



Dirk Kaiser

Treasury Management

Lessons in Finance and Investment at

Masarykova univerzita

Ekonomicko-správní fakulta

Fall Term 2008/2009





Prof. Dr. Dirk Kaiser

Short CV

1962	Born in Krefeld , Germany
1983 – 1989	UNIVERSITY OF BONN (studies in economics, diploma – the equivalent being a master’s degree)
1989 – 1994	UNIVERSITY FOR DISTANCE TEACHING, HAGEN (assistant at the chair of banking & finance; dissertation, doctorate – the equivalent being a „Ph. D.“)
1994 – 1996	Participations department of a bank in Düsseldorf
1996	Visiting lecturer at PORTLAND STATE UNIVERSITY, Oregon, USA
1996 – 1999	Director of the participations department of a tour operator in Düsseldorf
1999 – 2002	Director of the central mandate services department of a bank in Düsseldorf



since
2002

Professor for financial management, banks and insurances at
HOCHSCHULE BOCHUM (BOCHUM UNIVERSITY OF APPLIED SCIENCES)
[HTTP://WWW.HOCHSCHULE-BOCHUM.DE/FBW/PERSONEN/KAISER.HTML](http://www.hochschule-bochum.de/fbw/personen/kaiser.html)



Prof. Dr. Dirk Kaiser

Fall term 2008/2009
Consultancy hours

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3) +49 (234) 32-10604

Treasury Management

Recommended Reading and Further References

(recommended reading)

Brealey, R. A. / Myers, S. C. / Allen, F.

Principals of Corporate Finance, 8th Edition, New York: McGraw-Hill/Irwin (2006).

Bodie, Z. / Kane, A. / Marcus, A. J.

Essentials of Investments, 5th Edition. New York: McGraw-Hill/Irwin (2004).

(further references in English)

Copeland, T. E. / Weston, J. F. / Shastri, K.

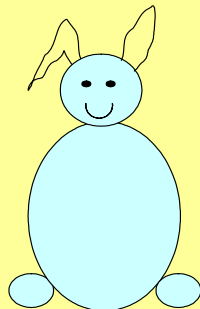
Financial Theory and Corporate Policy, 4th edition, Boston et al.: Pearson/Addison-Wesley (2005)

Feibel, B. J.

Investment Performance Measurement, New York: Wiley (2003).

(only for participants who are interested in an additional German textbook)

Kaiser, D.



Treasury Management. Betriebswirtschaftliche Grundlagen der Finanzierung und Investition, Wiesbaden: Gabler (2008).

Trilingual index

English–German–Czech index of key terms in finance

English	German	Czech
ability to pay	Zahlungsfähigkeit	platební schopnost



Any volunteers?

Treasury Management

Slides

Treasury Management
Recommended Reading and Further References

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<i>Brealey, R. A. / Myers, S. C. / Allen, F.</i>	<i>Principals of Corporate Finance, 8th Edition, New York: McGraw-Hill/Irwin (2006).</i>
<i>Bodie, Z. / Kane, A. / Marcus, A. J.</i>	<i>Essentials of Investments, 5th Edition. New York: McGraw-Hill/Irwin (2004).</i>

(further references in English)

<i>Copeland, T. E. / Weston, J. F. / Shastri, K.</i>	<i>Financial Theory and Corporate Policy, 4th edition, Boston et al.: Pearson/Addison-Wesley (2005)</i>
<i>Feibel, B. J.</i>	<i>Investment Performance Measurement, New York: Wiley (2003).</i>

(only for participants who are interested in an additional German textbook)

<i>Kaiser, D.</i>	<i>Treasury Management. Betriebswirtschaftliche Grundlagen der Finanzierung und Investition, Wiesbaden: Gabler (2008).</i>
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MUESF Brno / HS Bochum
Prof. Dr. Dirk Kaiser

120 Finanzwirtschaft
References



At disposal:

<http://www.is.muni.cz>

Treasury Management Survey

- 1 Basics (units 1–5, 10)
- 2 Cash flow from financing activities (units 6-8)
- 3 Cash flow from operating activities (unit 9)
- 4 Cash flow from investing activities (units 11-16)
 - Complete account of an investment
 - Dominance
 - Net present value
 - Annual equivalent
 - Internal rate of return
 - Payback period

3 *CF's*

6 *D's*

Treasury Management

Test

When? **First half of February 2009**
(including a preparatory unit)

Where? **MU ESF**

How long? **60 minutes in total**

Exercise 1 **Money cycle and cash flow statement (reproductive)**

Exercise 2 **Money cycle and cash flow statement (transfer)**

Exercise 3 **Investment calculus (reproductive)**

Exercise 4 **Investment calculus (transfer)**

(Relevant will be all what we have done in class, but nothing else.)

Treasury Management

Why?

1. Exchange

2. Money

„FINANCE“:
(1) the entirety of processes affecting the money stock by way of incoming and outgoing payments (“the financial system”);
(2) positive net effect of financial processes on the money stock.

