# Binary dependent variables

**LECTURE 8**

**28.04.2023**

# Lecture Outline

- The linear probability model
- Nonlinear probability models
  - Probit
  - Logit
- Brief introduction of maximum likelihood estimation
- Interpretation of coefficients in logit and probit models

# Introduction

- So far the dependent variable ($Y$) has been continuous:
  - Average hourly earnings
  - Birth weight of babies
- What if $Y$ is binary?
  - $Y$ = get into college, or not; $X$ = parental income.
  - $Y$ = person smokes, or not; $X$ = cigarette tax rate, income.
  - $Y$ = mortgage application is accepted, or not; $X$ = race, income, house characteristics, marital status ...

# The linear probability model

- Multiple regression model with continuous dependent variable

$$Y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki} + u_i$$

- The coefficient $\beta_j$ can be interpreted as the change in $Y$ associated with a unit change in $X_j$

- We will now discuss the case with a binary dependent variable

- We know that the expected value of a binary variable $Y$ is

$$E[Y] = 1 \cdot Pr(Y = 1) + 0 \cdot Pr(Y = 0) = Pr(Y = 1)$$

- In the multiple regression model with a binary dependent variable we have

$$E[Y_i|X_{1i}, \cdots, X_{ki}] = Pr(Y_i = 1|X_{1i}, \cdots, X_{ki})$$

- It is therefore called the **linear probability model.**

# Mortgage applications

Example:

- Most individuals who want to buy a house apply for a mortgage at a bank.
- Not all mortgage applications are approved.
- What determines whether or not a mortgage application is approved or denied?
- During this lecture we use a subset of the Boston HMDA data ($N = 2380$)
  - a data set on mortgage applications collected by the Federal Reserve Bank in Boston

| **Variable** | **Description** | **Mean** | **SD** |
|---|---|---|---|
| deny | $= 1$if mortgage application is denied | 0.120 | 0.325 |
| pi_ratio | anticipated monthly loan payments / monthly income | 0.331 | 0.107 |
| black | $= 1$if applicant is black, $= 0$ if applicant is white | 0.142 | 0.350 |

# Mortgage applications

- Does the payment to income ratio affect whether or not a mortgage application is denied?

```
. regress deny pi_ratio, robust

Linear regression                               Number of obs =      2380
                                                   F( 1, 2378) =     37.56
                                                   Prob > F      =    0.0000
                                                   R-squared     =    0.0397
                                                   Root MSE      =    .31828
```

| deny | Coef. | Robust Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| pi_ratio | .6035349 | .0984826 | 6.13 | 0.000 | .4104144 | .7966555 |
| _cons | -.0799096 | .0319666 | -2.50 | 0.012 | -.1425949 | -.0172243 |

- The estimated OLS coefficient on the payment to income ratio equals $\widehat{\beta_1} = 0.6$
- The estimated coefficient is significantly different from 0 at a 1% significance level.
- How should we interpret $\widehat{\beta_1}$?

# The linear probability model

- The conditional expectation equals the probability that $Y_i = 1$ conditional on $X_{1i}, \cdots, X_{ki}$:

$$E[Y_i|X_{1i}, \cdots, X_{ki}] = Pr(Y_i = 1|X_{1i}, \cdots, X_{ki}) = \beta_0 + \beta_1 X_{1i} + \cdots \beta_k X_{ki}$$

- The population coefficient $\beta_j$ equals the change in the probability that $Y_i = 1$ associated with a unit change in $X_j$.

$$\frac{\partial Pr(Y_i = 1|X_{1i}, \cdots, X_{ki})}{\partial X_j} = \beta_j$$

In the mortgage application example:

- $\widehat{\beta_1} = 0.6$

- A change in the payment to income ratio by 1 is estimated to increase the probability that the mortgage application is denied by 0.60.

- A change in the payment to income ratio by 0.10 is estimated to increase the probability that the application is denied by 6% (0.10*0.60*100).

# The linear probability model

**Assumptions are the same as for general multiple regression model:**

- $E(u_i | X_{1i}, X_{2i}, \ldots, X_{ki}) = 0$
- Big outliers are unlikely
- No perfect multicollinearity.

