PORTFOLIO THEORY – EXERCISES 3

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EXERCISE 1

An investor with risk-aversion $\gamma = 4$ *invests in a portfolio of one risk-free asset and two risky assets whose excess returns have mean vector, covariance matrix and inverse covariance matrix equal to:*

$$\mu = \begin{bmatrix} 0.006 \\ 0.004 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.003 & 0.002 \\ 0.002 & 0.0015 \end{bmatrix} \quad \boldsymbol{\Sigma^{-1}} \approx \begin{bmatrix} 3000 & -4000 \\ -4000 & 6000 \end{bmatrix}$$

Compute the portfolio weights that maximize mean-variance utility, and the corresponding utility.

EXERCISE 2

An investor with risk-aversion $\gamma = 2$ *invests in a portfolio of one risk-free asset and two risky assets whose excess returns have mean vector, covariance matrix and inverse covariance matrix equal to:*

$$\mu = \begin{bmatrix} 0.005 \\ 0.004 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.003 & 0.002 \\ 0.002 & 0.0015 \end{bmatrix} \quad \boldsymbol{\Sigma^{-1}} \approx \begin{bmatrix} 3000 & -4000 \\ -4000 & 6000 \end{bmatrix}$$

Compute the portfolio weights that maximize mean-variance utility given a full-investment constraint.

EXERCISE 3

Given a risk-free asset and two risky assets whose excess returns have mean vector, covariance matrix and inverse covariance matrix equal to

$$\mu = \begin{bmatrix} 0.01 \\ 0.008 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.004 & 0.002 \\ 0.002 & 0.003 \end{bmatrix} \quad \boldsymbol{\Sigma^{-1}} = \begin{bmatrix} 375 & -250 \\ -250 & 500 \end{bmatrix}$$

compute the weights for the optimal mean-variance portfolio with target return $Re = 0.01$.

EXERCISE 4

Given a risk-free asset and two risky assets whose excess returns have mean vector, covariance matrix and inverse covariance matrix equal to

$$\mu = \begin{bmatrix} 0.01 \\ 0.008 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.004 & 0.002 \\ 0.002 & 0.003 \end{bmatrix} \quad \boldsymbol{\Sigma^{-1}} = \begin{bmatrix} 375 & -250 \\ -250 & 500 \end{bmatrix}$$

compute the weights for the optimal mean-variance portfolio with target return $Re = 0.01$ *given a full-investment constraint.*

EXERCISE 5

Given a risk-free asset and two risky assets whose excess returns have mean vector, covariance matrix and inverse covariance matrix equal to

$$\mu = \begin{bmatrix} 0.01 \\ 0.008 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.004 & 0.002 \\ 0.002 & 0.003 \end{bmatrix} \quad \boldsymbol{\Sigma^{-1}} = \begin{bmatrix} 375 & -250 \\ -250 & 500 \end{bmatrix}$$

compute the tangency portfolio.

EXERCISE 6

Given two risky assets with mean return, covariance matrix and inverse covariance matrix equal to

$$\mu = \begin{bmatrix} 0.007 \\ 0.004 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.004 & 0.002 \\ 0.002 & 0.003 \end{bmatrix} \quad \boldsymbol{\Sigma}^{-1} = \begin{bmatrix} 375 & -250 \\ -250 & 500 \end{bmatrix}$$

compute the weights for the minimum variance portfolio.

EXERCISE 7

A mean-variance utility investor with risk-aversion coefficient $\gamma = 5$ can invest in two risky assets and one risk-free asset. The mean and covariance matrix of the excess returns of the two risky assets are:

$$\mu = \begin{bmatrix} 0.01 \\ 0.008 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 0.005 & 0.002 \\ 0.002 & 0.003 \end{bmatrix}$$

Compute the weights for the 1/N portfolio that optimally allocates the wealth between the risky assets and the risk-free asset.