**FINANCE,
INVESTMENT**

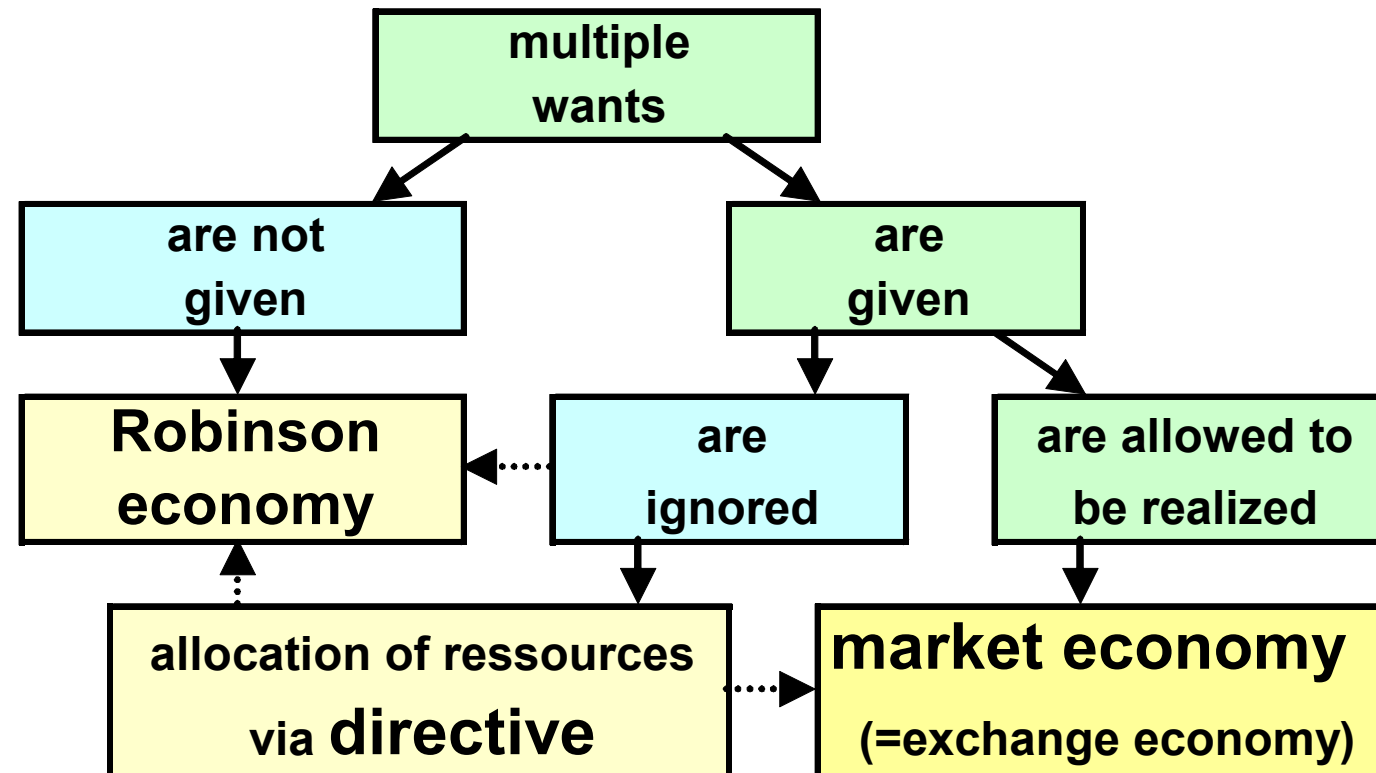
„INVESTMENT“:
a series of payments
(1) starting with an outgoing payment and
(2) showing at least one change of sign (“an investment measure”)

Preconditions of a Market Economy

(Fig. 1-1)



Walter Eucken, 1891-1950: Ordoliberalism



Timely Patterns of Exchange Contracts

(Fig. 1-3)

pacta sunt servanda
(Roman law vs.
common law)



	t=0	t=2
spot contract	signing consideration quid pro quo	
financial contract	signing consideration	
		quid pro quo
forward contract	signing	
		consideration quid pro quo

Exercise 1-1

Candice and Quentin find themselves in a simple exchange economy that is mostly equivalent to the one depicted in figure 1-3. In particular, two points in time (i.e.: $t=0$ and $t=2$) are at their disposal for signing, consideration and quid pro quo. However, Candice and Quentin are now allowed to sign exchange contracts not only in the present ($t=0$, early contracting), but also in the future ($t=2$, late contracting). To be sure, in both cases the signing has to take place until any goods or services are exchanged.

Find out the additional timely pattern(s) for exchange contracts that are induced by this weaker catalogue of assumptions!

Exercise 2-1

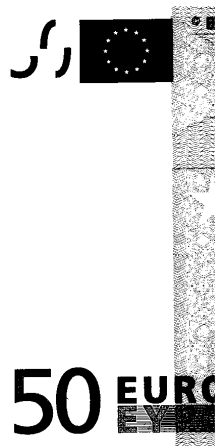
Candice and Quentin again find themselves in the simple market economy known from exercise 1-1 providing for two points in time ($t=0$ and $t=2$) and allowing for early and late ex ante contracting. In addition, both have agreed to introduce money as the generally accepted medium of exchange. In $t=2$, Quentin proposes to Candice a late spot contract providing for Candice to deliver one kilogram of marzipan and Quentin to pay CZK 1,000,000,000.

A good deal from Candice's perspective?

The Functions of Money

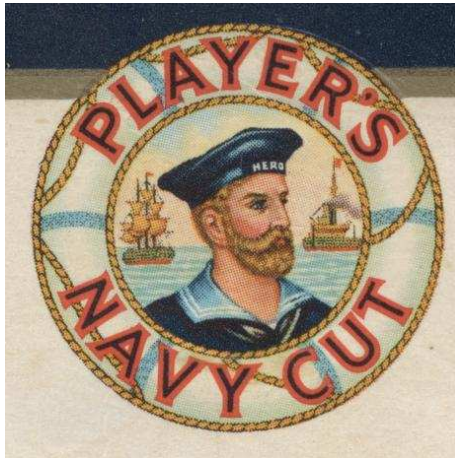
(Mn. 10)

- 1) Medium of exchange
- 2) Store of value
- 3) Unit of account
- 4) Legal tender etc.