**Advantages of the linear probability model:**

- Easy to estimate
- Coefficient estimates are easy to interpret

**Disadvantages of the linear probability model**

- Predicted probability can be above 1 or below 0!
- Error terms are heteroskedastic

# The linear probability model: heteroskedasticity

$$Y_i = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki} + u_i$$

- The variance of a Bernoulli random variable:

$$Var(Y) = Pr(Y = 1) \quad (1 - Pr(Y = 1))$$

- We can use this to find the conditional variance of the error term

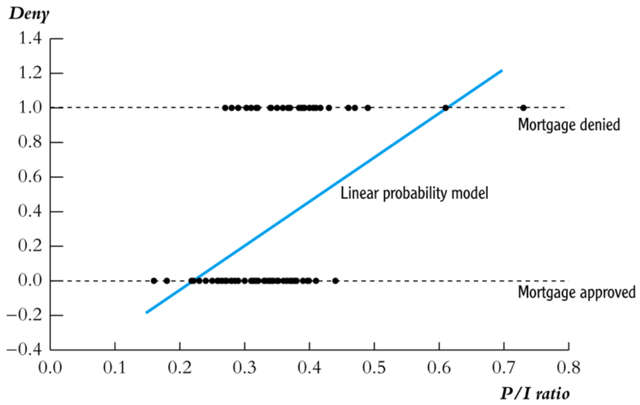
$$\begin{aligned}
Var(u_i|X_{1i},\dots,X_{ki}) &= Var(Y_i - (\beta_0 + \beta_1 X_{1i} + \dots + \beta_k X_{ki})|\, X_{1i},\dots, X_{ki}) \\
&= Var(Y_i|X_{1i},\dots, X_{ki}) \\
&= Pr(Y_i = 1|\, X_{1i}, \cdots, X_{ki}) \times (1 - Pr(Y_i = 1|\, X_{1i}, \cdots, X_{ki})) \\
&= (\beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}) \times (1 - \beta_0 - \beta_1 X_{1i} - \cdots - \beta_k X_{ki}) \\
&\neq \sigma_u^2
\end{aligned}$$

- Solution: Always use heteroskedasticity robust standard errors when estimating a linear probability model!

# The linear probability model: shortcomings

In the linear probability model the predicted probability can be below 0 or above 1!

***Example***: linear probability model, HMDA data
**Mortgage denial v. ratio of debt payments to income (P/I ratio) in a subset of the HMDA data set (*n* = 127)**



# Nonlinear probability models

- Probabilities cannot be less than 0 or greater than 1
- To address this problem we will consider nonlinear probability models

$$Pr(Y_i = 1) = G(Z)$$

$$\textit{with } Z = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}$$

$$\textit{and} \quad 0 \leq G(Z) \leq 1$$

- We will consider 2 nonlinear functions

1. Probit

$$G(Z) = \Phi(Z)$$

2. Logit

$$G(Z) = \frac{1}{1 + e^{-Z}}$$

# Probit

Probit regression models the probability that $Y = 1$

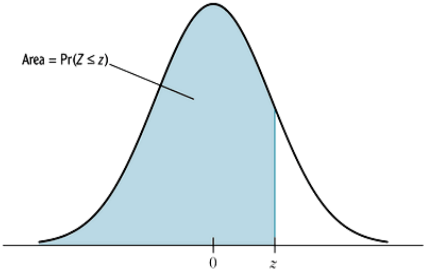
- Using the cumulative standard normal distribution function $\Phi(Z)$
- evaluated at $Z = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}$
- since $\Phi(z) = Pr(Z \leq z)$ we have that the predicted probabilities of the probit model are between 0 and 1

Example

- Suppose we have only 1 regressor and $Z = -2 + 3X_1$
- We want to know the probability that $Y = 1$ when $X_1 = 0.4$
- $z = -2 + 3 \cdot 0.4 = -0.8$
- $Pr(Y = 1) = Pr(Z \leq -0.8) = \Phi(-0.8)$

