## Exam test BPM_AOME, 5. 6. 2020

Name:
Score

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | $\sum$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |

Problem 1:[10b] Consider following LP problem:
$2 x_{1}+3 x_{2}+7 x_{3}+9 x_{4} \rightarrow \max$
subject to

$$
\begin{aligned}
& x_{1}+x_{2}+x_{3}+x_{4} \leq 9, \\
& x_{1}+2 x_{2}+4 x_{3}+8 x_{4} \leq 24, \\
& x_{i} \geq 0, i=1, \ldots 4 .
\end{aligned}
$$

After several steps of simplex method we got the table:

| $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ | $x_{5}$ | $x_{6}$ | $b_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $3 / 4$ | $1 / 2$ | 0 | -1 | 1 | $-1 / 4$ | 3 |
| $1 / 4$ | $1 / 2$ | 1 | 2 | 0 | $1 / 4$ | 6 |
| $-1 / 4$ | $5 / 4$ | 0 | 5 | 0 | $7 / 4$ | 42 |

- Check the optimality of the table and if it is not yet optimal, perform the next steps of the Simplex method.
- Find the basic variables in the optimal table.
- Find the optimal value of objective function.
- Determine which of the constraints are binding and find the shadow prices.

Problem 2:[10b] The office needs to outsource four jobs, selecting from the offer of five companies. Each company is able to complete only one job. Estimated costs are in the table. Use Hungarian method to assign jobs to companies so that the total cost is minimal.

|  | F1 | F2 | F3 | F4 | F5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Z1 | 15000 | 14000 | 20000 | 17000 | 16000 |
| Z2 | 18000 | 17000 | 19000 | 16000 | 15000 |
| Z3 | 17000 | 15000 | 18000 | 15000 | 14000 |
| Z4 | 17000 | 19000 | 20000 | 16000 | 18000 |

Problem 3: [10b] The project consists of nine activities. The duration of activities in days and their predecessors are listed in the following table:

| activity | A | B | C | D | E | F | G | H | I |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| duration | 7 | 9 | 6 | 5 | 5 | 4 | 9 | 8 | 6 |
| predecessors | - | - | A | A | B | C,D | C | C,D | E,F |

Draw a project network and use CPM to find minimal realization time of the project, critical activities and total time reserves for non-critical activities.

Problem 4:[10b] The DEA model for 6 units with two inputs and one output is defined by the table.

|  | $U_{1}$ | $U_{2}$ | $U_{3}$ | $U_{4}$ | $U_{5}$ | $U_{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{1}$ | 12 | 2 | 2 | 10 | 16 | 14 |
| $I_{2}$ | 9 | 16 | 12 | 10 | 4 | 2 |
| $O$ | 3 | 4 | 2 | 5 | 4 | 2 |

- Solve the problem graphically constant returns to scale and draw efficient frontier.
- Compute efficiency of $U 1$. Find virtual unit for $U 1$ (its projection on efficient frontier) and its peer units.

Problem 5:[15b] Suppose that you live in the city depicted in the plan.

a) Find the shortest path from home to other destinations.
b) Design which paths are to be repaired so there was a repaired connection between each of the two places and the total number of repaired km was minimal.

Problem 6:[15b] Optimize the delivery plan from three plants D1-D3 to four customers O1-O4. Plant capacities $[t]$, customer requirements $[t]$ and distance between suppliers and customers $[\mathrm{km}]$ are shown in the table below.

|  | O1 | O2 | O3 | O4 | capacities |
| :---: | :---: | :---: | :---: | :---: | :---: |
| D1 | 6 | 10 | 4 | 2 | 100 |
| D2 | 8 | 14 | 10 | 6 | 140 |
| D3 | 4 | 18 | 12 | 8 | 180 |
| requirements | 60 | 80 | 80 | 200 |  |

- Introduce a dummy row or column to balance the the transportation problem of minimizing total tonne-km.
- Find any feasible solution.
- Check the optimality of the solution using dual table. If the condition is not satisfied, perform steps of the MODI method necessary to obtain the optimal solution.

Problem 7:[15b] 5 cars are evaluated according to three criteria - price (CZK thousand), consumption (l per 100 km ) and safety (in points). The first two criteria are considered as minimizing, the last one is maximizing.

| Alternatives | price | consumption | safety |
| :---: | :---: | :---: | :---: |
| X1 | 400 | 7.5 | 7 |
| X2 | 500 | 7.2 | 9 |
| X3 | 340 | 6.8 | 6 |
| X4 | 300 | 6.5 | 4 |
| X5 | 380 | 7 | 8 |

- Find all dominated alternatives
- Find basal and ideal alternative.
- Normalize criteria matrix and rank the alternatives using WSA method for the weights vector $v=(0.4,0.2,0.4)$.

Problem 8:[15b] The investor decides to divide his investment between shares, mutual funds and government bonds. The total amount of the investment is equal to CZK 10 million. The investor assumes the return of individual assets of $6,3,2$ percent p. a. The risk of individual assets was rated by 10,6 and 2 points.

- Formulate a mathematical model to maximize expected return and minimize the risk.
- Find both partial optimal solutions.
- Write down the aggregate objective function and find the optimal solution using Solver when we consider the risk to be a three times more important criterion than the return.
- Convert the criterion return to the constraint by allowing a 10 -percent deviation from the optimal value. Identify the minimum risk of the resulting single-criterion problem.

