

Little's law basics

Ing. J.Skorkovský,CSc.
KPH-ESF-MU BRNO

Different times used in Little's law

- Lead time (LT)
- Flow time (FT)
- Cycle time (CT)

It is absolutely essential to define precisely above mentioned times in order to better understand principles of Little's law

$$WIP = TH \times CT$$

Other two variables of Little's law

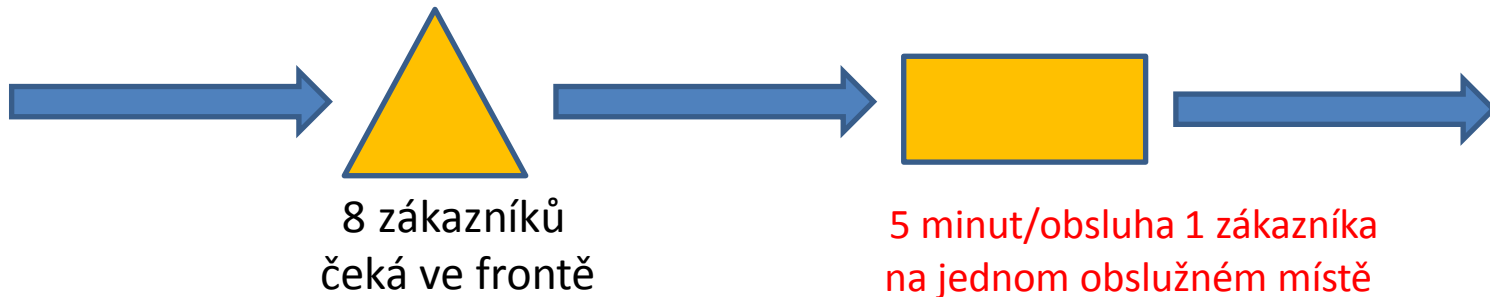
- **WIP**= Work in Process (Work in Progress)
- **TH**=Throughput=Throughput Rate = average output of production process (machine, workstations) per unit time

Definitions

- CT=average time from when the job is released into station (machine) or line to when it exits
- LT=management constant indicating the time allotted for production of a part on a given routing
- CT =FT (in different publications they use FT instead of CT)
- CT=Throughput Time (in different publications they use Throughput Time instead of CT)

Běžná situace, kterou je potřeba řešit

- 30 zákazníků/hodina – (maximální kapacita provozovny)
- 8 zákazníků čeká ve frontě (nárazník)
- 5 minut trvá doba obsluhy jednoho zákazníka



- Potřeba odstranit všechny časy, které nepřinášejí hodnotu
- 1 obslužné místo=12 zákazníků za hodinu ($60/5=12$), takže pro 30 zákazníků/hodinu je potřeba kapacita $2,5 = 30/12$ obslužného místa

Otázky

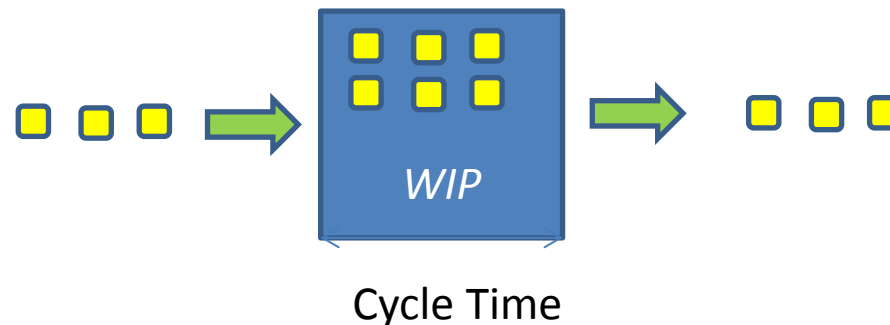
- Jak dlouho průměrně čeká zákazník ve frontě ?
- Kolik průměrně lidí může být naráz obslouženo ?
- Kolik zákazníků je v provozovně v jenom okamžiku (jak čekající, tak ti, které personál obsluhuje) ?
- Jaká je průměrná doba „průstupu“ (průtoku) zákazníka provozovnou (čekání i obsluha)
- **Zjednodušující podmínky**
 - „Vstupní tok“ (průměr) = „Výstupní tok“ (průměr)
 - Díky průměrování **neuvažujeme fluktuaci** (viz hody mincí)

Klíčová měřítka a proměnné (doplnění definic)

- **CT = Cycle Time** (jak dlouho trvá celý proces) = 5
- **Work in Process = WIP** (kolik jednotek je v procesu = nedokončená výroba = **W**ork **I**n **P**rogress)
- **Throughput = TH** (počet zákazníků/jednotka času) – např. 30/hod