# Probit

**TABLE 1** **The Cumulative Standard Normal Distribution Function, $\Phi(z)$ = Pr(Z ″ z)**




| z | Second Decimal Value of z | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| −0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| −0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |

$$Pr(Y = 1) = Pr(Z \leq -0.8) = \Phi(-0.8) = 0.2119$$

# Logit

Logit regression models the probability that $Y = 1$

- Using the cumulative standard logistic distribution function

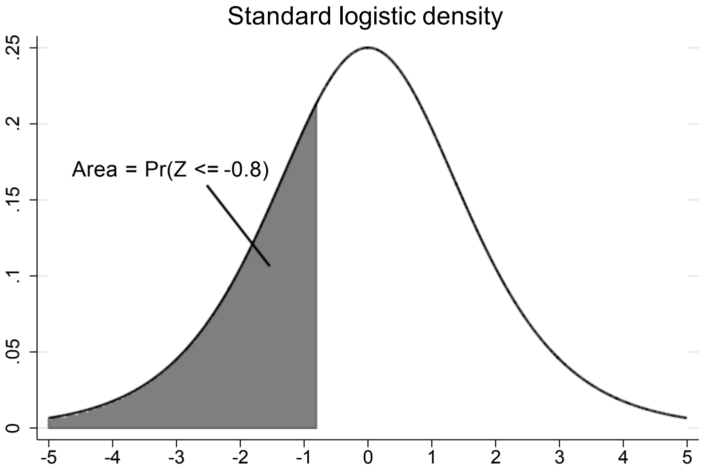
$$F(Z) = \frac{1}{1 + e^{-Z}}$$

- evaluated at $Z = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}$
- since $F(z) = Pr(Z \leq z)$ we have that the predicted probabilities of the probit model are between 0 and 1

Example

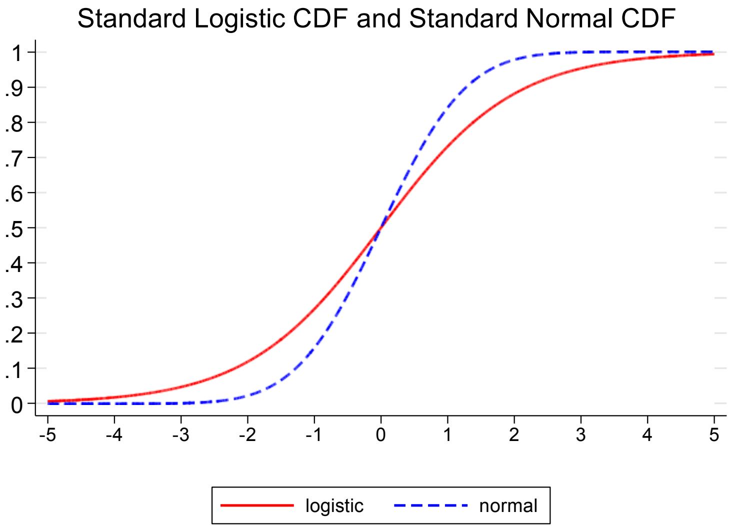
- Suppose we have only 1 regressor and $Z = -2 + 3X_1$
- We want to know the probability that $Y = 1$ when $X_1 = 0.4$
- $z = -2 + 3 \cdot 0.4 = -0.8$
- $Pr(Y = 1) = Pr(Z \leq -0.8) = F(-0.8)$

# Logit




- $Pr(Y = 1) = Pr(Z \leq -0.8) = \frac{1}{1+e^{0.8}} = 0.31$

# Logit & probit



# How to estimate logit and probit models

- In previous lectures we discussed regression models that are nonlinear in the independent variables
  - these models can be estimated by OLS
- Logit and Probit models are nonlinear in the coefficients $\beta_0, \beta_1, \cdots, \beta_k$
  - these models can't be estimated by OLS
- The method used to estimate logit and probit models is Maximum Likelihood Estimation (MLE).
- The MLE are the values of $(\beta_0, \beta_1, \cdots, \beta_k)$ that best describe the full distribution of the data.

# Maximum likelihood estimation

- The **likelihood function** is the joint probability distribution of the data, treated as a function of the unknown coefficients.
- The **maximum likelihood estimator (MLE)** are the values of the coefficients that maximize the likelihood function.
- MLE's are the parameter values "most likely" to have produced the data.

