

MUNI
ECON

Econometrics

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Lecture 2:

Trends and Seasonality

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Lecture 2

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Characterizing trending time series

- Many time series grow over time, they follow a trend. This is a key feature to consider when making inference. Why?
- Let us assume that we have two series characterized by a positive trend over time. If we ignore this, we might wrongly conclude that one series explains the other.
- Many time series appear to be correlated just because they are trending over time, but this is related to other unobserved factors, not to a relationship between them.
- So, we say there are a lot of time series that follow a trend. Can you think of any?

Characterizing trending time series

Do we have a trend here?

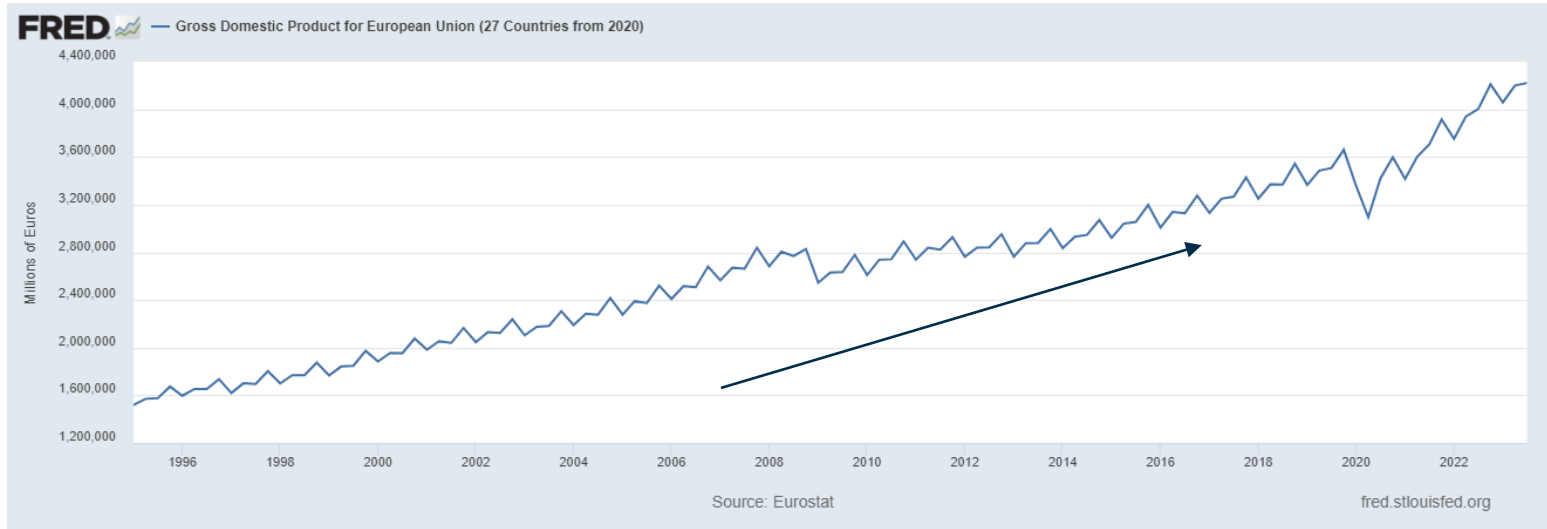


Figure 1: GDP for the European Union. **Not seasonally adjusted.**

Characterizing trending time series

And here?

