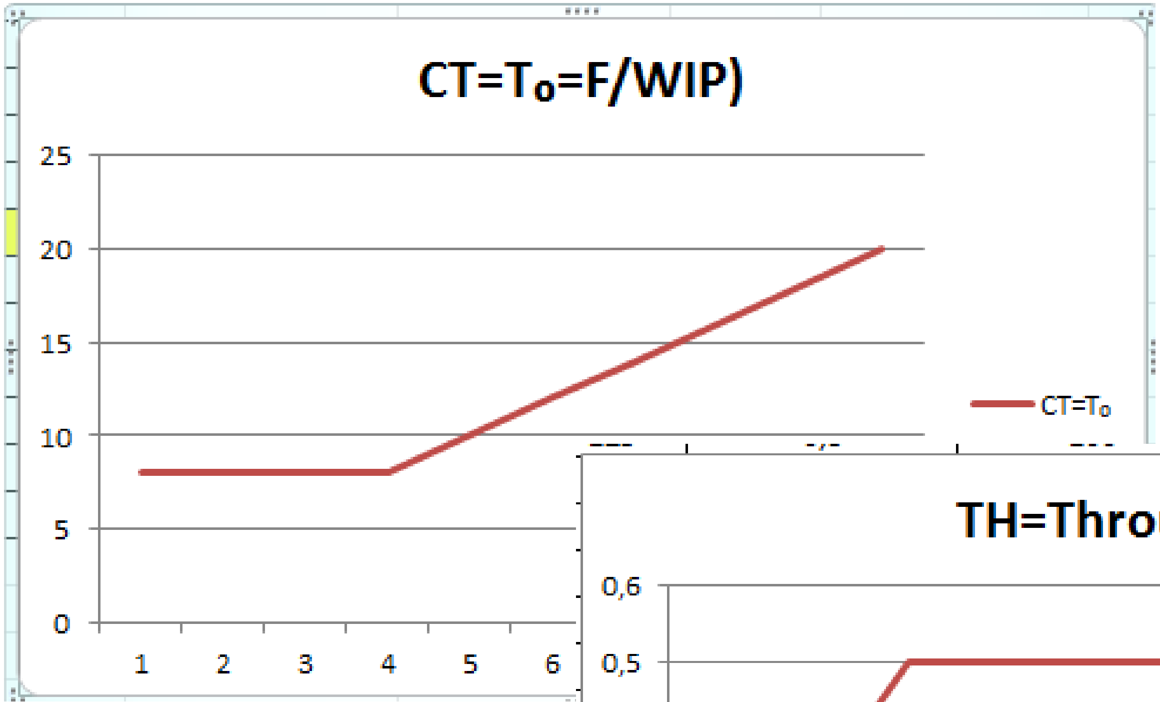
Table 1.