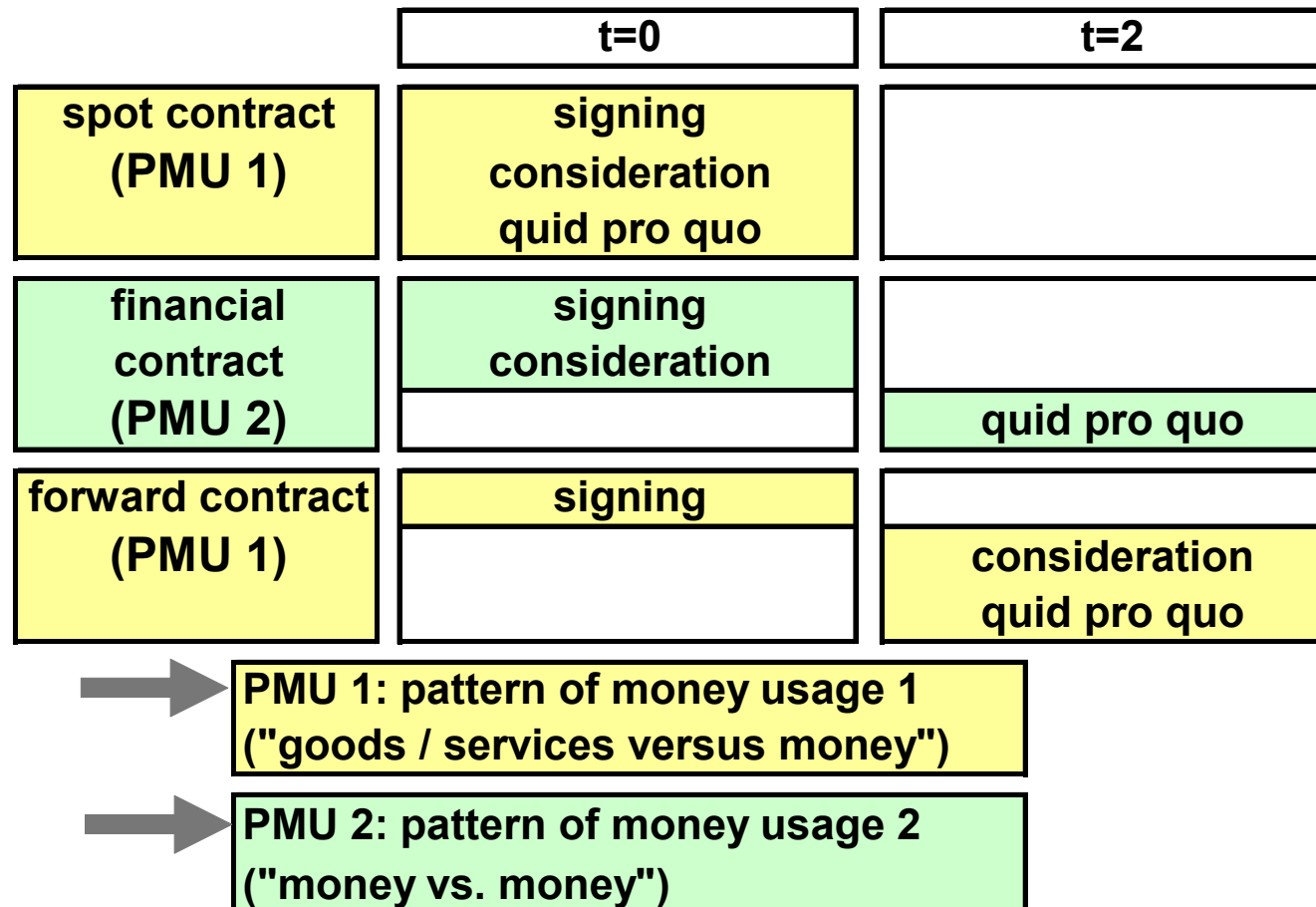
Correct? Complete?

In the Year 1946...



Patterns of Money Usage

(Fig. 2-1)



Exercise 2-2

The only good currently traded between Candice and Quentin is marzipan. Both want to express changes in their individual marzipan stocks by means of the letter m , changes in their individual money stocks by means of e . Furthermore, both have agreed to represent a spot contract by means of the three-letter code SpC , a financial contract by means of FiC and a forward contract by means of FoC . Changes in their individual stocks of money and marzipan as well as contracts are to be indexed by $t = 0, 2$ in order to point out their timely dimension.

Express the contractual patterns from exercise 1-1 by using the aforementioned symbols!

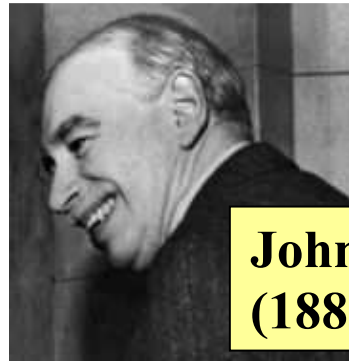
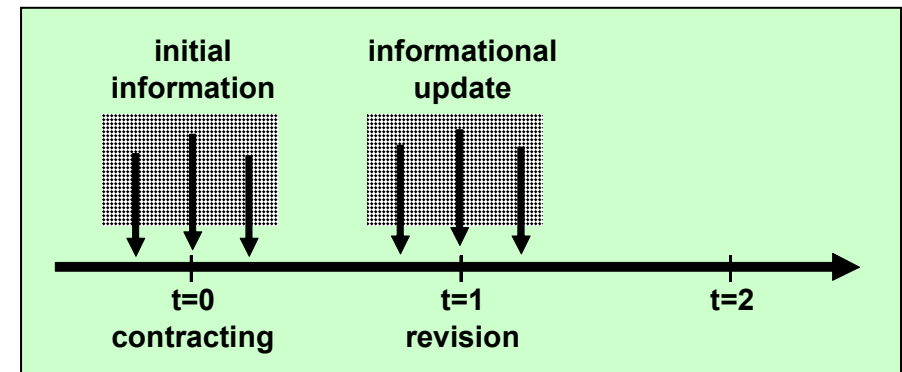


Liquidity

H. G. Moulton, 1918

„Liquidity is tantamount to shiftability.“

Austrian paradigm of informational updates
and revisions of decisions



John Maynard Keynes
(1883-1946)

Money is liquidity.

Ability to Pay



Stock-Flow Equation

	initial stock
-	cumulated outflows
+	cumulated inflows
=	final stock

Exercise 3-1



Candice and Quentin have found an internship with Masarykův pivovar a.s. (MP). Quentin is assessing the amounts of hop, malt and water coming in as well as the amount of beer leaving the brewery. Candice is driving the company's delivery truck in order to provide the local pubs with MP's famous lager. At the beginning of the 27th week of the year at 7:00 a.m. when they both clock in, 500 hectoliters of lager are in the container. During the week, Candice shows up 8 times with the truck, each time pumping off 20 hectoliters. On Tuesday, sladek Vaclav adds 300 hectoliters of fresh lager to the container.

What is the amount of beer in the container that Quentin will report to the company's controlling department on Friday at 3:45 p.m.?

