Derivatives II.

Option Pricing

Option price = Premium

- □ Intrinsic Value
 - Economic value of the option if it is execised immediately
- □ Time Value
 - Amount by which the premium exceeds its intrinsic value
 - "price of hope"

Intrinsic Value

- Difference between current price of underlying asset and strike price
- Maximum
 - Strike price
- Minimum
 - Zero
- "in the money"
- "out of money"

Time Value

Time Value = Option Premium – Intrinsic value

- The amount an investor is willing to pay for an option above its intrinsic value, in the hope that at some time prior to expiration its value will increase because of a favorable change in the price of the underlying security.
- The longer the amount of time for market conditions to work to an investor's benefit, the greater the time value.

Time value deterioration



Option Premium



Factors influence option prices

- Current price of underlying asset
- Strike Price
- Time to expiration
- Expected price volatility
- Short-term risk free rate
- Anticipated cash payment on the underlying asset

Current price of underlying asset

Call option

- Price o underlying asset increases = call option premium increases
- Put option
 - Price o underlying asset increases = put option premium decreases

Strike price

- The lower the strike price, the higher call option premium
- The lower the strike price, the lower put option premium

All other factors constant

Strike Price – Call





Share Price

Time to Expiration

Option as a "wasting asset"
 After expiration date has no value
 The longer the time to expiration, the greater the option premium

All other factors constant

Short term risk free rates

The higher short-term risk free rate, the greater call option premium

Anticipated cash payments of underlying assets

- Cash payments cause decrease of underlying asset price (dividends)
- Cash payment = call option premium decrease
- Cash payment = put option premium increase

Expected Price Volatility

The greater expected volatility of underlying asset, the higher option premium

Long Call



Short Call



Share Price

Short Put



Long Put



Underlying asset / Forward



Binomial Option Pricing Model

- An options valuation method developed by Cox in 1979
- The binomial option pricing model uses an iterative procedure, allowing for the specification of nodes, or points in time, during the time span between the valuation date and the option's expiration date



Beginning Value

Month End value





Beginning Value

Year End value

BOPM

Hedged Portfolio Long position in a certain amount of the asset Short call position in the underlying asset

Fully hedged portfolio

- Cost of Hedged Portfolio
 (investment made)
 - \Box H*S C
 - H Hedge ratio
 - S Current asset price
 - C current call option premium



• uHS - $C_u = dHS - C_d$

- u 1 plus percentage change in the asset price if the price goes up
- d 1 plus percentage change in the asset price if the price goes down
- \blacksquare C_u intrinsic value of call option if the price goes up
- \blacksquare C_d intrinsic value of call option if the price goes down

Hedge Ratio

$$uHS - C_u = dHS - C_d$$
$$H = (C_u - C_d) / ((u-d)S)$$

$$C_u = [Max 0, uS - E]$$

 $C_d = [Max 0, dS - E]$

Riskless Portfolio

Riskless portfolio = Riskles return

$$(1+r)(HS - C) = uHS - C_u$$

Substituing equatation for hedge ratio

$$C = \left(\frac{1+r-d}{u-d}\right) \frac{C_u}{\left(1+r\right)} + \left(\frac{u-1-r}{u-d}\right) \frac{C_d}{\left(1+r\right)}$$

Two period BOPM



Volatility

The up and down factors are calculated using the underlying volatility, σ and the time duration of a step, t, measured in years (using the day count convention of the underlying instrument).

$$u = e^{\sigma\sqrt{t}}$$

$$d = e^{-\sigma\sqrt{t}} = \frac{1}{u}.$$

Black – Scholes Formula

Fischer Black and Myron ScholesPublished in 1973

The key assumptions of the Black– Scholes model

The price of the underlying instrument St follows a geometric Brownian motion with constant drift μ and volatility σ :

$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t$

- It is possible to short sell the underlying stock.
- There are no arbitrage opportunities.
- Trading in the stock is continuous.
- There are no transaction costs or taxes.
- All securities are perfectly divisible (e.g. it is possible to buy 1/100th of a share).
- It is possible to borrow and lend cash at a constant risk-free interest rate

Black – Scholes Formula

$$C_0 = S_0 N(d_1) - X e^{-r_T T} N(d_2) \qquad \qquad d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + r_T T + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