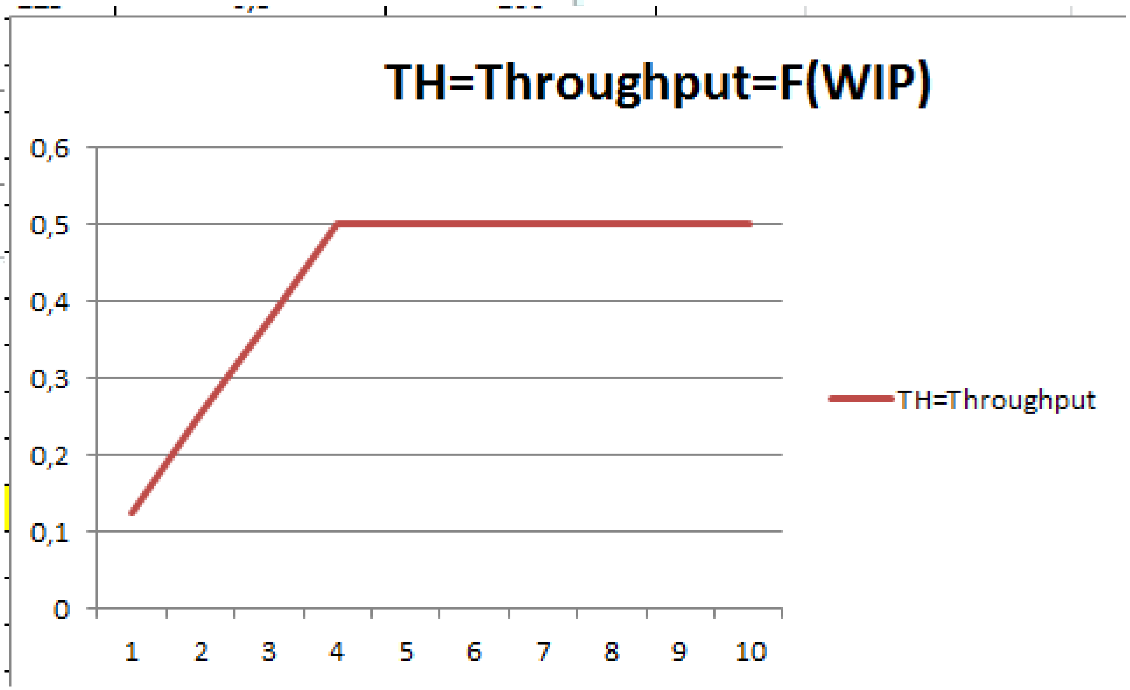
Parameters :	100 % rb=0,5			
WIP	CT=T_o	% T_o	TH=Throughput	% rb
1	8	100	0,125=1/8	25
2	8	100	0,25=2/8	50
3	8	100	0,375=3/8	75
4	8	100	0,5=4/8	100
5	10	125	0,5=5/10	100
6	12	150	0,5=6/12	100
7	14	175	0,5=7/14	100
8	16	200	0,5=8/16	100
9	18	225	0,5=9/18	100
10	20	250	0,5=10/20	100

20 hours = 12 hours waiting before line and 8 hours processing
All machines remains busy so rb=0,5 (2 hours per processing one penny)
% T_o =(CT/ T_o) *100, so e.g. 250=(20/8)*100
% rb =((WIP/ T_o)/0,5)*100, where rb=0,5

Explanation : Max rb=0,5 so if WIP=5, then T_o =5/0,5=10,
If WIP=6, then T_o=6/0,5=6/(5/10)=12 and so on

Best case performance IV





Conclusion

- Close examination of Table 1 reveals fundamental relationship among WIP, CT (LT) and T (Throughput)
-
- This relation is known as Little´s law.
- **WIP=TH * LT**
- *Source : Factory Physiscs, Wallace J Hopp and Mark L. Spearman ; ISBN 13: 978-1-57766-739-1   or   ISBN 10 :1-57766-739-5*
- http://www.factoryphysics.com/principle/littleslaw.htm

Example 1 (home study)

- **Estimating Waiting Times:** If are in a grocery queue behind 10 persons and estimate that the clerk is taking around 5 minutes/per customer, we can calculate that it will take us 50 minutes (10 persons x 5 minutes/person) to start service.
- This is essentially **Little's law.** We take the number of persons in the queue (10) as the "inventory".
- The inverse of the average time per customer (1/5 customers/minute) provides us the rate of service or the throughput.
- Finally, we obtain the waiting time as equal to number of persons in the queue divided by the processing rate 10/(1/5) = 50 minutes).

Example 2 (home study)

- **Planned Inventory Time:** Suppose a product is scheduled so that we expect it to wait for 2 days in finished goods inventory before shipping to the customer. This two days is called planned inventory time and is sometimes used as protection against system variability to ensure high delivery service. Using Little's law the total amount of inventory in finished goods can be computed as :
- **FGI = throughput × planned inventory time**

Youtube examples (6 minutes)

- http://www.youtube.com/watch?v=VU8TUSnQ-vw
- http://www.youtube.com/watch?v=rtGihR-bm-U