SEMINAR 7

Econ 4325

Problem 1

Assume that output is given by

$$1.1 : y_t = \gamma\left(\pi_t - E_{t-1}\pi_t\right) + u_t$$

where $E_{t-1}u_t = 0$.

Consider two alternative specifications of the preferences of the monetary authorities:

$$\begin{aligned}
1.2 &: \quad L_t = \frac{1}{2}\left[\left(\pi_t - \pi^*\right)^2 + \lambda\left(y_t - y^*\right)^2\right] \\
1.3 &: \quad L_t = \frac{1}{2}\left(\pi_t - \pi^*\right)^2 - \lambda y_t
\end{aligned}$$

1. **Given an economic interpretation of the difference between the two specifications.**
2. **Derive the solution for inflation and output under a discretionary policy the two loss functions. Compare and discuss the result.**

1. The two loss functions are similar with regard to inflation. They differ in how they regard output. The formulation in (1.2) implies that there is an optimal level of output. The quadratic formulation ensures that the marginal disutility increases in the distance from the target. Hence, output may be too high. The formulation in (1.3) implies that the government wants output to be as high as possible. The marginal utility (reduction in marginal disutility) from output is constant and equal to λ.

2. First we find the preferred monetary policy from the loss function. The government minimize the loss with respect to inflation.

(1.2)

$$\begin{aligned}
& \min_{\pi} \frac{1}{2}\left[\left(\pi_t - \pi^*\right)^2 + \lambda\left(y_t - y^*\right)^2\right] \\
s.t. \quad &: \quad y_t = \gamma\left(\pi_t - E_{t-1}\pi_t\right) + u_t \\
FOC \quad &: \quad \left(\pi_t - \pi^*\right) + \lambda\gamma\left(y_t - y^*\right) = 0
\end{aligned}$$

Solve this for π_t

$$\pi_t = \pi^* + \lambda\gamma y^* - \lambda\gamma y_t$$

Insert for y_t

$$\pi_t = \pi^* + \lambda\gamma y^* - \lambda\gamma\left[\gamma\left(\pi_t - E_{t-1}\pi_t\right) + u_t\right]$$

Collect terms

$$\pi_t\left(1 + \lambda\gamma^2\right) = \pi^* + \lambda\gamma y^* + \lambda\gamma^2 E_{t-1}\pi_t - \lambda\gamma u_t$$

We can find expected inflation from the FOC

$$\begin{aligned}
\pi_t &= \pi^* + \lambda\gamma y^* - \lambda\gamma y_t \\
E_{t-1}\pi_t &= E_{t-1}\pi^* + E_{t-1}\lambda\gamma y^* - E_{t-1}\lambda\gamma y_t \\
&= \pi^* + \lambda\gamma y^* - \lambda\gamma E_{t-1}\left[\gamma\left(\pi_t - E_{t-1}\pi_t\right) + u_t\right] \\
&= \pi^* + \lambda\gamma y^*
\end{aligned}$$

Insert above and solve for π_t

$$\begin{aligned}
\pi_t\left(1+\lambda\gamma^2\right) &= \pi^*+\lambda\gamma y^*+\lambda\gamma^2\left[\pi^*+\lambda\gamma y^*\right]-\lambda\gamma u_t \\
&= \pi^*\left(1+\lambda\gamma^2\right)+y^*\left(\lambda\gamma+\lambda^2\gamma^3\right)-\lambda\gamma u_t \\
\pi_t &= \pi^*+\frac{\lambda\gamma+\lambda^2\gamma^3}{1+\lambda\gamma^2}y^*-\frac{\lambda\gamma}{1+\lambda\gamma^2}u_t \\
&= \pi^*+\gamma\lambda y^*-\frac{\lambda\gamma}{1+\lambda\gamma^2}u_t
\end{aligned}$$

To find y_t we do the following: Start with the expression for y_t

$$y_t=\gamma\left(\pi_t-E_{t-1}\pi_t\right)+u_t$$

Insert for π_t

$$y_t=\gamma\left(\pi^*+\gamma\lambda y^*-\frac{\lambda\gamma}{1+\lambda\gamma^2}u_t-E_{t-1}\pi_t\right)+u_t$$

Insert for expected inflation $E_{t-1}\pi_t=\pi^*+\lambda\gamma y^*$

$$y_t=\left(1+\frac{\lambda\gamma^2}{1+\lambda\gamma^2}\right)u_t$$

Collect terms

$$y_t=\frac{1}{1+\lambda\gamma^2}u_t$$

(1.3)

$$\begin{aligned}
&\min_{\pi}\frac{1}{2}\left(\pi_t-\pi^*\right)^2-\lambda y_t \\
s.t. &: \quad y_t=\gamma\left(\pi_t-E_{t-1}\pi_t\right)+u_t \\
FOC &: \quad \left(\pi_t-\pi^*\right)-\lambda\gamma=0
\end{aligned}$$

Solve for π_t

$$\pi_t=\pi^*+\lambda\gamma$$

Insert into the expression for y_t

$$y_t=\gamma\left(\pi^*+\lambda\gamma-E_{t-1}\pi_t\right)+u_t$$

Note that from the FOC, the expected inflation is given by $\pi^*+\lambda\gamma$

$$y_t=u_t$$

Comparison and comment

There is an inflation bias under both loss functions. Expected inflation is the same for $y^*=1$. For (1.2) inflation is volatile. For (1.3) inflation is not. Mirroring this we see that output is more volatile under (1.3) than under (1.2). The reason is that under (1.2) the government wants to balance the effects of the shock on both inflation and output.