Monetary Stocks

money
(“cash & cash equivalents”)
= cash (bills and coins)
± balances of bank accounts payable / receivable on demand

net financial assets = money
+ other financial claims
- other financial obligations
(except for equity)

net assets (equity) = net financial assets
+ non-financial assets

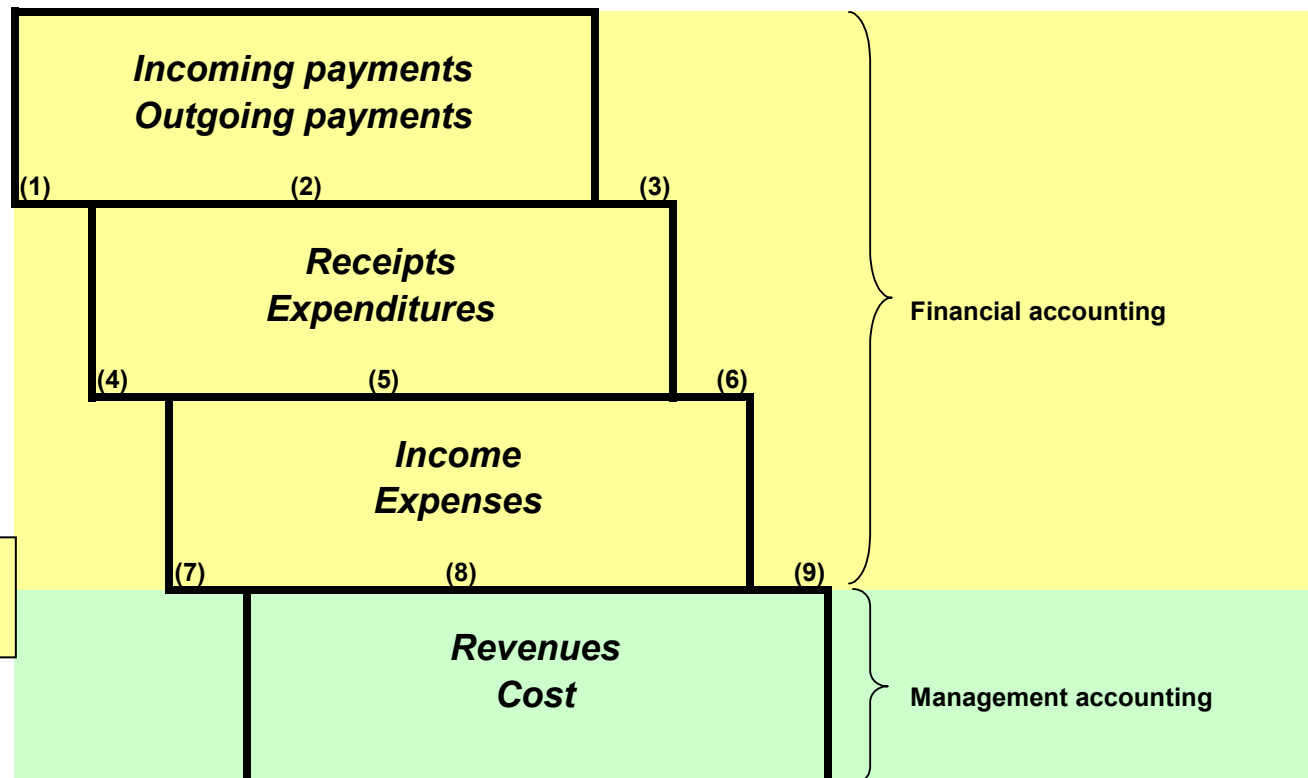
net operating assets

Monetary flows: Schmalenbach's Bar Graph

(Fig. 3-1)



Eugen Schmalenbach,
1873-1955



Exercise 3-2

During its financial year 2025, the Brněnské Marcipánové a Nugátové Kontor s.r.o. (“the company”) has among others registered the transactions (1) to (6) as described underneath.

Make up a three-column table and point out by means of this table to what extent these transactions have induced

- *incoming payments (+) or outgoing payments (-),*
- *receipts (+) or expenditures (-),*
- *income (+) or expenses (-).*

- (1) The Jemná Čokoláda a.s. makes a money transfer to the company’s current account amounting to CZK 6,500 for crude chocolate that was purchased on credit last year.**
- (2) Marzipan recorded at a book value of CZK 8,000 is sold for CZK 10,000 by the company. CZK 5,000 are paid instantaneously by the Jemná Čokoláda a.s. in cash, the remainder is purchased on credit.**

- (3) Magda Pavlová, one of the company's apprentices, is sent to Brněnská Spořitelná with CZK 1,365.75 in cash in order to deposit the money with the company's current account.**
- (4) Pablo Guerrero, a promising artist, transfers rent for his studio on the company's premises at the amount of CZK 1,200 covering the period from November 1, 2025, to October 31, 2026, to the company's current account.**
- (5) By means of a letter dated February 1, 2025, Brněnská věcná pojišťovna a.s. recognises a damage caused by a fire in 2024 amounting to CZK 7,200. The money transfer to the company's current account, however, will take some time.**
- (6) In 2025, a second fire destroys chocolate coating at a book value of CZK 1,800. The causes of the fire will supposedly be clarified in January 2026. Until that date, Brněnská věcná pojišťovna a.s. once more does not recognise the damage.**

Three Companies from Brno

(Case Studies I-III, Tab. 3-1)

	Cool, Fit & Partners	Brněnské Marcipánové a Nugátové Kontor	Jemná Čokoláda
Legal form	v.o.s.	s.r.o.	a.s.
Date of formation	January 1, 2025	September 1, 1873	July 1, 2025
Company purpose	Consulting; wholesale in personal computers	Wholesale in raw materials for the production of sweets	Production and sale of sweets
Slogan	„Stay cool – call the consultant“	„Raw materials for sweet producers - just in time“	„Not a day without chocolate!“
Domicile	Brno	Brno	Brno
Shareholders as of Dec. 31, 2025	Peter Cool (70%), Jane Fit (25%), Johnny B. Good (5%); all from Brno	Jiří Pátek (50%), Tomáš Pátek (50%), both from Brno	Všeobecné potraviny a.s., České Budějovice (100%)
Employees	5	15	150
Main bank relation; line of credit	Brněnská spořitelna (CZK 50,000)	Poštovní spořitelna (CZK 300,000)	Živnostenská spořitelna (CZK 1,000,000)
Further legal information	<ul style="list-style-type: none"> ○ Daddy Cool has guaranteed Brněnská spořitelna's loan ○ Mortgage on the company building in favour of Brněnská spořitelna ○ Faktorfinancování Česká republika a.s. has entered an agreement with CFP on the continuous purchase of accounts receivable up to CZK 100,000 against security cession of all A/R's 	<ul style="list-style-type: none"> ○ Pavel Sladký, managing director of the kontor, is member of the board of directors of Jemná Čokoláda ○ Kontor is delivering against reservation of proprietary rights and in places against additional acceptance of a bill of exchange. 	<ul style="list-style-type: none"> ○ Legally binding letter of comfort of sole shareholder in favour of Jemná Čokoláda ○ The board of directors has adopted rules of procedures for the officers of the company providing among other things that certain actions require BoD-approval.

