Portfolio Theory

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Lecture 10

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- Return distribution statistics
- Alpha
- Turnover and net returns
- Benchmark portfolios

Return distribution statistics

We first may start the results evaluation by computing some statistics that describe the return distribution:

- Mean: the higher the better
- Standard deviation: the lower the better
- Skewness: a positive value is preferable
- (Excess) Kurtosis: a lower value is preferable unless the skewness is significantly positive

We then compute the Sharpe ratio to quantify the risk-adjusted return.

Return distribution statistics

To evaluate downside risk, instead or in addition to the standard deviation and the Sharpe ratio we may compute some or all of these risk measures:

- Downside deviation: the lower the better
- CVaR: the lower (in absolute value) the better
- (Maximum) drawdown: the lower the better

We then quantify the risk-adjusted return with an appropriate measure, like the Sortino ratio.

Alpha

The **alpha** is used to check if the returns of the investment are explained by a given asset pricing model. It is obtained as the intercept in a regression of the portfolio returns over the returns of the factors of the model considered.

Usually, the CAPM (in which case the alpha is called “Jensen’s alpha”) or the Fama-French three-factor model are used. In the first case the regression takes the form:

$$R = \alpha + \beta\left(R_{Mkt} - R_f\right)$$

while in the second case it is:

$$R = \alpha + b_1\left(R_{Mkt} - R_f\right) + b_2 SMB + b_3 HML$$

Alpha

If α is significantly greater than zero, it means that our strategy achieves returns higher than those predicted by the model based on the portfolio exposure to the factors.

To check if we have a significant positive α, we compute its standard errors and then perform a test of hypothesis. The usual standard errors for the linear regression are generally not appropriate, as they assume homoskedasticity (i.e., constant variance) and no autocorrelation (i.e., no time dependency).

We may use instead the **Newey-West standard errors**, which can be easily computed in R.

Turnover and net returns

The **turnover** quantifies how much trading the strategy requires. The higher the turnover, the higher the transaction costs, which translates into lower net returns.

The turnover at a certain period t is given by:

$$TO_t = \sum_{i=1}^{N} \left| w_{i,t} - w_{i,t-1} \right|$$

For each stock we compute the absolute value of the change in its weight compared to the previous period, and we sum all these N values. We do this for each of the T periods in which we applied our strategy, and then we compute the mean to get the average turnover.

Turnover and net returns

To correctly use this formula, we need to account for the effect of realized returns during the previous period.

Suppose we have a portfolio with two assets updated monthly, and the weights at time $t - 1$ were 0.5 and 0.5, while now at time t we want to change them to 0.4 and 0.6.

If during that month the first asset experienced a +10% return, and the second one a -20% return, when we update the portfolio at time t we no longer have the two original weights, but $0.5 + 0.5 \times 0.1 = 0.55$ for the first asset and $0.5 - 0.5 \times 0.2 = 0.4$ for the second.

Turnover and net returns

These weights do not sum to 1 because the total value of the portfolio changed compared to period $t - 1$. We need to account for this by dividing both weights by their sum.

So in this example where they sum to $0.55 + 0.4 = 0.95$ we have $0.55/0.95 \approx 0.58$ for the first weight and $0.4/0.95 \approx 0.42$ for the second. Therefore, the actual turnover is $|0.4 - 0.58| + |0.6 - 0.42| = 0.36$.

Of course, the weights might not necessarily sum to 1, so the above example represents a special case.

Turnover and net returns

We rewrite the formula for the turnover as

$$TO_t = \sum_{i=1}^{N} \left| w_{i,t} - w_{i,t-1}^{+} \right|$$

where

$$w_{i,t-1}^{+} = \frac{\left(w_{i,t-1} + w_{i,t-1} \times R_{i,t-1}\right) \times \sum_{i=1}^{N} w_{i,t-1}}{\sum_{i=1}^{N}\left(w_{i,t-1} + w_{i,t-1} \times R_{i,t-1}\right)}$$

The $\sum_{i=1}^{N} w_{i,t-1}$ term allows us to correctly rescale the weights regardless whether they sum up to 1 or not (if they do, this term becomes equal to 1).

Turnover and net returns

A more accurate evaluation can be obtained by considering the portfolio **returns net of transaction costs**.

Transaction costs can be fixed or proportional to the amount of trading. The latter are more appropriate and can be accounted for by multiplying the turnover of each asset for the proportional cost.

Frazzini (2012) suggests using transaction costs equal to 10 basis points (bp). A basis point is equal to 0.01%.

Therefore, for example, if we need to buy or sell 5% of the positions in a certain asset, transaction costs are equal to $0.05 \times 0.001 = 0.00005$, which means that 0.005% of the money invested in that position is lost in transaction costs.

Turnover and net returns

The portfolio returns net of transaction costs can be used to compute all the other statistics we listed before.

Finally, it is useful to visualize the value V of the portfolio over time, which can be computed as

$$V_T = V_0 + \sum_{t=1}^{T} (V_{t-1} R_t)$$

It is appropriate to compute the value both ignoring and net of transaction costs, and to plot it.

Turnover and net returns

We might want to also compute the evolution of real wealth, i.e., accounting for the inflation. We can do it by dividing the value of the portfolio over time by the deflator.

We can compute the deflator D using a formula analogous to the one used to compute the value of the portfolio, simply replacing the return with the inflation rate I:

$$D_T = D_0 + \sum_{t=1}^{T} (D_{t-1} I_t)$$

The two series need to have the same starting value (e.g., 1 unit of wealth), and the same frequency (e.g., monthly).

Benchmark portfolios

We need to compare portfolios with appropriate **benchmarks** to know if the results are satisfying.

A benchmark portfolio surprisingly difficult to beat is the one created with a **naïve 1/N rule** that assigns equal weights to all the assets in all periods. Three strengths:

- immune to estimation errors (as it needs no inputs)
- does not require to perform any optimization procedure
- has a very low turnover, which translates into very low transaction costs

Benchmark portfolios

Another possible benchmark is a large **stock market index**, like the S&P 500. While it is not possible to buy an index, it is possible to buy an **ETF (Exchange-Traded Fund)**, i.e., a fund that is traded on the financial markets and which tries to replicate the index. Advantages:

- only one instrument (the ETF) needs to be bought; no need to trade all the stocks contained in the index
- over the long run stock markets provide good returns (in countries with a solid economy)
- no estimation and optimization procedures required