S = stock price

X = strike price

t = time remaining until expiration, expressed as a percent of a year

r = current continuously compounded risk-free interest rate

v = annual volatility of stock price (the standard deviation of the short-term returns over one year

In = natural logarithm

N(x) = standard normal cumulative distribution function

e = the exponential function

Put-Call parity

Relationship between the price of a call, and the price of a put
On the same underlying asset
With the same strike price
With the same expiration date

Put-Call parity

- A portfolio comprising a call option and an amount x of cash equal to the present value of the option's strike price has the same expiration value as a portfolio comprising the corresponding put option and the underlier.
- For European options, early exercise is not possible.
- If the expiration values of the two portfolios are the same, then their present values must also be the same.
- **1**969

Put-call parity


Put-Call parity

C + PV(x) = P + S

c = the current market value of the call;

- PV(x) = the present value of the strike price x, discounted from the expiration date at a suitable risk free rate;
- PV (cash) = the present value of cash distribution
- P = the current market value of the put;

S = the current market value of the underlying asset.

C = P + S - PV(x)

 A call is equivalent to a purchase of stock and a put financed by borrowing the PV(K)

Put-call Parity

- European options only
- Ignore time value of money
- It is not based on any option pricing model ;derived purely using arbitrage arguments
- Simple test of option pricing models

Bullish Option Trading Strategies

- Long Call
- Short Put
- Long Synthetic
- Call Backspread
- Call Bull Spread
- Put Bull Spread
- Covered Call
- Protective Put

Long Call

- A long call is simply the purchase of one call option.
- Maximum Loss: Limited to the premium paid up front for the option.
- Maximum Gain: Unlimited as the market rallies.
- When to use: When you are bullish on market direction and also bullish on market volatility.
- A long call option is the simplest way to benefit if you believe that the market will make an upward move and is the most common choice among first time investors.
- Being long a call option means that you will benefit if the stock/future rallies, however, you risk is limited on the downside if the market makes a correction.

Long Call



Short put

- A short put is simply the sale of a put option.
- Maximum Loss: Unlimited in a falling market.
- Maximum Gain: Limited to the premium received for selling the put option.
- When to use: When you are bullish on market direction and bearish on market volatility.
- Like the Short Call Option, selling naked puts can be a very risky strategy as your losses are unlimited in a falling market.

Short Put



Long Synthetic

- Buy one call option and sell one put option at the same strike price.
- Maximum Loss:Unlimited.
- Maximum Gain: Unlimited.
- Long Synthetic behaves exactly the same as being long the underlying security. You can use long synthetic's when you want the same payoff characteristics as holding a stock or futures contract. It has the benefit of being much cheaper than buying stock outright.

Long Synthetic



Call Backspread

- Short one ITM call option and long two OTM call options.
- Maximum Loss: Limited to the net premium.
- Maximum Gain: Unlimited on the upside and limited on the downside.
- Similar to a Short Straddle except the loss on the downside is limited.
- When to use: When you are bullish on volatility and bullish on market price. Note though, that you profit when prices fall, although the gains are greater if the market rallies.
- A Backspread looks a lot like a Long Straddle except the payoff flattens out on the downside. The other key difference is that Backspreads are usually done at a credit. That is, the net difference for both legs means that you receive money into your account up front instead of paying (debit) for the spread.
- Even though the payoff looks like a "long" type position, it is often referred to as a "short" strategy. Generally it is like this: if you receive money for the position up front it is called a "Short" position and when you pay for a position it is called being "Long".

Call Backspread



Call Bull Spread

- Long call option with a low strike price and short call option with a higher strike price.
- Maximum Loss: Limited to premium paid for the long option minus the premium received for the short option.
- Maximum Gain: Limited to the difference between the two strike prices minus the net premium paid for the spread.
- Call bull spread can only be worth as much as the difference between the two strike prices. So, when putting on a bull spread remember that the wider the strikes the more you can make. But the downside to this is that you will end up paying more for the spread. So, the deeper in the money calls you buy relative to the call options that you sell means a greater maximum loss if the market sells off.
- Call bull spread is a very cost effective way to take a position when you are bullish on market direction. The cost of the bought call option will be partially offset by the premium received by the sold call option. This does, however, limit your potential gain if the market does rally but also reduces the cost of entering into this position.
- This type of strategy is suited to investors who want to go long on market direction and also have an upside target in mind. The sold call acts as a profit target for the position. So, if the trader sees a short term move in an underlying but doesn't see the market going past \$X, then a bull spread is ideal.