Lets start with a special case: The MLE with no $X$

- We have $n$ i.i.d. observations $Y_1, \ldots, Y_n$ on a binary dependent variable
- $Y$ is a Bernoulli random variable
- There is only 1 unknown parameter to estimate:
  - The probability $\boldsymbol{p}$ that $Y = 1$,
  - which is also the mean of $Y$

# Maximum likelihood estimation (Optional)

Step 1: write down the likelihood function, the joint probability distribution of the data

- $Y_i$ is a Bernoulli random variable we therefore have

$$Pr(Y_i = y) = Pr(Y_i = 1)^y \cdot (1 - Pr(Y_i = 1))^{1-y} = p^y(1-p)^{1-y}$$

  - $Pr(Y_i = 1) = p^1(1 - p)^0 = p$
  - $Pr(Y_i = 0) = p^0(1 - p)^1 = 1 - p$

- $Y_1, \ldots, Y_n$ are i.i.d, the joint probability distribution is therefore the product of the individual distributions

$$\begin{aligned} Pr(Y_1 = y_1, \ldots . Y_n = y_n) &= Pr(Y_1 = y_1) \times \ldots \times Pr(Y_n = y_n) \\ &= \left[p^{y_1}(1-p)^{1-y_1}\right] \times \ldots \times \left[p^{y_n}(1-p)^{1-y_n}\right] \\ &= p^{(y_1+y_2+\ldots+y_n)}(1-p)^{n-(y_1+y_2+\ldots+y_n)} \end{aligned}$$

# Maximum likelihood estimation (Optional)

We have the likelihood function:

$$f_{Bernouilli}(p;\ Y_1 = y_1, \ldots .Y_n = y_n) = p^{\sum y_i}(1-p)^{n-\sum y_i}$$

Step 2: Maximize the likelihood function w.r.t *p*

- Easier to maximize the logarithm of the likelihood function

$$ln\left(f_{Bernouilli}(p;\ Y_1 = y_1, \ldots .Y_n = y_n)\right) = \left(\sum_{i=1}^{n} y_i\right) \cdot ln\,(p) + \left(n - \sum_{i=1}^{n} y_i\right) ln\,(1-p)$$

- Since the logarithm is a strictly increasing function, maximizing the likelihood or the log likelihood will give the same estimator.

# Maximum likelihood estimation (Optional)

- Taking the derivative w.r.t $p$ gives

$$\frac{d}{dp} ln\left(f_{Bernouilli}(p;\ Y_1 = y_1, \ldots . Y_n = y_n)\right) = \frac{\sum_{i=1}^{n} y_i}{p} - \frac{n - \sum_{i=1}^{n} y_i}{1-p}$$

- Setting to zero and rearranging gives

$$\begin{aligned}
(1-p) \times \sum_{i=1}^{n} y_i &= p \times (n - \sum_{i=1}^{n} y_i) \\
\sum_{i=1}^{n} y_i - p \sum_{i=1}^{n} y_i &= n \cdot p - p \sum_{i=1}^{n} y_i \\
\sum_{i=1}^{n} y_i &= n \cdot p
\end{aligned}$$

- Solving for $p$ gives the MLE

$$\widehat{p}_{MLE} = \frac{1}{n} \sum_{i=1}^{n} y_i = \overline{Y}$$

# MLE of the probit model (Optional)

Step 1: write down the likelihood function

$$\begin{aligned} Pr(Y_1 = y_1, \ldots . Y_n = y_n) &= Pr(Y_1 = y_1) \times \ldots \times Pr(Y_n = y_n) \\ &= \left[p_1^{y_1}(1-p_1)^{1-y_1}\right] \times \ldots \times \left[p_n^{y_n}(1-p_n)^{1-y_n}\right] \end{aligned}$$

- so far it is very similar as the case without explanatory variables except that $p_i$ depends on $X_{1i}, \ldots, X_{ki}$

$$p_i = \Phi\left(X_{1i}, \ldots, X_{ki}\right) = \Phi\left(\beta_0 + \beta_1 X_{1i} + \cdots + \beta_k X_{ki}\right)$$