2. The expected loss under discretion is given by:

$$\begin{aligned}
EL_t &= \frac{1}{2}E\left[\left(\pi_t-\pi^*\right)^2+\lambda\left(y_t-y^*\right)^2\right] \\
&= \frac{1}{2}E\left[\left(\pi^*+\gamma\lambda y^*-\frac{\lambda\gamma}{1+\lambda\gamma^2}u_t-\pi^*\right)^2+\lambda\left(\frac{1}{1+\lambda\gamma^2}u_t-y^*\right)^2\right] \\
&= \frac{1}{2}E\left[\left(\gamma\lambda y^*-\frac{\lambda\gamma}{1+\lambda\gamma^2}u_t\right)^2+\lambda\left(\frac{1}{1+\lambda\gamma^2}u_t-y^*\right)^2\right] \\
&= \frac{1}{2}E\left[\begin{array}{l}\left(\gamma\lambda y^*\right)^2-2\frac{\lambda^2\gamma^2}{1+\lambda\gamma^2}y^*u_t+\left(\frac{\lambda\gamma}{1+\lambda\gamma^2}u_t\right)^2+ \\ \lambda\left(\frac{1}{1+\lambda\gamma^2}u_t\right)^2+2\lambda\frac{1}{1+\lambda\gamma^2}u_ty^*+\lambda\left(y^*\right)^2\end{array}\right]
\end{aligned}$$

From $cov\left(u, y^*\right) = 0$

$$\begin{aligned} EL_t &= \frac{1}{2}\left[\begin{array}{c} (\gamma\lambda y^*)^2 + E\left(\frac{\lambda\gamma}{1+\lambda\gamma^2}u_t\right)^2 + \\ \lambda E\left(\frac{1}{1+\lambda\gamma^2}u_t\right)^2 + \lambda\left(y^*\right)^2 \end{array}\right] \\ &= \frac{1}{2}\left[(\gamma\lambda y^*)^2 + \left(\frac{\lambda\gamma}{1+\lambda\gamma^2}\right)^2\sigma^2 + \lambda\left(\frac{1}{1+\lambda\gamma^2}\right)^2\sigma^2 + \lambda\left(y^*\right)^2\right] \end{aligned}$$

The expected loss under strict inflation targeting is given by:

$$\begin{aligned} EL_t &= \frac{1}{2}E\left[(\pi_t - \pi^*)^2 + \lambda\left(y_t - y^*\right)^2\right] \\ &= \frac{1}{2}\lambda E\left[(u_t - y^*)^2\right] \\ &= \frac{1}{2}\lambda\left[\sigma^2 + (y^*)^2\right] \end{aligned}$$

We can try to find the value of y^* that makes the discretionary the preferred policy

$$\begin{aligned} \frac{1}{2}\left[(\gamma\lambda y^*)^2 + \left(\frac{\lambda\gamma}{1+\lambda\gamma^2}\right)^2\sigma^2 + \lambda\left(\frac{1}{1+\lambda\gamma^2}\right)^2\sigma^2 + \lambda\left(y^*\right)^2\right] &< \frac{1}{2}\lambda\left[\sigma^2 + (y^*)^2\right] \\ \lambda\left(\gamma y^*\right)^2 + \lambda\left(\frac{\gamma}{1+\lambda\gamma^2}\right)^2\sigma^2 + \left(\frac{1}{1+\lambda\gamma^2}\right)^2\sigma^2 &< \sigma^2 \\ \lambda\left(\gamma y^*\right)^2 &< \sigma^2\left[1 - \lambda\left(\frac{\gamma}{1+\lambda\gamma^2}\right)^2 - \left(\frac{1}{1+\lambda\gamma^2}\right)^2\right] \end{aligned}$$

$$\begin{aligned} & 1 - \lambda\left(\frac{\gamma}{1+\lambda\gamma^2}\right)^2 - \left(\frac{1}{1+\lambda\gamma^2}\right)^2 \\ =\ & \frac{\left(1+\lambda\gamma^2\right)^2}{\left(1+\lambda\gamma^2\right)^2} - \lambda\frac{\gamma^2}{\left(1+\lambda\gamma^2\right)^2} - \frac{1}{\left(1+\lambda\gamma^2\right)^2} \\ =\ & \frac{1 + 2\lambda\gamma^2 + \lambda^2\gamma^4 - \lambda\gamma^2 - 1}{\left(1+\lambda\gamma^2\right)^2} \\ =\ & \frac{\lambda\gamma^2 + \lambda^2\gamma^4}{\left(1+\lambda\gamma^2\right)^2} \\ =\ & \frac{\lambda\gamma^2 + \lambda^2\gamma^4}{\left(1+\lambda\gamma^2\right)\left(1+\lambda\gamma^2\right)} \\ =\ & \frac{\lambda\gamma^2}{\left(1+\lambda\gamma^2\right)} \end{aligned}$$

$$\begin{aligned} \lambda\left(\gamma y^*\right)^2 &< \sigma^2\frac{\lambda\gamma^2}{\left(1+\lambda\gamma^2\right)} \\ \left(y^*\right)^2 &< \frac{\sigma^2}{\left(1+\lambda\gamma^2\right)} \\ y^* &< \sqrt{\frac{\sigma^2}{\left(1+\lambda\gamma^2\right)}} = \frac{\sigma}{\sqrt{\left(1+\lambda\gamma^2\right)}} \end{aligned}$$

If the output target is higher than this, strict inflation targeting is better.

Interpretation:

If the output target is very ambitious (high), the inflation bias is large. Then strict inflation targeting is better because the cost in terms of too little stabilization of shocks is more than offset by the gain in terms of removing the inflation bias. If the output target is less ambitious, the cost of a discretionary policy is smaller.

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