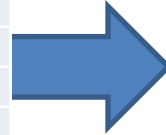
- Tyto měřítka jsou propojena Littlovým zákonem **WIP = TH x CT**

- Náš příklad : **TH = 30 zákazníků/hod**, obsluha jednoho je 5 minut, **WIP = (30/60) * (5/1) = (1/2) * 5 = 2,5**



Řešení (home study)

Proces	WIP	TH (Zák/hod)	CT
Buffer	8	30	
Obsluha		30	5
Celkem		30	



Proces	WIP	TH (Zák/hod)	CT (min/Zák)	Time
Buffer	8	30	5	
Obsluha		30	5	5
Celkem		30	5	

$$WIP = TH \times CT$$



Proces	WIP	TH (Zák/hod)	CT(min/Zák)	Time
Buffer	8	30	5	
Obsluha	2,5	30	5	5
Celkem	10,5	30	5	

$$CT = WIP / TH \text{ (třetí sloupec je kvůli jednotkám)}$$



Proces	WIP	TH(Zák/hod)	CT (/min/Zák)	Time
Buffer	8	30	5	16
Obsluha	2,5	30	5	5
Celkem	10,5	30	5	21

Zadání (z předchozích snímků)

30 zákazníků/hodina – (max kapacita provozovny) = Throughput = TH

8 zákazníků čeká ve frontě (nárazník) = WIP =buffer

5 minut trvá doba obsluhy jednoho zákazníka = CT

$WIP = TH \times CT = ((30/60) \times 5) = (3 \times 5) / 6 = 2,5$, tedy kolik zákazníků **naráz** může být obslouženo a celkem jich je v provozovně $10,5 = 8,0 + 2,5$ a dále pak :

$CT = WIP / TH = 8 / (3/6) = (8 \times 6) / 3 = 48 / 3 = 16$ (tak dlouho čeká zákazník ve

frontě) a nakonec pro kontrolu $CT = 5 = (2,5 / (3/6)) = 2,5 \times 6 / 3 = 15 / 3$ je doba obsluhy (už bylo zadáno)

Otázky

- Jak dlouho průměrně čeká zákazník ve frontě ?
- **Odpověď = 16**
- Kolik průměrně lidí může být naráz obslouženo ?
- **Odpověď = 2,5**
- Kolik zákazníků je v provozovně v jenom okamžiku (jak čekající tak ty, které personál obsluhuje) ?
- **Odpověď = 10,5**
- Jaká je průměrná doba „průstupu“ zákazníka provozovnou (čekání i obsluha) ?
- **Odpověď = 21 minut**

Little's law-2nd part

Skorkovský ,KPH,ESF.MU

Based on resource : Factory Physics (Hopp and Spearman)

Little's law - definition (formula)

- Fundamental relationships among :
 - WIP (Work In Process)
 - Cycle Time (CT)
 - Throughput (T or sometimes TH)
- Formula

$$WIP = TH \times CT$$

- Can be applied to :
 - Single machine station
 - Complex production line
 - Entire plant

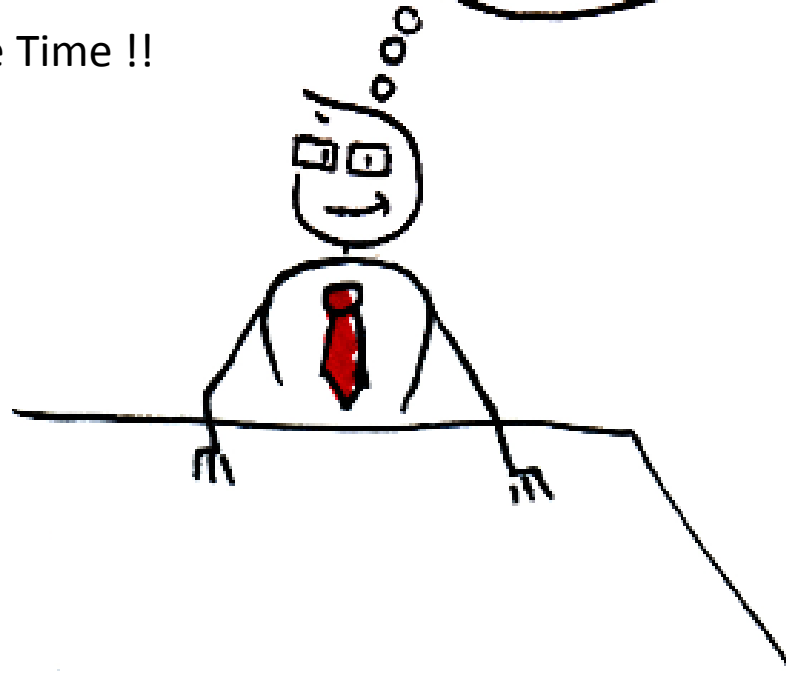
Relationships among these variables will serve to se clearly precise (quantitative) description of behaviour of the single production line . It helps user to use a given scale to benchmark actual production systems

I finally figured it out !!!!