Call Bull Spread



Put Bull Spread

 Long one put option and short another put option with a higher strike price.

- Maximum Loss: Limited to the difference between the two strike prices minus the net premium received for the position.
- Maximum Gain: Limited to the net credit received for the spread. I.e. the premium received for the short option less the premium paid for the long option.
- A Put Bull Spread has the same payoff as the Call Bull Spread except the contracts used are put options instead of call options. Even though bullish, a trader may decide to place a put spread instead of a call spread because the risk/reward profile may be more favourable. This may be the if the ITM call options have a higher implied volatility than the OTM put options. In this case, a call spread would be more expensive to initiate and hence the trader might prefer the lower cost option of a put spread.

Put Bull Spread



Covered Call

- Long the underlying asset and short call options.
- Maximum Loss: Unlimited on the downside.
- Maximum Gain: Limited to the premium received from the sold call option.
- When to use: When you own the underlying stock (or futures contract) and wish to lock in profits.
- This strategy is used by many investors who hold stock. It is also used by many large funds as a method of generating consistent income from the sold options.
- The idea behind a Covered Call (also called Covered Write) is to hold stock over a long period of time and every month or so sell out-of-the-money call options.
- Even though the payoff diagram shows an unlimited loss potential, you must remember that many investors implementing this type of strategy have bought the stock long ago and hence the call option's strike price may be a long way from the purchase price of the stock.
- For example, say you bought IBM last year at \$25 and today it is trading at \$40. You might decide write a \$45 call option. Even if the market sells off temporarily it will have a long way to go before you start seeing losses on the underlying. Meanwhile, the call option expires worthless and you pocket the premium received from the spread.

Covered Call



Protective Put

- Long the underlying asset and long put options.
- Maximum Loss: Limited to the premium paid for the put option.
- Maximum Gain: Unlimited as the market rallies.
- When to use: When you are long stock and want to protect yourself against a market correction.
- A Protective Put strategy has a very similar pay off profile to the Long Call. You
 maximum loss is limited to the premium paid for the option and you have an unlimited
 profit potential.
- Protective Puts are ideal for investors whom are very risk averse, i.e. they hold stock and are concerned about a stock market correction. So, if the market does sell off rapidly, the value of the put options that the trader holds will increase while the value of the stock will decrease. If the combined position is hedged then the profits of the put options will offset the losses of the stock and all the investor will loose will be the premium paid.
- However, if the market rises substantially past the exercise price of the put options, then the puts will expire worthless while the stock position increases. But, the loss of the put position is limited, while the profits gained from the increase in the stock position are unlimited. So, in this case the losses of the put option and the gains form the stock do not offset each other: the profits gained from the increase in the underlying out weight the loss sustained from the put option premium.

Protective Put



Bearish Option Trading Strategies

- Short Call
- Long Put
- Short Synthetic
- Put Backspread
- Call Bear Spread
- Put Bear Spread

Short Call

- A short call is simply the sale of one call option. Selling options is also known as "writing" an option.
- Maximum Loss: Unlimited as the market rises.
- Maximum Gain: Limited to the premium received for selling the option.
- When to use: When you are bearish on market direction and also bearish on market volatility.
- A short is also known as a Naked Call. Naked calls are considered very risky positions because your risk is unlimited.

Short Call



Long Put

- A long put is simply the purchase of one put option.
- Maximum Loss: Limited to the net premium paid for the option.
- Maximum Gain: Unlimited as the market sells off.
- When to use: When you are bearish on market direction and bullish on market volatility.
- Like the long call a long put is a nice simple way to take a position on market direction without risking everything. Except with a put option you want the market to decrease in value.
- Buying put options is a fantastic way to profit from a down turning market without shorting stock. Even though both methods will make money if the market sells off, buying put options can do this with limited risk.