- substituting for $p_i$ gives the likelihood function:

$$\left[\Phi\left(\beta_0 + \beta_1 X_{11} + \cdots + \beta_k X_{k1}\right)^{y_1}\left(1 - \Phi\left(\beta_0 + \beta_1 X_{11} + \cdots + \beta_k X_{k1}\right)\right)^{1-y_1}\right] \times \ldots$$

$$\times \left[\Phi\left(\beta_0 + \beta_1 X_{1n} + \cdots + \beta_k X_{kn}\right)^{y_n}\left(1 - \Phi\left(\beta_0 + \beta_1 X_{1n} + \cdots + \beta_k X_{kn}\right)\right)^{1-y_n}\right]$$

# MLE of the probit model (Optional)

Also with obtaining the MLE of the probit model it is easier to take the logarithm of the likelihood function

Step 2: Maximize the log likelihood function

$$
\begin{aligned}
& ln\left[f_{probit}\left(\beta_0, \ldots, \beta_k;\ Y_1, \ldots, Y_n \mid X_{1i}, \ldots, X_{ki}, i=1, \ldots, n\right)\right] \\
= \quad & \textstyle\sum_{i=1}^{n} Y_i ln\left[\Phi\left(\beta_0+\beta_1 X_{1i}+\cdots+\beta_k X_{ki}\right)\right] \\
& +\textstyle\sum_{i=1}^{n}(1-Y_i) ln\left[1-\Phi\left(\beta_0+\beta_1 X_{1i}+\cdots+\beta_k X_{ki}\right)\right]
\end{aligned}
$$

w.r.t $\beta_0, \ldots, \beta_1$

- There is no simple formula for the probit MLE, the maximization must be done using numerical algorithm on a computer.

# MLE of the logit model (Optional)

Step 1: write down the likelihood function

$$Pr(Y_1 = y_1, \ldots . Y_n = y_n) \quad = \quad \left[p_1^{y_1}(1-p_1)^{1-y_1}\right] \times \ldots \times \left[p_n^{y_n}(1-p_n)^{1-y_n}\right]$$

- very similar to the Probit model but with a different function for $p_i$

$$p_i = 1 / \left[1 + e^{-(\beta_0 + \beta_1 X_{1i} + \ldots + \beta_k X_{ki})}\right]$$

Step 2: Maximize the log likelihood function w.r.t $\beta_0, \ldots, \beta_1$

$$ln\left[f_{logit}\left(\beta_0, \ldots, \beta_k;\ Y_1, \ldots, Y_n | X_{1i}, \ldots, X_{ki}, i = 1, \ldots, n\right)\right]$$

$$= \quad \sum_{i=1}^{n} Y_i ln\left(1 / \left[1 + e^{-(\beta_0 + \beta_1 X_{1i} + \ldots + \beta_k X_{ki})}\right]\right)$$

$$+ \sum_{i=1}^{n}(1 - Y_i) ln\left(1 - \left(1 / \left[1 + e^{-(\beta_0 + \beta_1 X_{1i} + \ldots + \beta_k X_{ki})}\right]\right)\right)$$

- There is no simple formula for the logit MLE, the maximization must be done using numerical algorithm on a computer.

# Probit: mortgage applications

```
. probit deny pi_ratio

Iteration 0:  log likelihood =  -872.0853
Iteration 1:  log likelihood = -832.02975
Iteration 2:  log likelihood = -831.79239
Iteration 3: log likelihood =  -831.79234

Probit regression                          Number of obs   =      2380
                                           LR chi2(   1)   =     80.59
                                           Prob > chi2     =    0.0000
Log likelihood = -831.79234                 Pseudo R2      =    0.0462
```

| deny | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| pi_ratio | 2.967907 | .3591054 | 8.26 | 0.000 | 2.264073 | 3.67174 |
| _cons | -2.194159 | .12899 | -17.01 | 0.000 | -2.446974 | -1.941343 |

- The estimated MLE coefficient on the payment to income ratio equals $\widehat{\beta_1} = 2.97$
- The estimated coefficient is positive and significantly different from 0 at a 1% significance level.
- How should we interpret $\widehat{\beta_1}$?