Cool, Fit & Partners

Balance Sheets

(Tab. 3-2, 4-3)

Assets		CFP v.o.s., Balance Sheet as of January 01, 2025 TCZK		Liabilities	
Fixed assets	500	Equity	200		
<i>Property, plant</i>	300	<i>Equity Cool</i>	150		
<i>Equipment</i>	200	<i>Equity Fit</i>	50		
Current assets	400	Debt	700		
<i>Merchandise</i>	330	<i>Bank loan</i>	700		
<i>Current account</i>	70				
Total assets	900	Total liabilities	900		

Assets		CFP v.o.s., Balance Sheet as of December 31, 2025 before profit distribution, TCZK		Liabilities	
Fixed assets	550	Equity	300		
<i>Property, plant</i>	297	<i>Equity Cool</i>	130		
<i>Equipment</i>	223	<i>Equity Fit</i>	50		
<i>Financial assets 1</i>	10	<i>Equity Good</i>	20		
<i>Financial assets 2</i>	20	<i>Profit</i>	100		
Current assets	410	Debt	660		
<i>Merchandise</i>	205	<i>Bank loan</i>	650		
<i>Accounts receivable</i>	175	<i>Accounts payable</i>	10		
<i>Term account</i>	10				
<i>Current account</i>	10				
<i>Cash</i>	10				
Total assets	960	Total liabilities	960		

Brněnské Marcipánové a Nugátové Kontor

Income Statement

(Tab. 4-1)

Income statement for the Brněnské Marcipánové a Nugátové Kontor, Brno, for the time period from January 01, 2025, to December 31, 2025 (CZK mio)		
	2025	2024
1. Revenues	210	260
2. Changes in inventories of finished goods and work in progress	20	-10
3. Production for own fixed assets capitalized	6	4
4. Other operating income	16	2
5. Cost of purchased materials and services	142	151
6. Personnel expenses	42	31
7. Depreciation and amortization on tangible and intangible assets	38	42
8. Other operating expenses	10	4
9. Income from participations	2	2
10. Income from other financial assets	2	2
11. Other interest income	4	3
12. Depreciation and amortization on financial assets and financial current assets	0	2
13. Interest expenses	12	10
14. Profit before tax	16	23
15. Income tax	6	12
16. Other taxes	2	2
17. Net income / loss	8	9

Liquidity Reserves

Examples

- **Term deposits**
 - **Certificates of deposit**
 - **Commercial paper**
-
- **Bonds**
 - **Shares in investment funds**
 - **Ordinary and preference shares**
 - **Participation certificates**

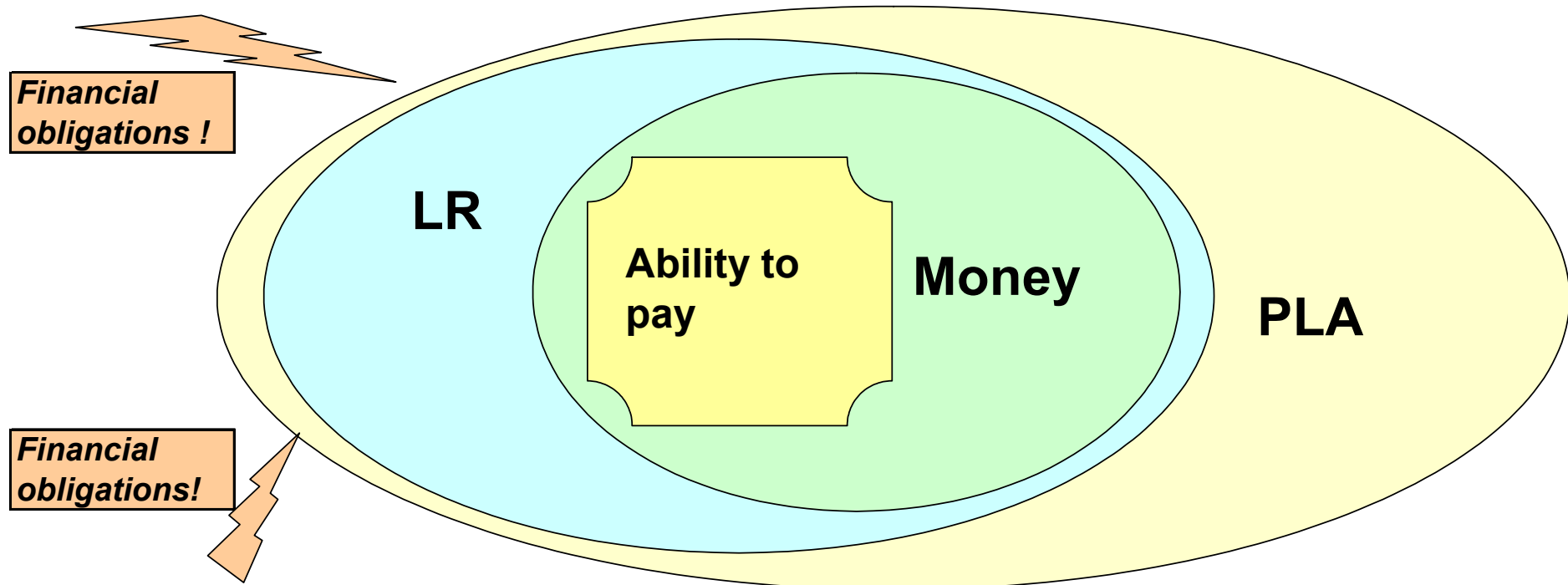
Potential Liquid Assets

Examples

- **Guarantee (or surety)**
- **Letter of comfort (if legally binding)**
- **Liquid assets of a full partner (general partner) or of a limited partner who has not yet paid in his capital completely**

Protecting the Company's Ability to Pay

(Fig. 3-3)



Exercise 3-3

In addition to tables 3-1, 3-2 and 4-3, here is further information concerning CFP:

- The shares of the Pazourek kamenolom a.s. (financial assets 1) and the Terra nostra pekarna a.s. (financial assets 2) are listed on the stock exchange; as of December 31, 2025, the quotations are 130% and 120%, respectively, over book value.
- The termination period remaining of the term account as of December 31, 2025, is 120 days.
- Private wealth of Peter Cool after contribution of his capital amounts to CZK 500,000; CZK 100,000 can be considered as liquid. For Jane Fit, the corresponding amounts are CZK 400,000 and CZK 200,000, respectively, for Johnny B. Good, CZK 100,000 and CZK 50,000, respectively.
- Daddy Cool's attitude towards the project of his son and his son's friends has become critical.

