PORTFOLIO THEORY – LECTURE NOTES 5

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PERFORMANCE EVALUATION

After we obtain the series of portfolio returns, we need to appropriately evaluate results in order to gauge how well our investment strategy performed. We may start with some basic statistics about the distribution of the returns:

- Mean: the higher the better
- Standard deviation: the lower the better
- Skewness: a positive value is preferable
- (Excess) Kurtosis: a lower value is preferable unless the skewness is significantly positive

We then compute the Sharpe ratio to quantify the risk-adjusted return.

If we are interested in evaluating downside risk, instead or in addition to the standard deviation and the Sharpe ratio, we may compute some or all of these measures:

- Downside deviation: the lower the better
- CVaR: the lower (in absolute value) the better
- (Maximum) drawdown: the lower the better

We then quantify the risk-adjusted return with an appropriate measure, like the Sortino ratio.

Another important financial indicator is the **alpha**, used to check if the returns of the investments are explained by a given asset pricing model. The alpha is obtained as the intercept in a regression of the portfolio returns over the returns of the factors of the model considered. Usually, the CAPM (in which case the alpha is called "Jensen's alpha") or the Fama-French three-factor model are used. In the first case the regression takes the form:

$$R = \alpha + \beta\left(R_{Mkt} - R_f\right)$$

while in the second case it is:

$$R = \alpha + b_1\left(R_{Mkt} - R_f\right) + b_2 SMB + b_3 HML$$

If α is significantly greater than zero, it means that our strategy achieves returns higher than those predicted by the model based on the portfolio exposure to the factors. In order to check if we have a significant positive α, we compute its standard errors and then perform a test of hypothesis. The usual standard errors for the linear regression are generally not appropriate, as they assume homoskedasticity (i.e., constant variance) and no autocorrelation (i.e., no temporal dependency in the standard errors). These conditions are usually not met in financial time series, where heteroskedasticity and/or autocorrelation are often observed. To account for this, we may use instead the **Newey-West standard errors**. The technical details of such estimator are somewhat complicated, but not relevant here, and Newey-West standard errors can be easily computed in R. Using them, we can test whether the α of our investment is significantly greater than zero.

A proper evaluation, however, should also account for the **turnover**, i.e., how much trading the strategy requires. The higher the turnover, the higher the transaction costs, which of course translates into lower net returns. To get an idea of the amount of trading required we can compute the average turnover. The turnover at a certain period t is given by:

$$TO_t = \sum_{i=1}^{N} \left|w_{i,t} - w_{i,t-1}\right|$$

Basically, for each stock we compute the absolute value of the change in the corresponding weight compared to the previous period, and we sum all these N values. We do this for each of the T periods in which we applied our strategy, and then we compute the mean. This gives us the average turnover.

However, applying this formula using the set of weights we selected for the previous period as the value for $w_{i,t-1}$ is not entirely correct. This is because when we update the weights at the beginning of each new period we need to account for the fact that, due to the realized returns during the period that just ended, the allocation of wealth changed compared to what was at the beginning of the previous period. We clarify this with an example.

Suppose we have a portfolio with two assets updated monthly, and the weights at time $t-1$ were 0.5 and 0.5, while now at time t we want to change them to 0.4 and 0.6. We might think that the turnover is $|0.4-0.5|+|0.6-0.5|=0.2$, which means we have to trade 20% of our wealth to update the portfolio. If the price of the two assets did not change over the month that just ended, this would indeed be correct. Suppose however that during that month the first asset experienced a +10% return, and the second one a -20% return. When we update the portfolio at time t, we no longer have the two original weights, but $0.5+0.5\times 0.1=0.55$ for the first asset and $0.5-0.5\times 0.2=0.4$ for the second. The weights computed like this do not sum up to 1 because the total value of the portfolio changed compared to period $t-1$. We need to account for this by dividing both weights by their sum. So in this example where they sum to $0.55+0.4=0.95$ we have $0.55/0.95\approx 0.58$ for the first weight and $0.4/0.95\approx 0.42$ for the second. Therefore, the actual turnover is $|0.4-0.58|+|0.6-0.42|=0.36$.

We might express this concept by rewriting the formula for the turnover as

$$TO_t=\sum_{i=1}^{N}\left|w_{i,t}-w_{i,t-1}^{+}\right|$$

where $w_{i,t-1}^{+}$ indicates that we are considering the returns from the previous period after accounting for the redistributing effect of the realized returns. We can then compute the average turnover as before.

While the turnover certainly provides some useful information, we can get an even better figure by considering the portfolio **returns net of transaction costs**. Transaction costs can be fixed or proportional to the amount of trading. It is generally considered more appropriate to use proportional transaction costs. These can be accounted for by multiplying the turnover of each asset for the proportional cost. Frazzini (2012) suggests using transaction costs equal to 10 basis points (bp). A basis point is equal to 0.01%. Therefore, for example, if we need to buy or sell 5% of the positions we have in a certain asset, we face transaction costs equal to $0.05\times 0.001=0.00005$, which means that 0.005% of the money invested in that position is lost in transaction costs.

Once we have the portfolio returns net of transaction costs, we can use them to compute all the other statistics we listed before.

Finally, it is useful to visualize the value V of the portfolio over time, which can be computed as

$$V_T=V_0+\sum_{t=1}^{T}(V_{t-1}R_t)$$

It is appropriate to compute the value both ignoring and net of transaction costs. The result is then plotted in a graph, which provides a visual representation of the effectiveness of our strategy.

We might want to also compute the evolution of real wealth in addition to the nominal wealth. In other words, we might want to account for the inflation. We can do this by dividing the value of the portfolio over time by the deflator. We can compute the deflator D using a formula analogous to the one used to compute the value of the portfolio, simply replacing the return with the inflation rate I:

$$D_T = D_0 + \sum_{t=1}^{T} (D_{t-1} I_t)$$

Of course, the two series need to have the same starting value (e.g., 1 unit of wealth), and the same frequency (e.g., monthly).