Portfolio Theory

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Lecture 8

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Long-short portfolios

Suppose we have N assets. Investing in a **long-short portfolio** involves:

1. Ranking the assets according to their expected return
2. Divide them in two legs:
   - a long leg with a given number of assets with the highest predicted return
   - a short leg with a given number of assets with the lowest predicted return

Typically, equal weights are used within each leg, but alternatives are of course possible

Long-short portfolios

Advantages of long-short portfolios:

- No optimization procedures required
- Completely or partially self-financing: money obtained from shorting can be used for the long positions

Disadvantages of long-short portfolios:

- N has to be relatively small, as it is difficult to short large number of stocks in practice → less diversification
- Placing a substantial amount of wealth in short positions can be risky

Mean-variance utility maximization

How to rank the assets based on their expected return?

Using the sample mean is NOT appropriate.

A factor-based approach is often used.

This is one typical way **factor investing** is performed.

It generally involves computing expected returns using a multifactor model.

Mean-variance utility maximization

Remember that the formula of a multifactor model with k factors is:

$$R_i = \alpha_i + b_{i1}f_1 + b_{i2}f_2 + \cdots + b_{ik}f_k + \varepsilon_i$$

In practice, the expected return of a stock given a certain multifactor model is computed as:

$$E[R_i] = \alpha_i + b_{i1}\gamma_1 + b_{i2}\gamma_2 + \cdots + b_{ik}f\gamma_k$$

where γ is the factor risk premium.

Fama-MacBeth regression

The loadings and the risk premia are typically estimated using the **Fama-MacBeth regression**, which is a two-stage multiple linear regression.

Consider N assets over T periods.

In the first stage, the loadings b_{ik} are estimated by regressing the returns of each asset i on the k factors, using the entire set of T periods:

$$R_{1t} = \alpha_1 + b_{11}f_{1t} + b_{12}f_{2t} + \cdots + b_{1k}f_k$$
$$R_{2t} = \alpha_2 + b_{21}f_{1t} + b_{22}f_{2t} + \cdots + b_{2k}f_k$$
$$\vdots$$
$$R_{it} = \alpha_i + b_{i1}f_{1t} + b_{i2}f_{2t} + \cdots + b_{ik}f_k$$
$$\vdots$$
$$R_{Nt} = \alpha_N + b_{N1}f_{1t} + b_{N2}f_{2t} + \cdots + b_{Nk}f_{kt}$$

Fama-MacBeth regression

In the second stage, the estimated loadings are then used as explanatory variables in a second regression that, for each period t, regresses the asset returns of the entire set of N assets:

$$R_{i1} = \gamma_{10} + \gamma_{11}\widehat{b_{i1}} + \gamma_{12}\widehat{b_{i2}} + \cdots + \gamma_{1k}\widehat{b_{ik}}$$
$$R_{i2} = \gamma_{20} + \gamma_{21}\widehat{b_{i1}} + \gamma_{22}\widehat{b_{i2}} + \cdots + \gamma_{2k}\widehat{b_{ik}}$$
$$\vdots$$
$$R_{it} = \gamma_{t0} + \gamma_{t1}\widehat{b_{i1}} + \gamma_{t2}\widehat{b_{i2}} + \cdots + \gamma_{tk}\widehat{b_{ik}}$$
$$\vdots$$
$$R_{iT} = \gamma_{T0} + \gamma_{T1}\widehat{b_{i1}} + \gamma_{T2}\widehat{b_{i2}} + \cdots + \gamma_{Tk}\widehat{b_{ik}}$$

Fama-MacBeth regression

The risk premia are time-varying. A common approach is to compute their average value over the T periods.

The expected return of each asset i according to the chosen multifactor model is given by:

$$E[R_i] = b_{i1}\gamma_1 + b_{i2}\gamma_2 + \cdots + b_{ik}\gamma_k$$

Example with the Fama-French three-factor model

For greater clarity, let us consider how this works with the Fama-French three-factor model:

$$R_i = R_f + b_{i1}\left(R_m - R_f\right) + b_{i2}SMB + b_{i3}HML$$

In practice the expected return of asset i is computed as:

$$E[R_i] = R_f + b_{i1}\gamma_{(R_m - R_f)} + b_{i2}\gamma_{SMB} + b_{i3}\gamma_{HML}$$

We use the Fama-MacBeth regression to estimate the loadings and the risk premia. Usually, the excess return is used as dependent variable, to focus on the component of the return that is dependent on factor exposure.

Example with the Fama-French three-factor model

Therefore, the first stage regression for each asset i is:

$$R_{it} - R_{ft} = \alpha_i + b_{i1}\left(R_{mt} - R_{ft}\right) + b_{i2}SMB_t + b_{i3}HML_t$$

To simplify the notation, we indicate $\left(R_{mt} - R_{ft}\right)$ as MKT:

$$R_{it} - R_{ft} = \alpha_i + b_{i1}MKT_t + b_{i2}SMB_t + b_{i3}HML_t$$

This regression needs to be carried out separately for each of the N assets.

We can now set up the second stage regression:

$$R_i - R_f = \gamma_{t0} + \gamma_{t1}\widehat{b_{i1}} + \gamma_{t2}\widehat{b_{i2}} + \gamma_{t3}\widehat{b_{i3}}$$

This regression needs to be carried out separately for each of the T periods in the estimation window.

Example with the Fama-French three-factor model

The second regression gave us T values for γ_{t1}, γ_{t2} and γ_{t3}.

We compute their average in order to have a single value. We rename the average of γ_{t1}, γ_{t2} and γ_{t3} as γ_{MKT}, γ_{SMB} and γ_{HML} respectively, for better clarity.

We also compute the average risk-free rate in order to have a single value for R_f.

We can now compute the expected return of each asset i:

$$E[R_i] = R_f + b_{i1}\gamma_{MKT} + b_{i2}\gamma_{SMB} + b_{i3}\gamma_{HML}$$

Example with the Fama-French three-factor model

It is now straightforward to create the long-short portfolio:

1. Rank the assets according to their expected return and take a long position on those positioned in the upper part of the ranking, and a short position on those in lower part of the ranking.
2. Each time the portfolio has to be updated, compute new estimates of the expected returns. For example, if we want to update the portfolio monthly, we ne need to repeat the procedure every month, using the up-to-date data.