-
- i) *Determine the amounts of CFP's money stock, financial assets, non-financial assets and net assets!*
- ii) *As of December 31, 2025: Which assets might be taken into consideration as liquidity reserves, which ones as potential liquid assets? Which amounts will the Treasury of the company most probably attribute to LR and PLA, respectively?*

Structuring the Production Factors: Classical and for Management Purposes

(Fig. 4-1)



Adam Smith, 1723-1790



David Ricardo, 1773-1823

Land

Capital

Labor

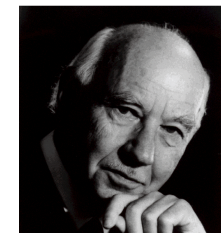
Classical economics

Fixed assets

Management
Labor
other

Management Science

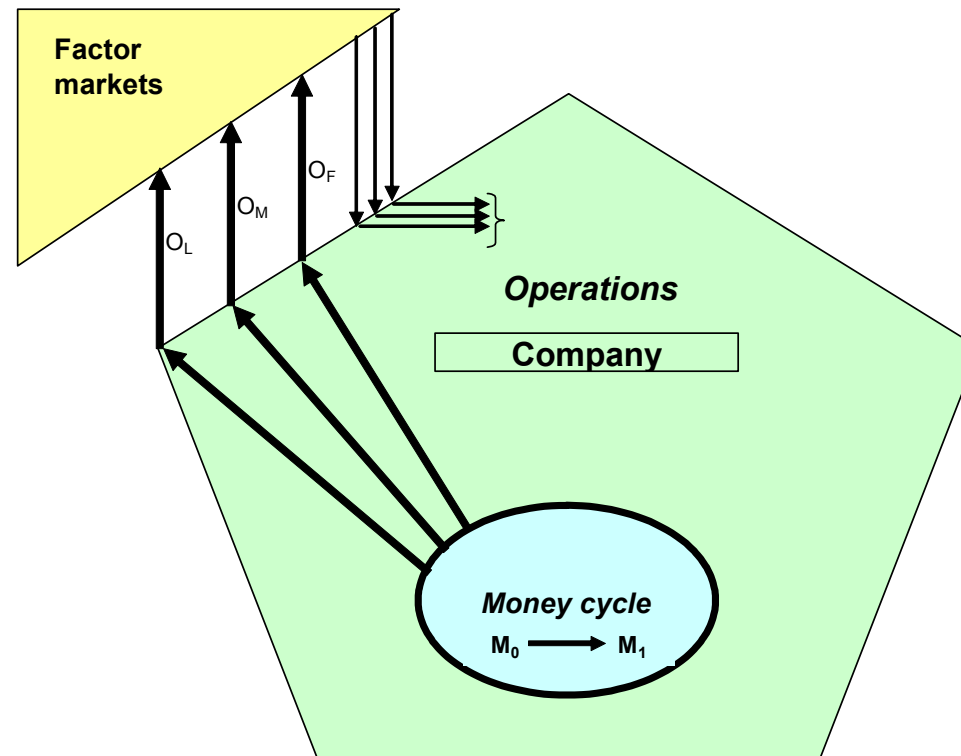
Other



Erich Gutenberg, 1897-1984

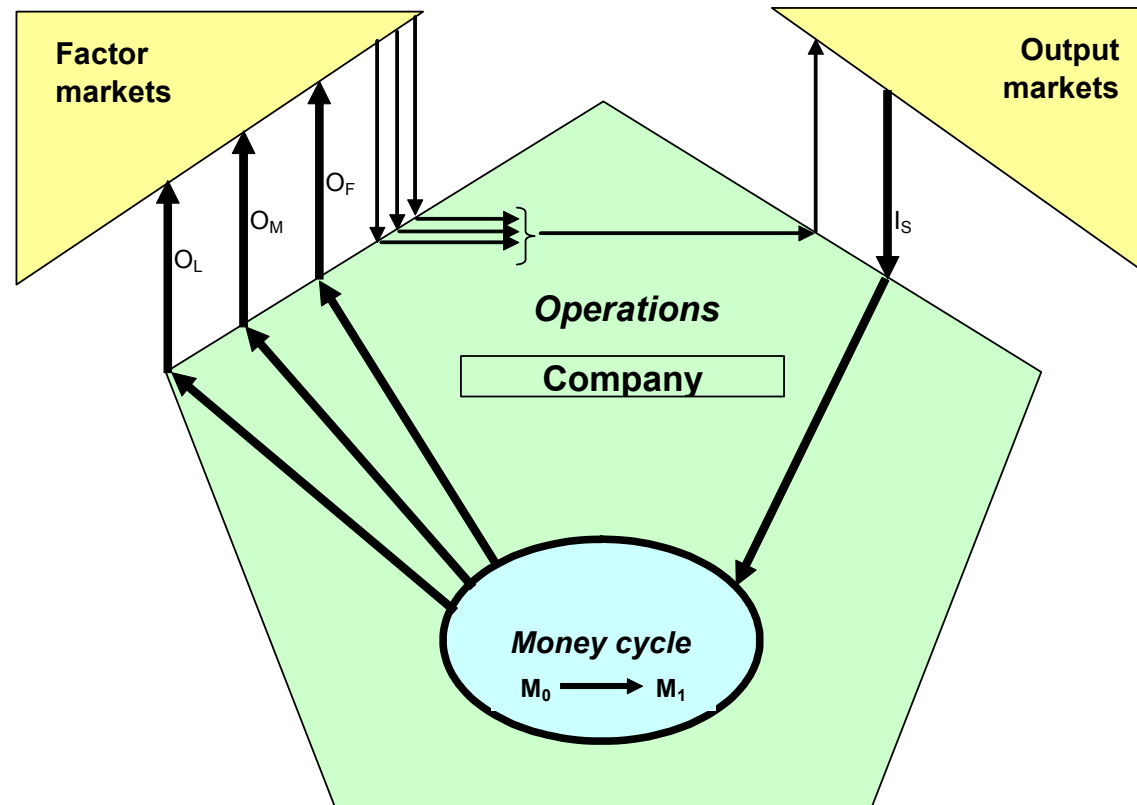
Constructing the Money Cycle

Step 1



Constructing the Money Cycle

Step 2

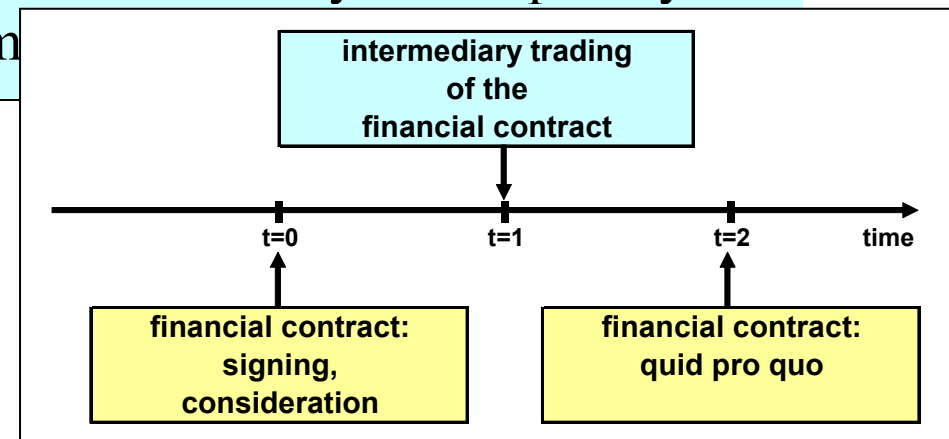


Primary and Secondary Markets

(Fig. 4-2)

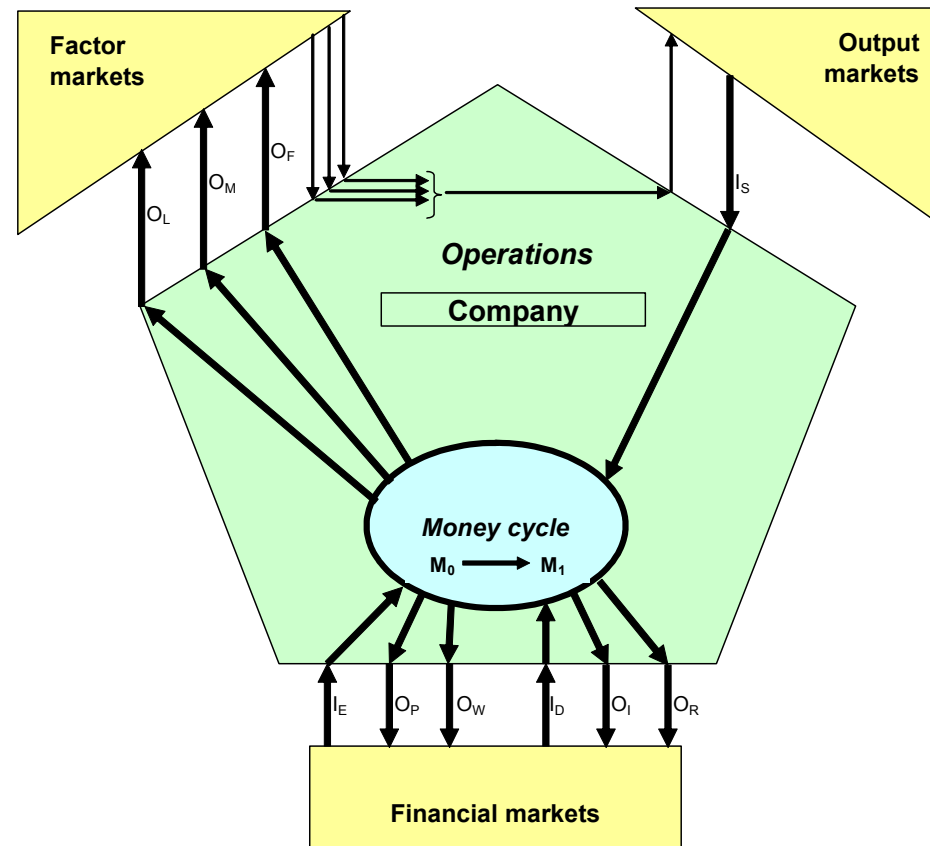
Definition 4-1

- a) PRIMARY MARKETS are markets where exchange contracts are signed ($t=0$) and where they are by means of consideration and quid pro quo ($t=0$ and $t=2$) fulfilled.
- b) SECONDARY MARKETS are markets where exchange contracts that were signed already, but that are not yet completely fulfilled, are traded in the meantime.



Constructing the Money Cycle

Step 3



Exercise 4-1

Candice (like “consideration”) and Quentin (like “quid pro quo”) would both like to sign a financial contract; formally speaking: $FiC_0 = FiC_0(e_0, e_2)$. Both have agreed on an ordinary debt contract. Still in $t=0$, Candice pays the consideration amounting to CZK 10,000 to Quentin, so that from her point of view the following is valid: $e_0^{Candice} = -10,000.00 [CZK]$. The debt contract obliges Quentin to the following payments in $t=2$: (1) redemption of the consideration at face value; (2) a fixed interest rate amounting to $r_D = 0.068$ per period (interest formally due in $t=1$ has to be capitalized).

- i) *What is the amount due to Candice in $t=2$?*
- ii) *Now imagine it had not been Quentin but the Cool, Fit & Partners v.o.s. (CFP) that had received the consideration of this financial contract. Assume CFP’s perspective and “translate” the contract into the language of the money cycle!*