# Probit: mortgage applications

*The estimate of $\beta_1$ in the probit model CANNOT be interpreted as the change in the probability that $Y_i = 1$ associated with a unit change in $X_1$!!*

- In general the effect on $Y$ of a change in $X$ is the expected change in $Y$ resulting from the change in $X$
- Since $Y$ is binary the expected change in $Y$ is the change in the probability that $Y = 1$

In the probit model the predicted change the probability that the mortgage application is denied when the payment to income ratio increases from

**0.10 to 0.20:**

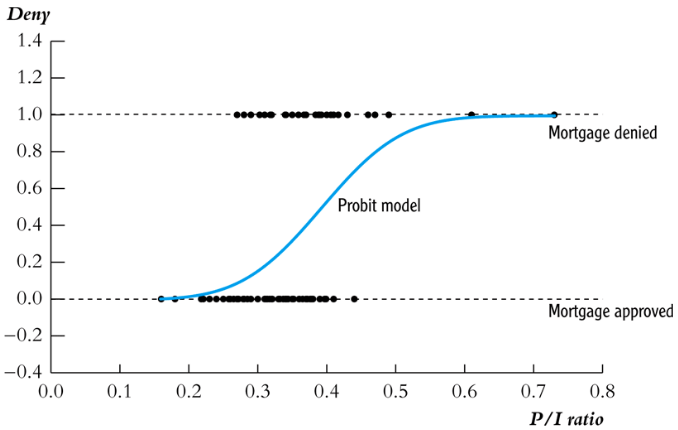
$$\triangle \widehat{Pr(Y_i = 1)} = \Phi\left(-2.19 + 2.97 \cdot 0.20\right) - \Phi\left(-2.19 + 2.97 \cdot 0.10\right) = 0.0495$$

**0.30 to 0.40:**

$$\triangle \widehat{Pr(Y_i = 1)} = \Phi\left(-2.19 + 2.97 \cdot 0.40\right) - \Phi\left(-2.19 + 2.97 \cdot 0.30\right) = 0.0619$$

# Probit: mortgage applications

Predicted values in the probit model:




- All predicted probabilities are between 0 and 1!

# Logit: mortgage applications

```
. logit deny pi_ratio

Iteration  0:  log  likelihood  =   -872.0853
Iteration  1:  log  likelihood  =  -830.96071
Iteration  2:  log  likelihood  =  -830.09497
Iteration  3:  log  likelihood  =  -830.09403
Iteration 4: log likelihood =      -830.09403

Logistic regression                          Number of obs   =         2380
                                             LR chi2(    1)      =    83.98
                                             Prob > chi2     =       0.0000
Log likelihood = -830.09403                    Pseudo R2        =    0.0482
```

| deny | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| pi_ratio | 5.884498 | .7336006 | 8.02 | 0.000 | 4.446667 | 7.322328 |
| _cons | -4.028432 | .2685763 | -15.00 | 0.000 | -4.554832 | -3.502032 |

- The estimated MLE coefficient on the payment to income ratio equals $\widehat{\beta_1} = 5.88$
- **The estimated coefficient is positive and significantly different from 0 at a 1% significance level.**
- **How should we interpret $\widehat{\beta_1}$?**

# Logit: mortgage applications

*Also in the Logit model:*

*The estimate of $\beta_1$ CANNOT be interpreted as the change in the probability that $Y_i = 1$ associated with a unit change in $X_1$!!*

In the logit model the predicted change the probability that the mortgage application is denied when the payment to income ratio increases from