Long Put



Short Synthetic



- Short one call option and long one put option at the same strike price.
- Maximum Loss:Unlimited.
- Maximum Gain: Unlimited.
- When to use: When you are bearish on market direction.
- A Short Synthetic is just the reverse of the Long Synthetic i.e. this option combination behaves exactly the same way as being short the underlying security. So, if you are very bearish on an asset and want the same characteristics as if you were short the asset then you might want to consider using a Short Synthetic.

Put Backspread



- Long two OTM put options and short one ITM put optic
- Maximum Loss: Limited to the difference between the two strikes less the premium received for the spread.
- Maximum Gain: Limited on the upside to the net premium received for the spread. Unlimited on the downside.
- When to use: When you are bearish on market direction and bullish on volatility.
- This strategy could also be referred to as a Short Put Backspread,
- A Put Backspread should be done as a credit. This means that after you buy 2 OTM puts and sell 1 ITM put the net effect should be a credit to you. I.e. you should receive money for this spread as your are short more than you are long.
- Put Backspread's are a great strategy if you are bullish and bearish at the same time, however, have a bias to the downside. Looking from the payoff, you can see that if the market sells off you make unlimited profits below the break even point. If, however, you are wrong about the direction and the market stages a rally instead, you still win though your profits are limited.
- You might say that this type of strategy is similar to a Long Straddle and you would be right. The difference is that 1) the profits are limited on one side and 2) Backspread's are cheaper to put on.

Call Bear Spread



- Short one call option with a low strike price and long one call option with a higher strike price.
- Maximum Loss: Limited to the difference between the two strikes minus the net premium.
- Maximum Gain: Limited to the net premium received for the position. I.e. the premium received for the short call minus the premium paid for the long call.
- When to use: When you are mildly bearish on market direction.
- A call bear spread is usually a credit spread. A credit spread is where the net cost of the position results in you receiving money up front for the trade. I.e. you sell one call option (receive \$5) and the buy one call option (\$4). The net effect is a credit of \$1.
- This type of spread is used when you are mildly bearish on market direction. Same idea as the Call Bull Spread but reversed i.e. you think the market will go down but think that the cost of a short stock or long put is too expensive.

Put Bear Spread



- Short one put option at a lower strike price and long one put option at a higher strike price.
- Maximum Loss: Limited to the net amount paid for the spread. I.e. the premium paid for the long position less the premium received for the short position.
- Maximum Gain: Limited to the difference between the two strike prices minus the net paid for the position.
- When to use: When you are bearish on market direction.
- A Put Bear Spread has the same payoff as the Call Bear Spread as both strategies hope for a decrease in market prices. The choice as to which spread to use, however, comes down to risk/reward.
- A good tip is to compare the market prices of both spreads to determine which has the better payoff for you.

Neutral Option Trading Strategies

- Long Straddle
- Short Straddle
- Long Strangle
- Short Strangle
- Call Time Spread
- Put Time Spread
- Call Ratio Vertical Spread
- Put Ratio Vertical Spread
- Long Call Butterfly
- Short Call Butterfly
- Long Put Butterfly
- Short Put Butterfly

Long Straddle

- Buy one call option and buy one put option at the same strike price.
- Maximum Loss: Limited to the total premium paid for the call and put options.
- Maximum Gain: Unlimited as the market moves in either direction.
- When to use: When you are bullish on volatility but are unsure of market direction.
- A long straddle is an excellent strategy to use when you think the market is going to move but don't know which way. A long straddle is like placing an each-way bet on price action: you make money if the market goes up or down.
- But, the market must move enough in either direction to cover the cost of buying both options.
- Buying straddles is best when implied volatility is low or you expect the market to make a substantial move before the expiration date for example, before an earnings announcement.

Long Straddle



Short Straddle



- Short one call option and short one put option at the same strike price.
- Maximum Loss: Unlimited as the market moves in either direction.
- Maximum Gain: Limited to the net premium received for selling the options.
- When to use: When you are bearish on volatility and think market prices will remain stable.
- Short straddles are a great way to take advantage of time decay and also if you think the market price will trade sideways over the life of the option.