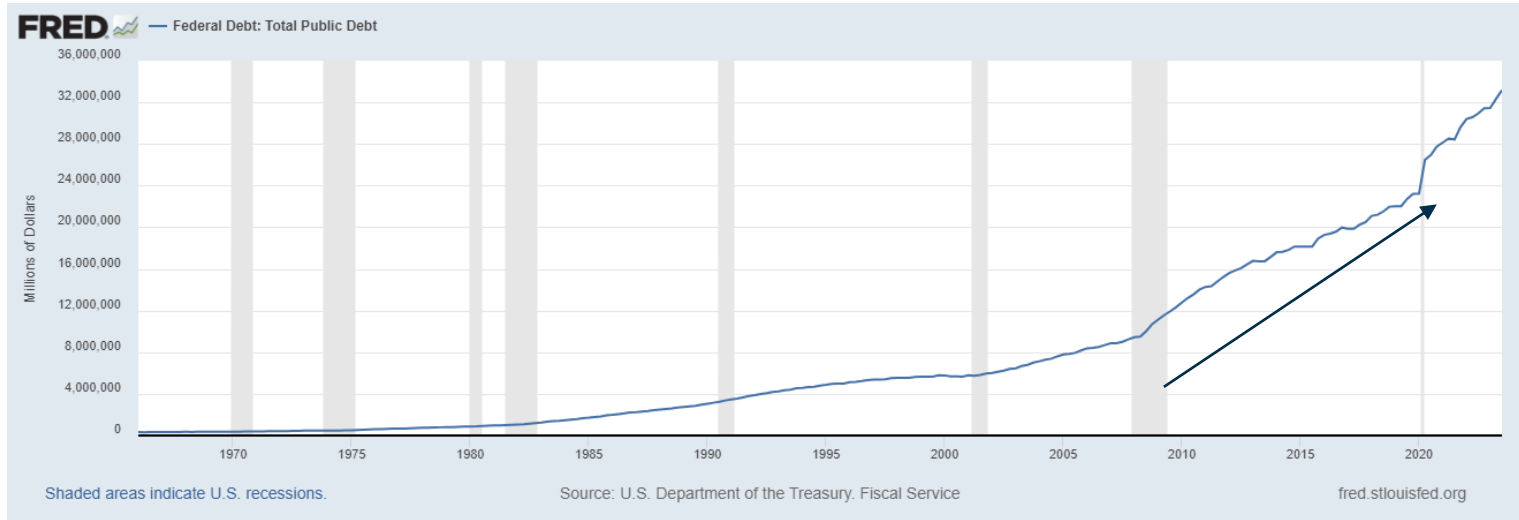


Figure 2: United States Federal Debt. **Not seasonally adjusted.**

Characterizing trending time series

What about here? Becomes difficult here...

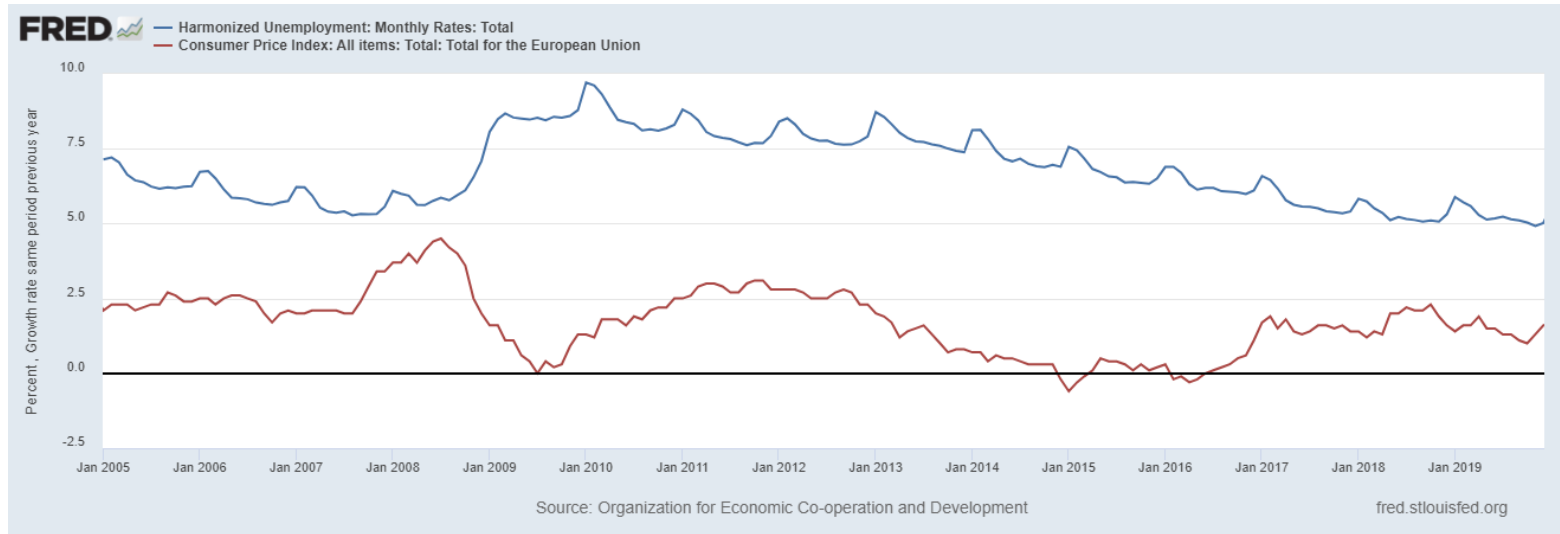


Figure 3: Unemployment and inflation in the European Union. Not seasonally adjusted.

