PORTFOLIO THEORY – EXERCISES 1

EXERCISE 1

The security S pays 990 euro after eight months and it costs 900 euro.

If there is a 20% tax on financial profits and the inflation rate is 2% per year, what is the real annual return paid by security S after taxes?

After eight months we earn $990 - 900 = 90$ euro.

On this we have to pay a 20% tax, so we actually earn $90 - 90 * 0.2 = 72$ euro

The nominal return net of taxes is therefore:

$$R = \frac{972 - 900}{900} = 0.08 = 8\%$$

The annual nominal return is:

$$R_{12} = (1 + R)^{\frac{12}{8}} - 1 = (1 + 0.08)^{\frac{3}{2}} - 1 \approx 0.122$$

We now need to account for the inflation. The real return is:

$$R_r = \frac{1 + R_{12}}{1 + i} - 1 = \frac{1 + 0.122}{1 + 0.02} - 1 = 0.1 = 10\%$$

EXERCISE 2

The log-return of the four assets included in an equally weighted portfolio is:

$r_1 = 0.1,\ r_2 = -0.06,\ r_3 = 0.07,\ r_4 = 0.05$

What is the return of the portfolio?

Log-returns are not asset additive. We first need to convert them to simple returns:

$$R_1 = \exp(r_1) - 1 = \exp(0.1) - 1 = 0.1052$$

$$R_2 = \exp(r_2) - 1 = \exp(-0.06) - 1 = -0.0582$$

$$R_3 = \exp(r_3) - 1 = \exp(0.07) - 1 = 0.0725$$

$$R_4 = \exp(r_4) - 1 = \exp(0.05) - 1 = 0.0513$$

We can now compute the return of the portfolio:

$$R_p = w_1 R_1 + w_2 R_2 + w_3 R_3 + w_4 R_4 =$$

$$= 0.25 * 0.1052 + 0.25 * (-0.0582) + 0.25 * 0.0725 + 0.25 * 0.0513 = 0.0427$$

EXERCISE 3

The returns of a security over four periods are:

$R_{t=1} = 0.2,\ R_{t=2} = -0.1,\ R_{t=3} = 0.08,\ R_{t=4} = 0.04$

If we invested 1000 euro in this asset at t=0, how much is our investment worth at t=4?

The value of the investment is:

$$V_4 = V_0 + V_0\left[\prod_{t=1}^{4}(1+R_t) - 1\right] = 1000 + 1000[(1+0.2)(1-0.1)(1+0.08)(1+0.04) - 1]$$

$$\approx 1000 + 1000 * [1.213 - 1] = 1213$$

Alternatively, we can transform the returns in log-returns, which are time-additive:

$$r_{t=1} = \ln(R_{t=1} + 1) = \ln(0.2 + 1) \approx 0.1823$$

$$r_{t=2} = \ln(R_{t=2} + 1) = \ln(-0.1 + 1) \approx -0.1054$$

$$r_{t=3} = \ln(R_{t=3} + 1) = \ln(0.08 + 1) \approx 0.0770$$

$$r_{t=4} = \ln(R_{t=4} + 1) = \ln(0.04 + 1) \approx 0.0392$$

The cumulative log-return from $t = 1$ to $t = 4$ is:

$$cumret_{1-4} = 0.1823 - 0.1054 + 0.0770 + 0.0392 = 0.1931$$

We need to convert this into a simple return:

$$cumRet_{1-4} = \exp(cumret_{1-4}) - 1 = \exp(0.1931) - 1 \approx 0.213$$

And the value of the investment at $t = 4$ is therefore:

$$V_4 = V_0 + V_0 * cumRet_{1-4} = 1000 + 1000 * 0.231 = 1231$$

EXERCISE 4

The vector of weights and the covariance matrix of a portfolio with three assets are:

$$\boldsymbol{w} = \begin{bmatrix} 0.5 \\ 0.7 \\ -0.2 \end{bmatrix} \qquad \boldsymbol{\Sigma} = \begin{bmatrix} 0.004 & 0.006 & 0.003 \\ 0.006 & 0.008 & 0.007 \\ 0.003 & 0.007 & 0.005 \end{bmatrix}$$

Compute, using matrix form, the variance of the portfolio.