Long Strangle

- Buy one call option with a lower strike price and buy one put option at a higher strike price.
- Maximum Loss: Limited to the total premium paid for the call and put options.
- Maximum Gain: Unlimited as the market moves in either direction.
- When to use: When you are bullish on volatility but are unsure of market direction.
- A long strangle is similar to a straddle except the strike prices are further apart, which lowers the cost of putting on the spread but also widens the gap needed for the market to rise/fall beyond in order to be profitable.
- Like long straddles, buying strangles is best when implied volatility is low or you expect a large movement of market price in either direction.

Long Strangle



Short Strangle



- Short one call option with a lower strike price and short one put option with a higher strike price.
- Maximum Loss: Unlimited as the market moves in either direction.
- Maximum Gain: Limited to the net premium received for selling the options.
- When to use: When you are bearish on volatility and think market prices will remain stable.
- A short strangle is similar to the Short Straddle except the strike prices are further apart, which lowers the premium received but also increases the chance of a profitable trade.

Call Time Spread



- Short one front month call option and long one far month call option. (i.e. the option you sell is to be closer to expiration than the option you are buying).
- Maximum Loss: Limited on both down and upside for market direction.
- Maximum Gain: Limited.
- When to use: When you are bearish on volatility and neutral to bearish on market price.
- Note that with this payoff graph I have shown the net theoretical result only at the first expiration date when with the underlying trading at 100, which is the best result: the near month call will expire worthless and you will still have a long call ATM position.
- Traders use time spreads to take advantage of time decay the property of options being a decaying asset. However, due to the risk involved in selling naked options, a time spread protects the position buy buying an option in the next month.
- The long back month option position offsets large losses that can result from being short options when the underlying market moves unfavourably.
- It is best to implement a time spread when there is < 30 days to expiration in the front month. That is for the short side i.e. selling an option with 30 days or less to expiration.
Put Time Spread



- Short one front month put option and long one far month put option. (i.e. the option you sell is to be closer to expiry than the option you are buying).
- Maximum Loss: Limited.
- Maximum Gain: Limited.
- When to use: When you are bearish on volatility and neutral to bearish on market price.
- Note that with the payoff graph above I have shown the net theoretical result only at the first expiration date when with the underlying trading at 70, which is the best result: the near month call will expire worthless and you will still have a long AT put position.
- A put time spread is similar to Call Time Spread except that you want the market to decrease rather than increase. So, a put time spread is used to take advantage of time decay. However, due to the risk involved in selling naked options, a time spread protects the position buy buying an option in the next month.
- It is best to implement a time spread when there is < 30 days to expiration in the front month. Also, look to sell options that are out-of-the-money.

Call Ratio Vertical Spread



- Long one ITM call option and short two OTM call options.
- Maximum Loss: Unlimited on the upside and limited on the downside.
- Maximum Gain: Limited to the premium received.
- When to use: When you are bearish on volatility and neutral on market direction.
- Even though a Call Ratio Vertical Spread is the reverse of a Call Backspread, it is generally not referred to as being short a Call Backspread as a Call Ratio Spread requires up front payment and is hence a long strategy.
- You will notice that it is very similar to a Short Strangle except the risk is limited on the downside.

Put Ratio Vertical Spread



- Short two OTM put options and long one ITM put option.
- Maximum Loss: Unlimited on the downside and limited to the net premium paid on the upside.
- Maximum Gain: The difference between the two strike prices less the premium paid for the position.
- When to use: When you are neutral on market direction and bearish on volatility.

Long Call Butterfly



- Short two ATM call options, long one ITM call option and long one OTM call option.
- Maximum Loss: Limited to the ATM strike less the ITM strike less the net premium paid for the spread.
- Maximum Gain: Limited to the net premium received from the spread.
- When to use: When you are neutral on market direction and bearish on volatility.
- A long butterfly is similar to a short straddle except your losses are limited. This means that you make money when the market remains flat over the life of the options.
- You might be thinking that it looks like a "short" strategy because of the similarity to the short straddle. You are right in thinking that they have similar characteristics, however, the difference between a Long Butterfly and a Short Straddle is the premium - a Long Butterfly will cost you money (or premium) to establish whereas a Short Straddle won't cost you anything as you receive money (premium) up front for putting on the position.