Characterizing trending time series

Definitely not here, right?

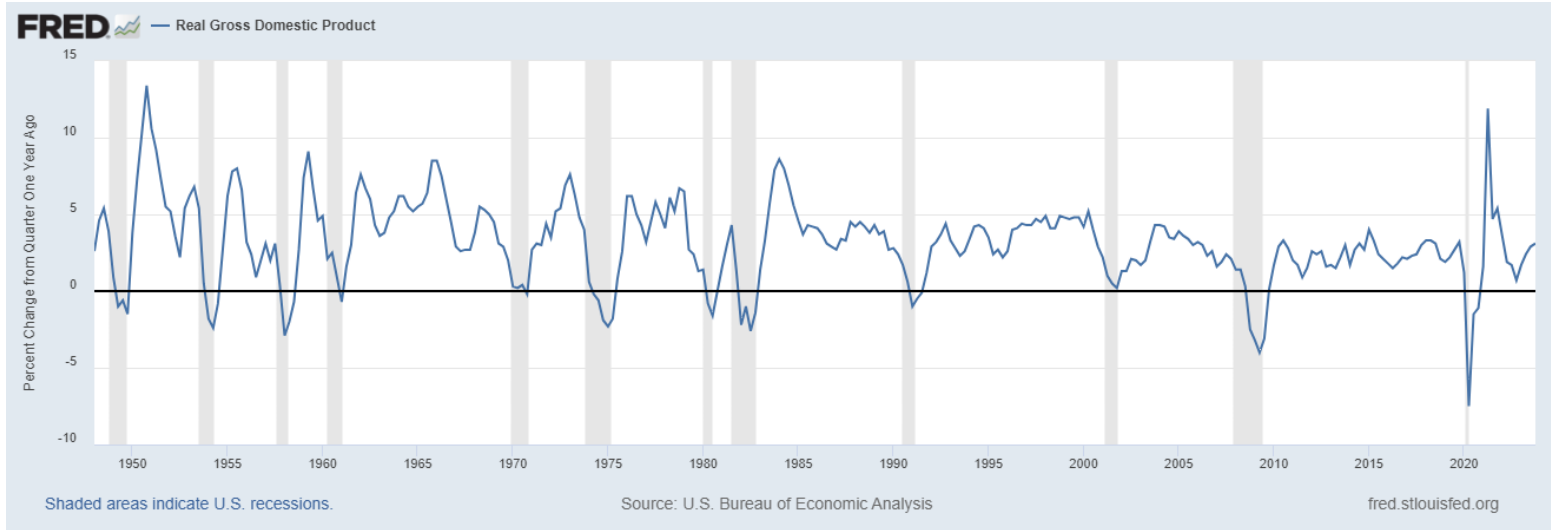


Figure 4: Real GDP growth for the United States. **Seasonally adjusted.**

Characterizing trending time series

The simplest way to capture a time trend could be:

$$y_t = \alpha_0 + \alpha_1 t + e_t, \quad t = 1, 2, \dots \quad (1)$$

where $\{e_t\}$ is an independent, identically distributed (i.i.d) sequence with $E(e_t) = 0$ and $\text{Var}(e_t) = \sigma_e^2$.

The parameter α_1 multiplies time, t , and this results in adding a linear time trend to the model. If we hold all $\{e_t\}$ fixed, α_1 measures the change in y_t due to the passage of time. Indeed:

$$\Delta y_t = y_t - y_{t-1} = \alpha_1 \quad (2)$$

Characterizing trending time series

We could also think of the trend as:

$$E[y_t] = \alpha_0 + \alpha_1 t \quad (3)$$

Note that if $\alpha_1 > 0$, then on average y_t is growing over time and, therefore, has an upward trend. if $\alpha_1 < 0$, the trend is downward.

Of course, if we check some of the series we were just checking, and we draw a trend, we will note that they do not fall exactly over the time-series, however, it does on average.

