PORTFOLIO THEORY – EXERCISES 2

EXERCISE 1

Suppose that the conditions for the CAPM are fully respected. The market portfolio expected return is 5% and the risk-free rate is 1%. We create a portfolio in which 80% of the wealth is placed in the security A whose beta is $\beta_A = 0.5$*, 50% in the security B whose beta is* $\beta_B = 1.5$*, and we short the risk-free asset. What is the expected return of the portfolio?*

As the weights for the risky assets sum to $0.8 + 0.5 = 1.3$, it means that the weight for the risk-free asset is -0.3. Since the CAPM holds, the expected return of A:

$$E[R_A] = R_f + \beta_A * \left(E[R_M] - R_f\right) = 0.01 + 0.5 * (0.05 - 0.01) = 0.01 + 0.02 = 0.03$$

The expected return of B is:

$$E[R_B] = R_f + \beta_B * \left(E[R_M] - R_f\right) = 0.01 + 1.5 * (0.05 - 0.01) = 0.01 + 0.06 = 0.07$$

Hence, the expected return of the portfolio is:

$$E[R_P] = 0.8 * 0.03 + 0.5 * 0.07 - 0.3 * 0.01 = 0.056 = 5.6\%$$

EXERCISE 2

The market portfolio has an expected return of 5%, and the risk-free rate is 1%. Suppose that the conditions for the CAPM are fully respected. The expected return of a portfolio in which 40% of the wealth if placed in stocks S and 60% in an ETF that replicates the market portfolio with zero tracking error is 7%. What is the beta of the stocks S?

First, we determine the beta of the portfolio:

$$E[R_P] = R_f + \beta_P * \left(E[R_M] - R_f\right)$$

$$0.07 = 0.01 + \beta_P * (0.05 - 0.01)$$

$$0.06 = 0.04\beta_P$$

$$\beta_P = \frac{0.06}{0.04} = 1.5$$

The beta of a portfolio is equal to the weighted average of the beta of its components. The beta of the ETF is equal to 1 (because it replicates the market portfolio). Therefore, the beta of the stocks S is:

$$1.5 = 0.4 * \beta_S + 0.6 * 1$$

$$0.9 = 0.4 * \beta_S$$

$$\beta_S = \frac{0.9}{0.4} = 2.25$$