# **Portfolio Theory**

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# Lecture 1

Content:

- The corporation and the financial market
- General evaluation principles
- Present value of some common investments

In the US there are four major types of firms:

- Sole proprietorships: owned and run by one person who has unlimited personal liability for the debts.
- Partnerships: has more owners, who are fully liable, except for limited partnership, where general partners are fully liable while limited partners' liability is limited to their investment.
- Limited liability companies (LLC): it is a limited partnership without a general partner.
- <u>Corporations</u>

The corporation is a legal entity separate from its owners (i.e., the latter are not liable for the obligations of the former, and vice versa).

The ownership is divided into **shares**. The set of shares owned by an investor is called "stock", but these terms are often used interchangeably to indicate shares.

The collection of all the outstanding shares of a corporation is known as the **equity** of the corporation.

The owners can profit from:

- dividends payment;
- capital gain.

The direct control of the corporation is held by the board of directors and the CEO.



These issues are dealt with by corporate finance.

Some or all the shares of a corporation can be privately held, and they are traded on private markets.

Publicly traded stocks are traded on the **stock market**.

**Primary market**: where new shares are first issued.

Secondary market: where existing shares are traded.

**Liquid** stocks can be easily converted into cash without affecting its market price. This allows for flexible investments and efficient pricing of the investment.

**Bid-ask spread**: the difference between the price at which it is possible to buy the stock (bid price) and the price at which it is possible to sell the stock (ask price).

Bid price is always higher than ask price, but this spread is small if the market is liquid.

Market makers are intermediaries that quote both a buy and a sell price to profit on the bid-ask spread. They increase the liquidity.

**Limit order book**: the collection of all limit orders (orders to buy or sell a set amount at a fixed price). They provide liquidity.

These and other issue are studied by a branch of finance called market microstructure.

**Time value of money**: the difference in value between money today and money in the future.

**Interest rate:** The rate at which we can exchange money today for money in the future

**Risk-free interest rate (R\_f)**: the rate at which money can be borrowed or lent without risk over a certain period.

**Arbitrage**: the practice of buying and selling equivalent goods in different markets to profit from a price difference

**Security**: an investment opportunity that trades in a financial market

**Short selling**: selling a security you do not own by borrowing it and returning it to the owner at a later date.

Trading securities implies facing **transaction costs**:

- the commissions paid to the broker
- the bid-ask spread

Arbitrage keeps the prices of equivalent goods and securities close to each other: prices cannot deviate more than the transaction costs of the arbitrage.

Given the interest rate R and the number of periods n, the value of an investment:

• in terms of dollars today is the **present value (PV)**:

$$PV = \frac{FV}{(1+R)^n}$$

• in terms of dollars in the future is the **future value (FV)**:

$$FV = PV * (1+R)^n$$

(Risky cash flows must be discounted at a rate equal to the risk-free rate plus an appropriate risk premium.)

General process for pricing securities (in a market without arbitrage opportunities):

- 1. Identify the cash flows that will be paid by the security
- 2. Determine the PV of the security's cash flows

Unless the price of the security equals this present value, there is an arbitrage opportunity:

No-Arbitrage Price = PV(All cash flows paid by the security)

It is only possible to compare or combine values at the same point in time.

To move a cash flow *C* forward in time you must compound it (compute the future value):

$$FV_n = C * (1+R)^n$$

To find the equivalent value today of a future cash flow C you must discount it (compute the present value):

$$PV = \frac{C}{(1+R)^n}$$

#### **Present Value of a Cash Flow Stream:**

$$PV = \sum_{n=0}^{N} PV(C_n) = \sum_{n=0}^{N} \frac{C_n}{(1+R)^n}$$

#### Future Value of a Cash Flow Stream with Present Value PV:

$$FV_n = PV * (1+R)^n$$

**A perpetuity** is a stream of equal cash flows that occur at regular intervals and last forever.

The present value of a perpetuity with payment *C* and interest rate *R* is given by

$$PV = \sum_{n=1}^{\infty} \frac{C}{(1+R)^n}$$

Because the first cash flow is in one period,  $C_0 = 0$ . It can be shown that:

$$PV(C \text{ in perpetuity}) = \frac{C}{R}$$

An **annuity** is a stream of N equal cash flows paid at regular intervals (the first one at date 1). At the end of it, you get your initial investment back.

The present value of an *N*-period annuity with payment *C* and interest rate *R* is  $PV = \sum_{n=1}^{N} \frac{C}{(1+R)^n}$ 

It can be shown that the **PV of an annuity** is:

$$PV = C \frac{1}{R} \left( 1 - \frac{1}{(1+R)^N} \right)$$

The Future Value of an Annuity is:

$$FV = C\frac{1}{R}((1+R)^N - 1)$$

A **growing perpetuity** is a stream of cash flows that occur at regular intervals and grow at a constant rate forever. A growing perpetuity with a first payment *C* and a growth rate *g* has present value

$$PV = \sum_{n=1}^{\infty} \frac{C(1+g)^{n-1}}{(1+R)^n}$$

It must be g < R, so that each successive term in the sum is less than the previous term and the overall sum is finite.

It can be shown that:

$$PV(growing \, perpetuity) = \frac{C}{R-g}$$

A growing annuity is a stream of *N* growing cash flows, paid at regular intervals. The first cash flow arrives at the end of the first period, and the first cash flow does not grow. The present value of an *N*-period growing annuity with initial cash flow *C*, growth rate *g*, and interest rate r is given by

$$PV = C \frac{1}{R - g} \left( 1 - \left(\frac{1 + g}{1 + R}\right)^N \right)$$

Because the annuity has only a finite number of terms, we can have g > R. The formula does not work for g = R, but in that case growth and discounting cancel out:  $PV = C \frac{N}{1+R}$