Do all time series have a linear time trend? No... Can you think of a series with exponential trends?

Characterizing trending time series

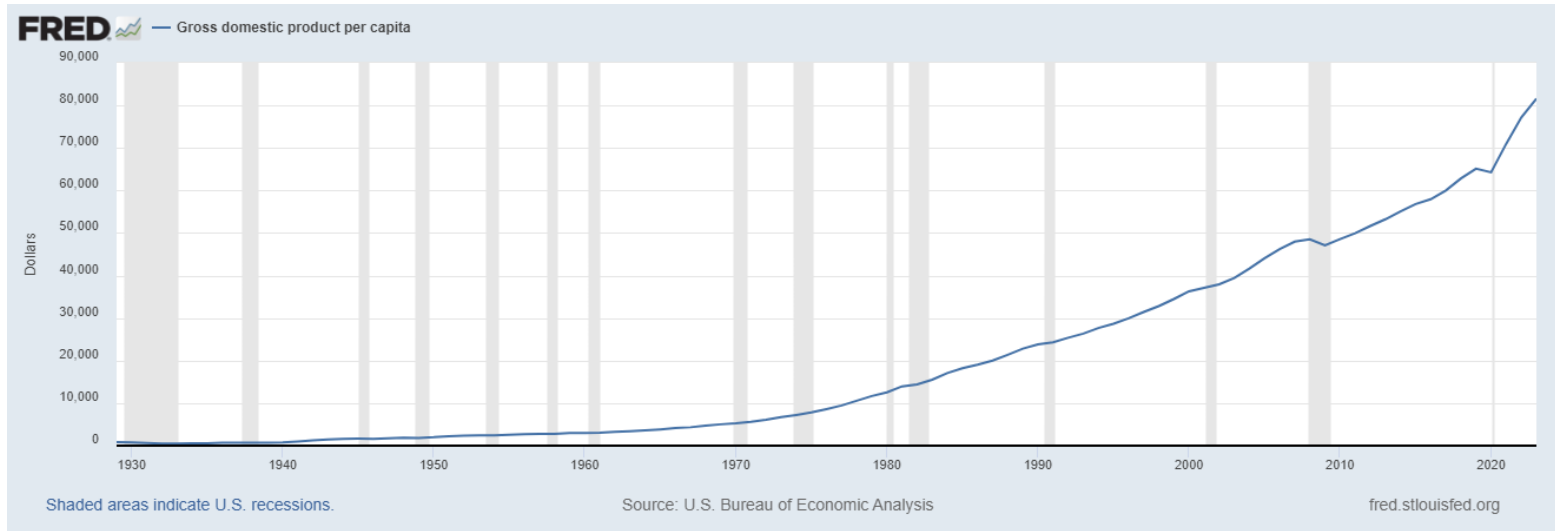


Figure 5: US GDP per capita. **Not seasonally adjusted.**

Series that have the same average growth rate from period to period follow an exponential trends. (Do you know the rule of 70?).

Characterizing trending time series

How do we model an exponential trend?

Well, we could model the natural logarithm of the dependent variable as a linear trend:

$$\log(y_t) = \beta_0 + \beta_1 t + e_t, \quad t = 1, 2, \dots \quad (4)$$

Remembering that $\Delta \log(y_t) \approx \frac{y_t - y_{t-1}}{y_t}$, we can take changes in (4) and assume that $\Delta e_t = 0$, in order to get:

$$\Delta \log(y_t) = \beta_1, \text{ for all } t \quad (5)$$

This implies that β accounts for a rate of change from one period to the next. For example, if y_t is real GDP and t accounts for years, then β is the yearly rate of growth of real GDP.

Characterizing trending time series

Let us think of more complicated trends... What about a quadratic time trend like this...

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + e_t, \quad t = 1, 2, \dots \quad (6)$$

Assuming that e_t remains fixed, can you calculate the slope of Eq. (6)?

$$\frac{\Delta y_t}{\Delta t} = \beta_1 + 2\beta_2 t \quad (7)$$

Can you find the slope of the following time series process with a cubic trend?

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + e_t, \quad t = 1, 2, \dots \quad (8)$$

$$\frac{\Delta y_t}{\Delta t} = \beta_1 + 2\beta_2 t + 3\beta_3 t^2 \quad (9)$$

Using trending variables in regression analysis

Why is important to consider trends in regression analysis?

