Portfolio Theory Lecture 7

Luděk Benada

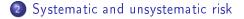
Department of Finance - 402, benada.esf@gmail.com

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Luděk Benada MPF APOT









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Testing of the model

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• The model is based on expectations

- The variables are expressed in future value
- ...but the calculation is on observed values
- Prices of assets will be variing around equilibrium
- The equilibrium return of an individual asset:

•
$$r_i^e = r_f + (r_M - r_f) * \beta_i$$

• ...but the real return:

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Random component of the model

• Properties of random error:

- $E(\varepsilon_i) = 0$; for $\forall_i = 1, 23, \dots$
- $Cov(\varepsilon_i, r_i) = 0$; for $\forall_i = 1, 23, ...$
- $Cov(\varepsilon_i, \varepsilon_j) = 0$; for $\forall_i = 1, 23, ... \land i \neq j$
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• Parameters could be estimated:

• The set of this points is described by empirical regresion function:

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$$\hat{y}_i = f(a, b, x) = a + b * x$$

• Residue...
$$\varepsilon_i = y_i - \hat{y}_i$$

- The methodology of OLS minimalize the errors
- Thus the objective function:

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$$S_r = \sum_{i=1}^N e_i^2 \Rightarrow min$$

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Solution

• Partial derivatives with respect to a, b

- ⇒system of equiations:
 - $na+b\sum X_i=\sum Y_i$
 - $a \sum X_i + b \sum X_i^2 = \sum X_i Y_i$
 - ...matrix calculation...

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• Variance of excess return:

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$$\operatorname{var}(r_i - r_f) = \operatorname{var}(r_i) + \operatorname{var}(r_f) = \sigma_i^2$$

• $\operatorname{var}[(r_M - r_f) + \varepsilon_i] = \beta_i^2 * \sigma_{i,i}^2 + \varepsilon_i^2$

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$$[(r_M - r_f) + \varepsilon_i] = \beta_i^2 * \sigma_M^2 + \varepsilon_i^2$$

Market and unique risk

ε_i²...concerns only an individual company or industry, could be diversified!

- $\beta_i^2 * \sigma_M^2$...undiversified part of risk, concerns all securities on the market
- Ratio of the systematic risk is given by $R^2...$ is explaining how fits the model

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- Investors are looking for investment opportunities ...securities in unequilibrium
- A security is undervalueted if the return is higher then the equilibrium return
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Investment decision

• If $\delta_i > 0$ \Rightarrow the security is over the SML \dots undervalueted

- If $\delta_i <$ 0 \Rightarrow the security is under the SML \ldots overvalueted
- If $\delta_i = 0 \Rightarrow$ the security is on the SML ...in equilibrium

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