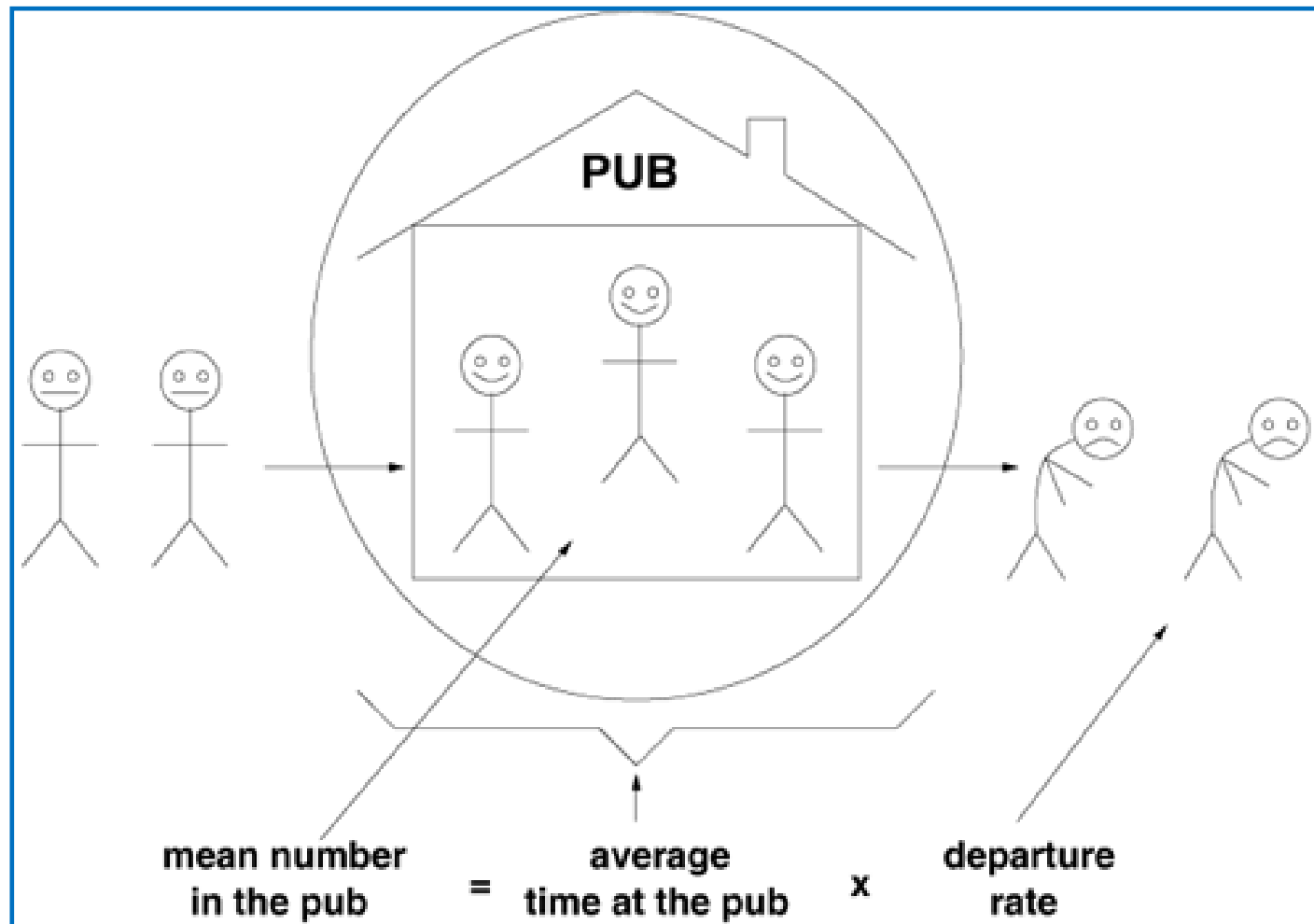
Little's Law

$$\text{avg. Lead Time} = \frac{\text{avg. Work in Progress}}{\text{avg. Throughput}}$$

Avg. Lead Time = Cycle Time !!



Daily application of the law....