- Remember our 6 assumptions from last week? Well, adding a trend does not necessarily violate any of these.
- If we ignore trends and have that both explanatory and dependent variables follow one (increasing or decreasing) we might wrongly conclude that these two are related in a way they are not.
- This is what we call a **spurious regressions!**
- But, what is really the problem? What happens if we omit the trend? Can you guess?

Using trending variables in regression analysis

- If you think of a simple regression like the following:

$$y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \beta_3 t + u_t \quad (10)$$

- What happens if we estimate this one instead?

$$y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + u_t \quad (11)$$

- **Allowing for this trend recognizes explicitly that y_t may be growing (if $\beta_3 > 0$) or falling (if $\beta_3 < 0$) for reasons unrelated to X_{t1} and X_{t2} . Otherwise, we would be omitting a relevant variables that goes directly into our error term, and our estimated would be biased!**

Using trending variables in regression analysis

Let us check a simple way to add a trend in a simple linear regression!

Seasonality

What is seasonality?



Figure 6: US unemployment. Both **seasonally and not seasonally adjusted**.

Seasonality

What is seasonality?

- When we work with **data at quarterly, monthly, weekly, or daily data**, our time series may exhibit seasonality. Can you think of any examples?
- For example, retail sales are higher in December, due to Christmas; wages are usually set in the beginning of the year in many countries; the real estate market for student housing is high in September; the ski resorts have high sales in winter.
- Of course, not all the series at lower frequency than yearly display seasonality (interest rates, for instance, do not exhibit seasonality).
- However, we need to find a way to adjust our time series in order to account for the existence of seasonal patterns.

Seasonality

How do we deal with seasonality?

- A simple approach would be to work with dummy variables.
- This is quite straightforward. If we work with monthly data, for example, we can estimate a model of the following kind:

$$y_t = \beta_0 + \beta_1 X_{t1} + \dots + \beta_k X_{tk} + \delta_1 feb_t + \delta_2 mar_t + \dots + \delta_{11} dec_t + u_t \quad (12)$$

where $feb_t, mar_t, \dots, dec_t$ are dummy variables indicating whether time period t corresponds to the corresponding month.

- Note that we always need to leave one dummy variable out of our model. In Eq.(11), we have left January out (base month) and β_0 is the intercept for January.
- If there is no seasonality in y_t , our estimators from δ_1 to δ_{11} would be non-significant (test F).

Seasonality

The most straightforward method, however, is called **seasonal decomposition!**

- Seasonal decomposition decomposes seasonal time series data into its components.
- What components?
 1. Trend
 2. Seasonality
 3. Remainder
- How do we decompose the time series?

Seasonality

Additive method

- Add components up
- Simple method
- Gives a quick idea (first exploration)

Drawbacks from the additive model

- Good for constant seasonality but problematic with long series that suffer changes in the seasonal component.
- In this case it might be better to move towards alternative methods (x12, Seats, etc.).

Multiplicative method

- Multiplies components

Seasonality

Let us see how this works on R:

- The nottem dataset consists of 20 years of monthly temperature measurements at Nottingham Castle.
- **Temperatures, as you can observe, are highly seasonal.**
- We do not have trends and **temperature amplitude remains the same. Which model should we use?**
- We can certainly go with the **additive model!**