Short Call Butterfly



- Long two ATM call options, short one ITM call option and short one OTM call option.
- Maximum Loss: Limited to the net difference between the ATM strike less the ITM strike less the premium received for the position.
- Maximum Gain: Limited to the net premium received for the option spread.
- When to use: When you are neutral on market direction and bullish on volatility. Neutral on market direction meaning that you want the market to move in either direction - i.e. bullish and bearish at the same time.
- Short Call Butterfly's have a similar pay off to the Short Straddle except the downside risk is limited. Short Straddles have unlimited downside risk: a Short Butterfly's risk is limited to the premium paid for the three options.

Long Put Butterfly



- Sell two ATM put options, buy one ITM put option and buy one OTM put option.
- Maximum Loss: Limited to the ATM strike less the ITM strike less the net premium paid for the spread.
- Maximum Gain: Limited to the net premium received from the spread.
- When to use: When you are neutral on market direction and bearish on volatility.
- This strategy is the same as the Long Call Butterfly except we use put options instead of call options.
- A Long Put Butterfly is used with similar intentions to the Short Straddle except your losses are limited if the market moves out of your favour. Whereas a Short Straddle has unlimited losses if the market moves.

Short Put Butterfly



- Long two ATM put options, short one ITM put option and short one OTM put option.
- Maximum Loss: Limited to the net difference between the ATM strike less the ITM strike less the premium received for the position.
- Maximum Gain: Limited to the net premium received for the option spread.
- When to use: When you are bullish or bearish on market direction and bullish on volatility.
- Short put butterfly's have the same characteristics as the Short Call Butterfly the only difference is that we use put options instead of call options.
- Short butterfly's are an excellent strategy if you expect the market to move, however, you are unsure about what direction the market will move. For example, say there is an announcement due regarding earnings or a Government figure to be released. You might be nervous about market activity and expecting a large move in either direction.
- In these types of situations you might want to consider implementing a short butterfly strategy - even though your profits are limited they are inexpensive to establish therefore giving you a higher return on investment.

"Greeks"

- Delta
- Gamma
- Theta
- Vega
- Rho
- These terms refer to calculations made using an option pricing model and tell an option trader how the price of an option will change given certain market variables.

Delta

- The sensitivity of an option price relative to changes in the price of the underlying asset.
- It tells option traders how fast the price of the option will change as the underlying stock/future moves.
- Option delta is represented as the price change given a 1 point move in the underlying asset and is usually displayed as a decimal value.
- Delta values range between 0 and 1 for call options and -1 to 0 for put options.

Delta



Call Delta

1.

Call Delta Vs Underlying Price



Put Delta

Put Delta Vs Underlying Price



Gamma

- Indicates how the delta of an option will change relative to a 1 point move in the underlying asset.
- The Gamma shows the option delta's sensitivity to market price changes.
- Gamma shows how volatile an option is relative to movements in the underlying asset

Gamma vs Underlying Price





- Gamma increases as the option moves from being in-the-money reaching its peak when the option is at-the-money.
- As the option moves out-of-the-money the Gamma then decreases.

Vega

- The Vega of an option indicates how much the price of the option will change as the volatility of the underlying asset changes.
- Vega is quoted to show the theoretical price change for every 1 percentage point change in volatility.
- For example, if the theoretical price is 2.5 and the Vega is showing 0.25, then if the volatility moves from 20% to 21% the theoretical price will increase to 2.75.
- Vega is most sensitive when the option is at-the-money and tapers off either side as the market trades above/below the strike.



Vega vs Underlying Price



Theta

- Theta shows how much value the option price will lose for every day that passes
- Theta is the calcuation that shows how much of this time value is eroding as each trading day passes - assuming all other inputs remain unchanged.
- Because of this negative impact on an option price, the Theta will always be a negative number.
- For example, say an option has a theoretical price of 3.50 and is showing a Theta value of -0.20. Tomorrow, if the underlying market opens unchanged (opens at the same price as the previous days close) then the theoretical value of the option will now be worth 3.30 (3.50 - 0.20).

Theta



Rho

- measures the sensitivity of the option value to the interest rate.
- It is the derivative of the option value with respect to the interest rate.
- The higher the interest rate, the greater the time value of the option.
- Rho is positive for calls and negative for puts.
- For both calls and puts, the longer the time to expiration, the larger is the effect of the interest rate on the option value.