**0.10 to 0.20:**

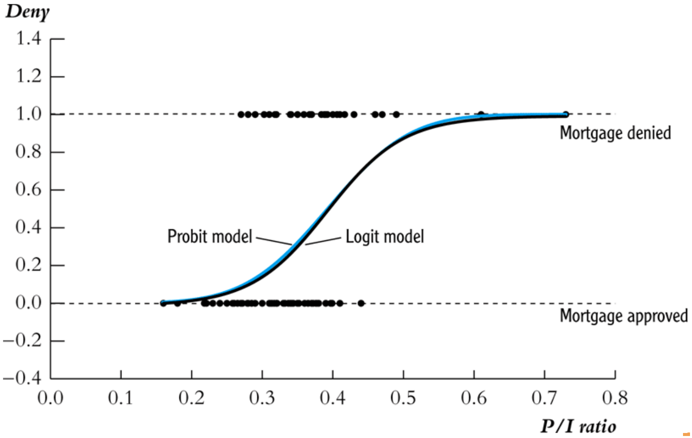
$$\triangle \widehat{Pr(Y_i = 1)} = \left(1/1 + e^{-(-4.03+5.88\cdot 0.20)}\right) - \left(1/1 + e^{-(-4.03+5.88\cdot 0.10)}\right) = 0.023$$

**0.30 to 0.40:**

$$\triangle \widehat{Pr(Y_i = 1)} = \left(1/1 + e^{-(-4.03+5.88\cdot 0.40)}\right) - \left(1/1 + e^{-(-4.03+5.88\cdot 0.30)}\right) = 0.063$$

# Logit: mortgage applications

**The predicted probabilities from the probit and logit models are very close in these HMDA regressions:**



# Probit & Logit with multiple regressors

- We can easily extend the Logit and Probit regression models, by including additional regressors
- Suppose we want to know whether white and black applications are treated differentially
- Is there a significant difference in the probability of denial between black and white applicants conditional on the payment to income ratio?
- To answer this question we need to include two regressors
  - P/I ratio
  - Black

# Probit with multiple regressors

```
Probit regression                               Number of obs =          2380
                                                LR chi2(    2)     =   149.90
                                                Prob > chi2      =     0.0000
Log likelihood =  -797.13604                     Pseudo R2        =     0.0859
```

| deny | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| black | .7081579 | .0834327 | 8.49 | 0.000 | .5446328 | .8716831 |
| pi_ratio | 2.741637 | .3595888 | 7.62 | 0.000 | 2.036856 | 3.446418 |
| _cons | -2.258738 | .129882 | -17.39 | 0.000 | -2.513302 | -2.004174 |

- To say something about the size of the impact of race we need to specify a value for the payment to income ratio
- Predicted denial probability for a white application with a P/I-ratio of 0.3 is

$$\Phi(-2.26 + 0.71 \cdot 0 + 2.74 \cdot 0.3) = 0.0749$$

- Predicted denial probability for a black application with a P/I-ratio of 0.3 is

$$\Phi(-2.26 + 0.71 \cdot 1 + 2.74 \cdot 0.3) = 0.2327$$

- Difference is 15.8%

# Logit with multiple regressors

```
Logistic regression                               Number of obs =        2380
                                                  LR chi2(    2)    =    152.78
                                                  Prob > chi2     =    0.0000
Log likelihood =  -795.69521                        Pseudo R2      =    0.0876
```

| deny | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| black | 1.272782 | .1461983 | 8.71 | 0.000 | .9862385 | 1.559325 |
| pi_ratio | 5.370362 | .7283192 | 7.37 | 0.000 | 3.942883 | 6.797841 |
| _cons | -4.125558 | .2684161 | -15.37 | 0.000 | -4.651644 | -3.599472 |

- To say something about the size of the impact of race we need to specify a value for the payment to income ratio
- Predicted denial probability for a white application with a P/I-ratio of 0.3 is
  $$1/1 + e^{-(-4.13+5.37\cdot 0.30)} = 0.075$$
- Predicted denial probability for a black application with a P/I-ratio of 0.3 is
  $$1/1 + e^{-(-4.13+5.37\cdot 0.30+1.27)} = 0.224$$
- Difference is 14.8%

# LPM, Probit & Logit

Table 1: Mortgage denial regression using the Boston HMDA Data

Dependent variable: deny = 1 if mortgage application is denied, = 0 if accepted

| regression model | LPM | Probit | Logit |
|---|---|---|---|
| black | 0.177*** | 0.71*** | 1.27*** |
| | (0.025) | (0.083) | (0.15) |
| P/I ratio | 0.559*** | 2.74*** | 5.37*** |
| | (0.089) | (0.44) | (0.96) |
| constant | -0.091*** | -2.26*** | -4.13*** |
| | (0.029) | (0.16) | (0.35) |
| difference Pr(deny=1) between black and white applicant when P/I ratio=0.3 | 17.7% | 15.8% | 14.8% |