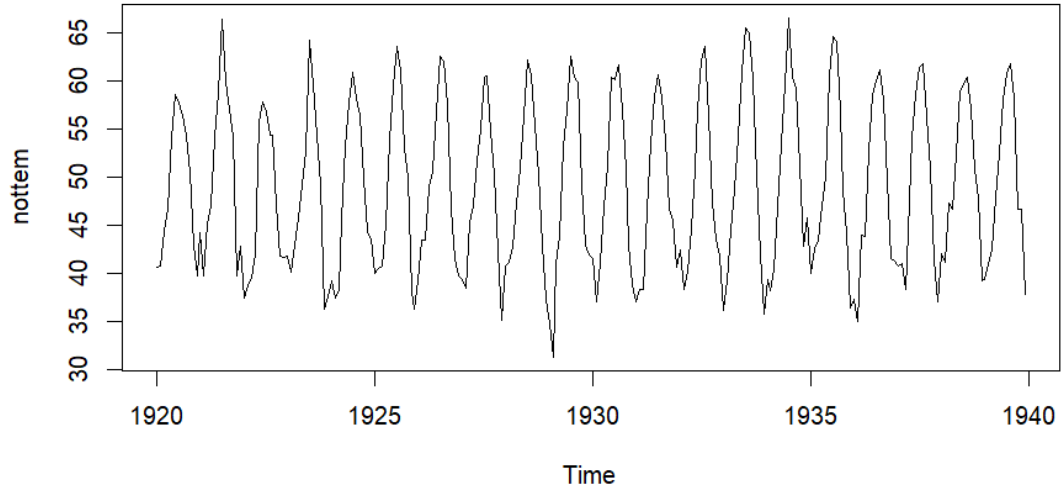


Figure 7: Nottem dataset

Seasonality

How does it look the seasonal decomposition with our additive model?

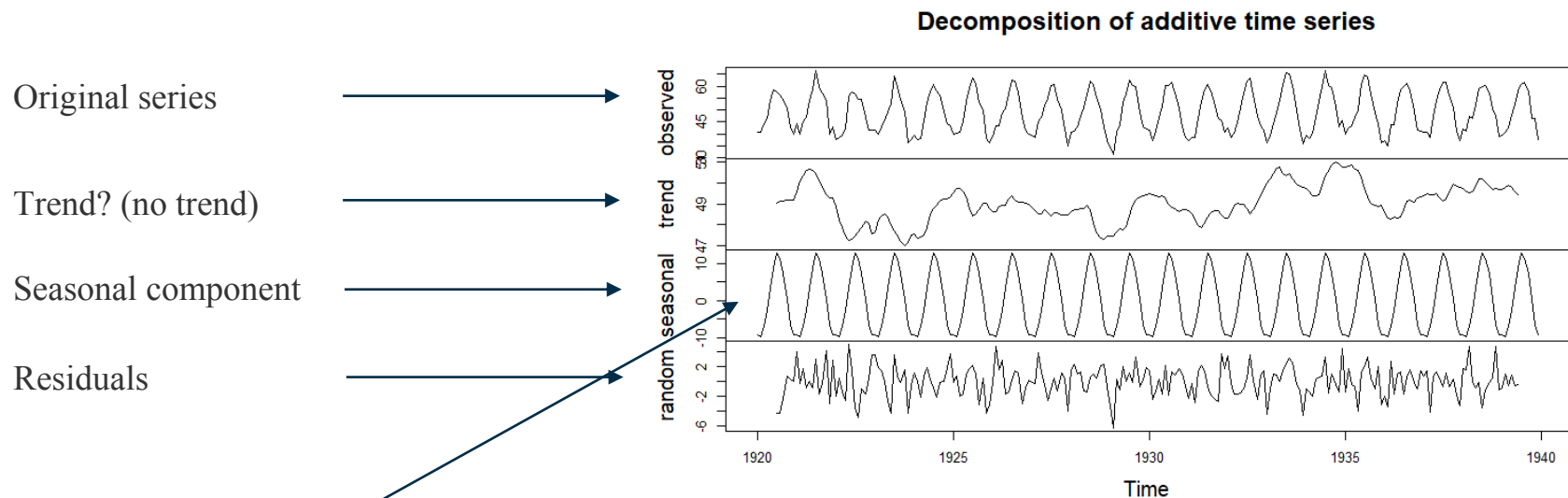


Figure 8: Decomposition of additive time series

Seasonality

So how do we get our seasonal adjusted data?

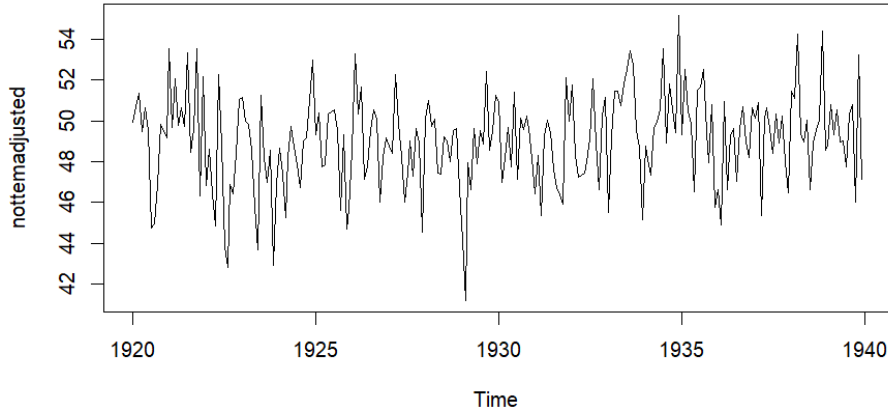


Figure 9: Notttem dataset seasonally adjusted

Well, basically, we have to subtract the seasonal component from the original series, and it looks like Figure 9.