We just need to apply the formula:

$$Var(R_P) = \boldsymbol{w}'\Sigma\boldsymbol{w} = [0.5 \quad 0.7 \quad -0.2] \begin{bmatrix} 0.004 & 0.006 & 0.003 \\ 0.006 & 0.008 & 0.007 \\ 0.003 & 0.007 & 0.005 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.7 \\ -0.2 \end{bmatrix} =$$

$$[0.5*0.004+0.7*0.006-0.2*0.003 \quad 0.5*0.006+0.7*0.008-0.2*0.007 \quad 0.5*0.003+0.7*0.007-0.2*0.005]\begin{bmatrix}0.5\\0.7\\-0.2\end{bmatrix}$$

$$=[0.0056 \quad 0.0072 \quad 0.0054]\begin{bmatrix}0.5\\0.7\\-0.2\end{bmatrix}=0.0056*0.5+0.0072*0.7+0.0054*(-0.2)=0.00676$$

EXERCISE 5

A risky investment is estimated to deliver the following returns.

After 9 months:

- $R=-0.15$ *with a 20% probability*
- $R=0.1$ *with a 70% probability*
- $R=0.25$ *with a 10% probability*

After 24 months:

- $R=-0.2$ *with a 20% probability*
- $R=0.15$ *with a 60% probability*
- $R=0.3$ *with a 20% probability*

The annual inflation rate is 3%.

What is the real cumulative expected return after 24 months?

First we need to compute the expected return at 9 months and at 24 months.

$$E[R_{9m}]=\sum_R p_R R=0.2*(-0.15)+0.7*0.1+0.1*0.25=0.065$$

$$E[R_{24m}]=\sum_R p_R R=0.2*(-0.2)+0.6*0.15+0.2*0.3=0.11$$

The cumulative expected return at 24 months is:

$$R_{24m,cum}=\prod_{t=1}^{2}(1+R_t)-1=(1+0.065)(1+0.11)-1\approx 0.182$$

The 24-month inflation rate is:

$$i_{24}=(1+i_{12})^2-1=(1+0.03)^2-1=0.0609$$

So the real cumulative expected return after 24 months is:

$$R_r=\frac{1+R}{1+i}-1=\frac{1+0.182}{1+0.0609}-1\approx 0.114$$

EXERCISE 6

Consider the following series of <u>unadjusted</u> monthly closing prices (in euro) of a stock that undergoes the corporate events indicated next to the price.

January: 7

February: 6.5 Dividend of 1 euro per share is paid

March: 7.5

April: 7.2

May: 4 2 for 1 stock split

June: 4.5

Compute the adjusted stock returns.

First, we adjust the prices to account for the stock split:

January: $7/2 = 3.5$

February: $6.5/2 = 3.25$ Dividend: $1/2 = 0.5$

March: $7.5/2 = 3.75$

April: $7.2/2 = 3.6$

May: 4

June: 4.5

Now we compute the returns, accounting for the dividend in February:

$$R_{Feb} = \frac{3.25 + 0.5 - 3.5}{3.5} \approx 0.071$$

$$R_{Mar} = \frac{3.75 - 3.25}{3.25} \approx 0.154$$

$$R_{Apr} = \frac{3.6 - 3.75}{3.75} = -0.04$$

$$R_{May} = \frac{4 - 3.6}{3.6} \approx 0.111$$

$$R_{Jun} = \frac{4.5 - 4}{4} = 0.125$$

Alternatively, we can use the Cumulative Adjustment Factor:

$$CAF_t = CAF_{t-1} \times Split\,ratio_t \times \left(1 + \frac{Dividend_t}{Price_t}\right)$$

January: 7 $\quad CAF_{Jan} = 1$

February: 6.5 $\quad CAF_{Feb} = 1 \times 1 \times \left(1 + \frac{1}{6.5}\right) \approx 1.1538$

March: 7.5 $\quad CAF_{Mar} = 1.1538 \times 1 \times (1 + 0) = 1.1538$

April: 7.2 $\quad CAF_{Apr} = 1.1538 \times 1 \times (1 + 0) = 1.1538$

May: 4 $\quad CAF_{May} = 1.1538 \times 2 \times (1 + 0) = 2.3076$

June: 4.5 $\quad CAF_{Jun} = 2.3076 \times 1 \times (1 + 0) = 2.3076$

Now we adjusted the prices:

January: $7 * 1 = 7$

February: $6.5 * 1.1538 = 7.4997$

March: $7.5 * 1.1538 = 8.6535$

April: $7.2 * 1.1538 \approx 8.3074$

May: $4 * 2.3076 = 9.2304$

June: $4.5 * 2.3076 = 10.3842$

And finally we compute the returns:

$$R_{Feb} = \frac{7.4997 - 7}{7} \approx 0.071$$

$$R_{Mar} = \frac{8.6535 - 7.4997}{7.4997} \approx 0.154$$

$$R_{Apr} = \frac{8.3074 - 8.6535}{8.6535} \approx -0.04$$

$$R_{May} = \frac{9.2304 - 8.3074}{8.3074} \approx 0.111$$

$$R_{Jun} = \frac{10.3842 - 9.2304}{9.2304} = 0.125$$