PORTFOLIO THEORY – MOCK EXAM 14/05/2024

Dr. Andrea Rigamonti

EXERCISE 1

In a perfectly competitive market where the interest rate r remains constant over time, a security pays 600 euro after one year and another 600 euro after two years, and it costs 1000 euro. What is the interest rate?

The interest rate is:

$$1000 = \frac{600}{1+r} + \frac{600}{(1+r)^2}$$

$$1000(1+r)^2 = 600(1+r) + 600$$

$$1000(1+2r+r^2) = 600 + 600r + 600$$

$$1000 + 2000r + 1000r^2 = 1200 + 600r$$

$$1000r^2 + 1400r - 200 = 0$$

$$5r^2 + 7r - 1 = 0$$

$$r = \frac{-7 \pm \sqrt{49+20}}{10} \approx \frac{-7 \pm 8.3}{10}$$

The equation is solved for $r = \frac{-7+8.3}{10} = 0.13$ and for $r = \frac{-7-8.3}{10} = -1.53$

The second one obviously does not make sense, so the interest rate is 13%

EXERCISE 2

The log-return of the four assets included in an equally weighted portfolio is:

$r_1 = 0.1,\ r_2 = -0.06,\ r_3 = 0.07,\ r_4 = 0.05$

What is the return of the portfolio?

Log-returns are not asset additive. We first need to convert them to simple returns:

$$R_1 = \exp(r_1) - 1 = \exp(0.1) - 1 = 0.1052$$

$$R_2 = \exp(r_2) - 1 = \exp(-0.06) - 1 = -0.0582$$

$$R_3 = \exp(r_3) - 1 = \exp(0.07) - 1 = 0.0725$$

$$R_4 = \exp(r_4) - 1 = \exp(0.05) - 1 = 0.0513$$

We can now compute the return of the portfolio:

$$R_p = w_1 R_1 + w_2 R_2 + w_3 R_3 + w_4 R_4 =$$
$$= 0.25 * 0.1052 + 0.25 * (-0.0582) + 0.25 * 0.0725 + 0.25 * 0.0513 = 0.0427$$

EXERCISE 3

Suppose that the conditions for the CAPM are fully respected. The market portfolio expected return is 5% and the risk-free rate is 1%. We create a portfolio in which 80% of the wealth is placed in the security A whose beta is $\beta_A = 0.5$, 50% in the security B whose beta is $\beta_B = 1.5$, and we short the risk-free asset. What is the expected return of the portfolio?

As the weights for the risky assets sum to $0.8 + 0.5 = 1.3$, it means that the weight for the risk-free asset is -0.3. Since the CAPM holds, the expected return of A:

$$E[R_A] = R_f + \beta_A * \left(E[R_M] - R_f\right) = 0.01 + 0.5 * (0.05 - 0.01) = 0.01 + 0.02 = 0.03$$

The expected return of B is:

$$E[R_B] = R_f + \beta_B * \left(E[R_M] - R_f\right) = 0.01 + 1.5 * (0.05 - 0.01) = 0.01 + 0.06 = 0.07$$

Hence, the expected return of the portfolio is:

$$E[R_P] = 0.8 * 0.03 + 0.5 * 0.07 - 0.3 * 0.01 = 0.056 = 5.6\%$$

EXERCISE 4

The vector of weights and the covariance matrix of a portfolio with two assets are:

$$\boldsymbol{w} = \begin{bmatrix} 1.2 \\ -0.2 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.002 & 0.001 \\ 0.001 & 0.003 \end{bmatrix}$$

Compute, using matrix form, the standard deviation of the portfolio.

$$Var(R_P) = [1.2 \quad -0.2] \begin{bmatrix} 0.002 & 0.001 \\ 0.001 & 0.003 \end{bmatrix} \begin{bmatrix} 1.2 \\ -0.2 \end{bmatrix} =$$

$$[1.2 * 0.002 - 0.2 * 0.001 \quad 1.2 * 0.001 - 0.2 * 0.003] \begin{bmatrix} 1.2 \\ -0.2 \end{bmatrix} =$$

$$[0.0022 \quad 0.0006] \begin{bmatrix} 1.2 \\ -0.2 \end{bmatrix} = 0.0022 * 1.2 - 0.0006 * 0.2 = 0.00252$$

The standard deviation is the square root of the variance, therefore it is $\sqrt{0.00252} = 0.0501996$