Since the main component of nottem was the seasonal component, we do not see any trend and it pretty much looks like a series of residuals.

Seasonality

A simple forecasting exercise...

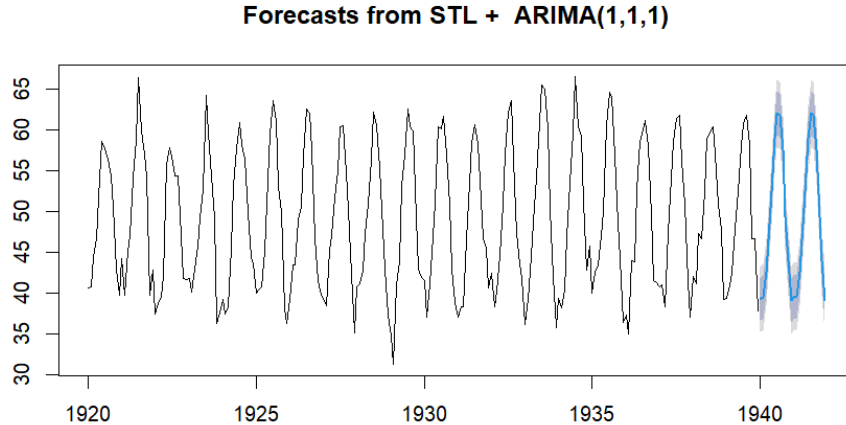


Figure 10: Notten dataset forecast 24 months ahead

We will see in a simple code that it is possible to run a forecast using the data from the seasonal decomposition.

With this kind of regular data, (no trend and constant seasonal component, the forecast looks quite accurate).

Seasonality

Exponential smoothing with function ETS

- Combines multiplicative and additive methods. It describes the time series using three parameters, each of them with interesting options:
 1. Error (additive or multiplicative)
 2. Trend (non-present, additive or multiplicative)
 3. Seasonality (non-present, additive or multiplicative)
- Interestingly, with exponential smoothing it is possible to combine the components, each of them having different multiplicative, additive or none components.
- Exponential smoothing gives more weight to recent data (makes sense and cover changes in seasonal patterns).

Seasonality

Exponential smoothing with function ETS

- A simple function in R is `ets()`.
- Applying `ets()` produces the following smoothing coefficients:
 1. Alpha: initial level
 2. Beta: trend
 3. Gamma: seasonality
 4. Phi: dumped parameter (usually between 0.8 and 1, the lower it is the more important the dumping effect is; allows for the model to dump the trend since this may change along the series).
- The lower the coefficients are, the higher the level of smoothness, the closer to 1, the more it relies on recent data.

Main takeaways from Lecture 2

- Visual inspect the data, identify trends. Why do we need to consider trends in our analysis?
- What is the easiest way to detrend the data? How can you check whether it works or not?
- What are the consequences of not detrending our data for regression analysis?
- When can we find seasonality in the data?
- When do additive and multiplicative methods work best?
- What the potential problems of working with seasonal data?

Exercise

Let us solve the exercise together and prepare for the seminar...

Seminar 2

Seminar exercise

1. Plot the dataset 'AirPassengers' (you can get it directly from R database), describe briefly the dataset and the patterns you observe. (1 point)
2. Create two models, model1 and model2, decomposing the series with additive and multiplicative functions. Plot and compare both models. Which one fits the data better? Why do you think this is the case? (1 point)
3. Create a third model (model3) using the ets() function, report the parameters identified by the function and explain them (1 point).
4. Plot both the original and the seasonally adjusted series from point 3. (1 point)
5. Deliver the codes and a brief one-page report including points 1-4 before Thursday 7th of March. (1 point)
6. Extra point: Remember the tbillmodel from seminar 1? Create a quadratic trend and estimate the tbillmodel including this trend. Is it significant? (1 point)