# Threats to internal and external validity

Both for the Linear Probability as for the Probit & Logit models we have to consider threats to

1. Internal validity
   - Is there omitted variable bias?
   - Is the functional form correct?
     - Probit model: is assumption of a Normal distribution correct?
     - Logit model: is assumption of a Logistic distribution correct?
   - Is there measurement error?
   - Is there sample selection bias?
   - is there a problem of simultaneous causality?
2. External validity
   - These data are from Boston in 1990-91.
   - Do you think the results also apply today, where you live?

# Distance to college & probability of obtaining a college degree

```
Linear regression                               Number of obs =      3796
                                                   F( 1, 3794) =     15.77
                                                 Prob > F      =    0.0001
                                                 R-squared     =    0.0036
                                                 Root MSE      =    .44302
```

| college | Coef. | Robust Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| dist | -.012471 | .0031403 | -3.97 | 0.000 | -.0186278 | -.0063142 |
| _cons | .2910057 | .0093045 | 31.28 | 0.000 | .2727633 | .3092481 |

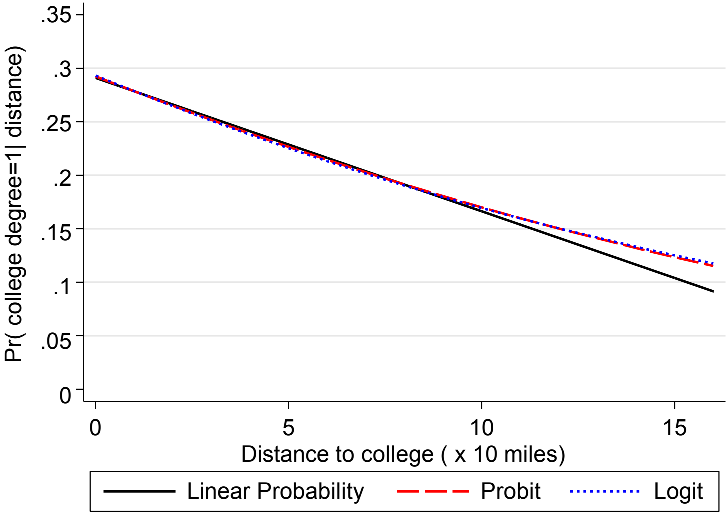
```
Probit regression                               Number of obs =      3796
                                                LR chi2(    1)  =     14.48
                                                Prob > chi2     =    0.0001
Log likelihood =  -2204.8977                     Pseudo R2      =    0.0033
```

| college | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| dist | -.0407873 | .0109263 | -3.73 | 0.000 | -.0622025 | -.0193721 |
| _cons | -.5464198 | .028192 | -19.38 | 0.000 | -.6016752 | -.4911645 |

```
Logistic regression                             Number of obs =      3796
                                                LR chi2(    1)  =     14.68
                                                Prob > chi2     =    0.0001
Log likelihood =  -2204.8006                     Pseudo R2      =    0.0033
```

| college | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| dist | -.0709896 | .0193593 | -3.67 | 0.000 | -.1089332 | -.033046 |
| _cons | -.8801555 | .0476434 | -18.47 | 0.000 | -.9735349 | -.786776 |

# Distance to college & probability of obtaining a college degree




- The 3 different models produce very similar results.

# Summary

- If $Y_i$ is binary, then $E(Y_i|X_i) = Pr(Y_i = 1|X_i)$
- Three models:

1. linear probability model (linear multiple regression)
2. probit (cumulative standard normal distribution)
3. logit (cumulative standard logistic distribution)

- LPM, probit, logit all produce predicted probabilities
- Effect of $\Delta X$ is a change in conditional probability that $Y = 1$
- For logit and probit, this depends on the initial $X$
- Probit and logit are estimated via maximum likelihood
  - Coefficients are normally distributed for large n
  - Large-$n$ hypothesis testing, conf. intervals is as usual