EXERCISE 5

Given the following series of returns

0.05 , -0.01 , 0.02 , 0.03 , 0

and the following series of risk-free rates

0 , 0.005 , 0.005 , 0 , 0.01

Compute the Sortino ratio for an investor that sets the benchmark equal to the risk-free rate

As the investor sets the benchmark equal to the risk-free rate, we can use the excess returns and set the benchmark to zero. So first we compute the excess returns:

$$0.05 - 0 = 0.05$$

$$-0.01 - 0.005 = -0.015$$

$$0.02 - 0.005 = 0.015$$

$$0.03 - 0 = 0.03$$

$$0 - 0.01 = -0.01$$

We then compute the semivariance:

$$\sigma_B^2 = \frac{1}{T}\sum_{t=1}^{T}[\mathrm{Min}(R_t - B, 0)]^2 = \frac{1}{5}[0 + (-0.015)^2 + 0 + 0 + (-0.01)^2] =$$

$$= \frac{1}{5}0.000325 = 0.000065$$

Then we compute the downside deviation, which is the square root of the semivariance:

$$\sigma_B = \sqrt{0.000065} \approx 0.008$$

Finally we need the mean excess return:

$$\overline{R} = \frac{0.05 - 0.015 + 0.015 + 0.03 - 0.01}{5} = \frac{0.07}{5} = 0.014$$

Now we can compute the Sortino ratio:

$$\mathrm{Sortino} = \frac{\overline{R} - B}{\sigma_B} = \frac{0.014 - 0}{0.008} = 1.75$$

EXERCISE 6

Compute the turnover from $t-1$ to t of an equally weighted portfolio of two assets, taking into account the effect of the realized returns $R_t = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}$

We have to use the formula

$$TO_t = \sum_{i=1}^{N} \left| w_{i,t} - w_{i,t-1}^{+} \right|$$

To apply it correctly we first need to compute $w_{i,t-1}^{+}$.

The first asset experienced a +20% returns, therefore we have $0.5 + 0.5 \times 0.2 = 0.6$

The second asset experienced a -10% returns, therefore we have $0.5 - 0.5 \times 0.1 = 0.45$

The weights now sum to 1.05. We need to normalize them so that they sum up to 1 again:

$$w_{i,t-1}^{+} = \begin{bmatrix} \frac{0.6}{1.05} \\ \frac{0.45}{1.05} \end{bmatrix} \approx \begin{bmatrix} 0.57 \\ 0.43 \end{bmatrix}$$

Now we can apply the formula and compute the turnover:

$$TO_t = |0.5 - 0.57| + |0.5 - 0.43| = 0.07 + 0.07 = 0.14$$

This means we need to trade 14% of our wealth in order to update the weights.

EXERCISE 7

1. *How does the price of a bond on the secondary market change if the interest rates rise?*
2. *If the conditions of the CAPM are fully respected, what is the value of the Jensen's alpha of a security?*
3. *What happens to the bid-ask spread if the market gets more liquid?*
4. *What is the beta of an ETF that perfectly replicates the market portfolio?*
5. *If the law of one price holds, how much can the price of two equivalent securities deviate from each other on different markets?*
6. *What are the three assumptions of the Arbitrage Pricing Theory?*
7. *Consider a mean-variance investor. If the risk aversion parameter of the investor increases from 3 to 5, how does the optimal portfolio selected by the investor change in terms of expected return and standard deviation?*
8. *What does it mean if a portfolio has a $VaR = 0.08$ with a 99% confidence level?*
9. *We compute the alpha of a portfolio using the Fama-French three-factor model, and we find that it is not significantly different from zero. What does this mean?*

1. The price of the bond decreases.
2. It is equal to zero.
3. The spread gets smaller.
4. It is equal to 1
5. It can deviate at most by the amount of the transaction costs.

6. The three assumptions are: (1) Capital markets are perfectly competitive (2) Investors always prefer more wealth for a given risk (3) The stochastic process generating asset returns can be expressed as a linear function of a set of K risk factors, and all unsystematic risk is diversified away.
7. If the risk-aversion parameter increases, the investor will select a portfolio with a lower expected return but also lower standard deviation.
8. It means that, with a 99% confidence, you lose at most 8% in a given period
9. It means that the returns of the portfolio are explained by the exposure of the portfolio to the systematic risk as proxied by the three factors used in that linear model.