Definition of basic parameters (supplements)

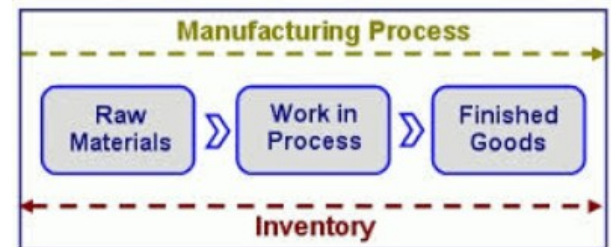
- **Throughput (Throughput rate, TH)** : production per unit time that is sold (see TOC definition)
- If **TH** is measured in cost dollars rather than in prices, it is typically called :

Cost of good sold (COGS)

- **Upper limit** of TH in production process is capacity
- If you release more raw material above capacity of the line (machine), **system** become unstable → **WIP** goes up !! See later ...

Definition of basic parameters (supplements)

- **WIP (Work In Process)** : inventory between start and end points of the product routing
- **WIP** can be used as one parameter to calculate (measure) an **efficiency**
- **Efficiency** can be defined as **Turnover Ratio** = TH/FGI for warehouses or $TH/(FGI+WIP)$ for production plants where **FGI**=Finished goods inventory
- **WIP** : inventory still in line
- **FGI** : inventory waiting for dispatch (shipping)

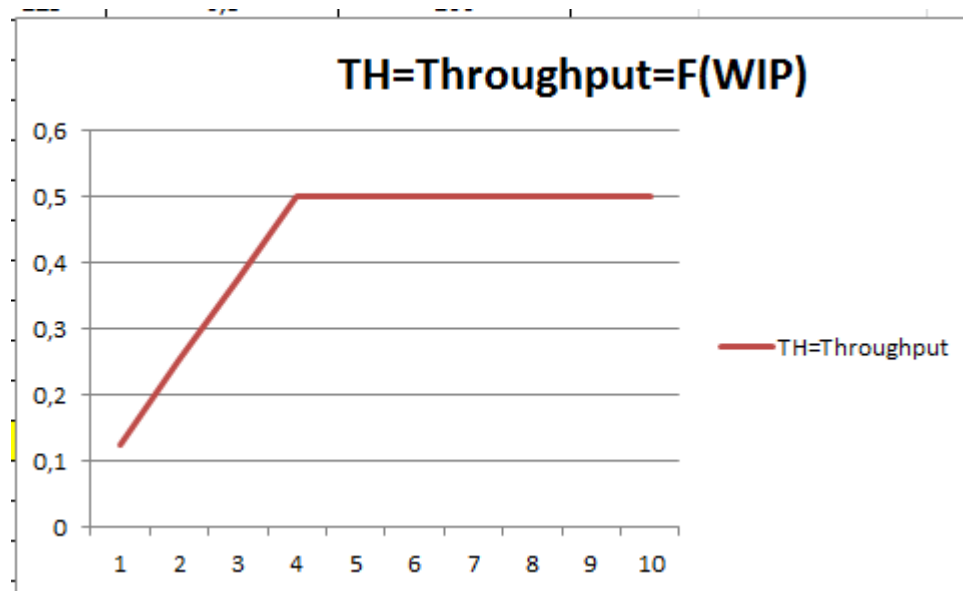


Definition of basic parameters (supplements)

- **CT (Cycle Time) or so called Throughput Rate** : average time from release of the job of the beginning of the routing until it reaches an inventory point at the end of the routing or time that part spends as a WIP.
- **LT (Lead Time)** : managerial constant used for planning of production
- **Service Level** (especially for M-T-O lines, where plant have to satisfy orders with specific due dates) :

Service level $P\{\text{Cycle time} \leq \text{Lead Time}\}$

Best case performance



Resources

- **WIP=TH * CT**

- *Source : Factory Physics, Wallace J Hopp and Mark L. Spearman ; ISBN 13: 978-1-57766-739-1 or ISBN 10 :1-57766-739-5*

<http://www.factoryphysics.com/principle/littleslaw.htm>

Example 1 (home study)

- **Estimating Waiting Times:** If are in a grocery queue behind 10 persons and estimate that the clerk is taking around 5 minutes/per customer, we can calculate that it will take us 50 minutes (10 persons x 5 minutes/person) to start service.
- This is essentially **Little's law**. We take the number of persons in the queue (10) as the "inventory".
- The inverse of the average time per customer (1/5 customers/minute) provides us the rate of service or the Throughput.
- Finally, we obtain the waiting time as equal to number of persons in the queue divided by the processing rate $10/(1/5) = 50$ minutes).

Example 2 (home study)

- **Planned Inventory Time:** Suppose a product is scheduled so that we expect it to wait for 2 days in finished goods inventory before shipping to the customer. This two days is called **planned inventory time** and is sometimes used as protection against system variability to ensure high delivery service. Using Little's law the total amount of inventory in finished goods can be computed as :
- **FGI = throughput × planned inventory time**

Youtube examples (6 minutes)

- <http://www.youtube.com/watch?v=VU8TUSnQ-vw>
- <http://www.youtube.com/watch?v=rtGihR-bm-U>