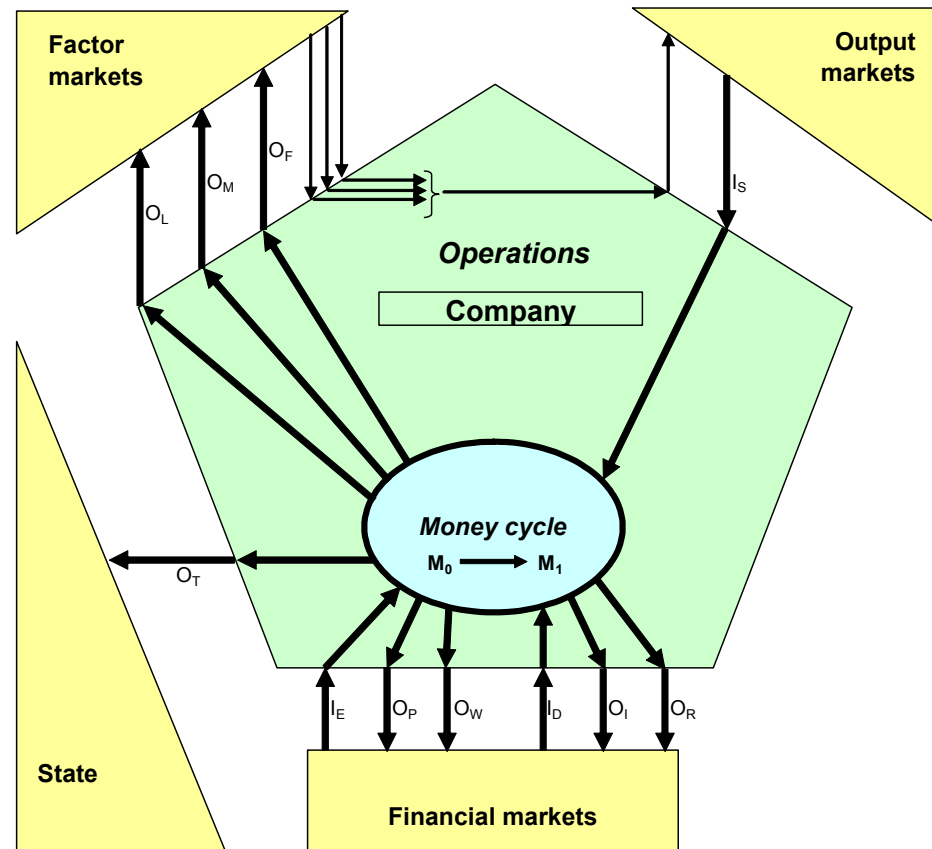
Exercise 4-2

Starting point is the debt contract from exercise 4-1. Due to new information, Candice finds herself in a state of increased demand for for money in $t=1$. For this reason, she revises her initial decision and sells the contract for an instant payment of CZK 10,720.15 in $t=1$ to a third party.

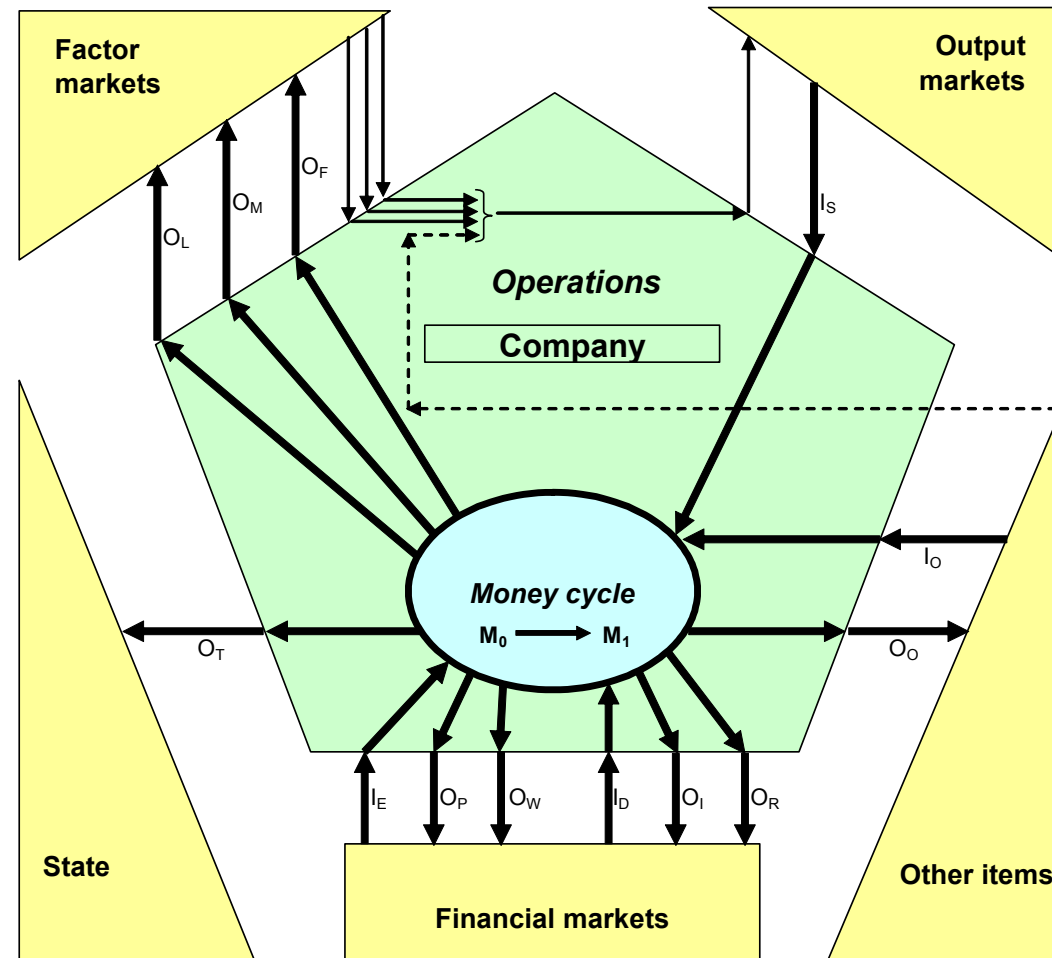
- i) *Associate the contractual actions described above with the concepts of primary market and secondary market!*
- ii) *Express the secondary trading described above by means of the symbols known to you from exercise 2-2!*
- iii) *What is the current interest rate on the secondary market in $t=1$?*

Constructing the Money Cycle

Step 4



Ops and Money Cycle (Fig. 4-3)



Application of the Stock-Flow Equation

	Initial stock of money	M_0
\pm	Incoming / outgoing payments factor markets	$+I_F - O_F - O_M - O_L$
$+$	Incoming payments output markets	$+I_S$
\pm	Incoming / outgoing payments other items	$+I_O - O_O$
\pm	Incoming / outgoing payments financial markets	$+I_E - O_P - O_W + I_D - O_I - O_R$ $- O_E + I_P + I_W - O_D + I_I + I_R$
$-$	Taxes	$-O_T$
$=$	Final stock of money	M_1

Theoretical Cash Flow Statement

CF from operating activities ("Internal funding")	$+I_S + I_O - O_M - O_L - O_O - O_T$
+ CF from investing activities ("Investments")	$-O_F + I_F - O_E + I_P + I_W$ $-O_D + I_I + I_R$
+ CF from financing activities ("External funding")	$+I_E - O_P - O_W + I_D - O_I - O_R$
= Net increase / decrease in money stock	$=M_1 - M_0$

Exercise 4-4

The Cool, Fit & Partners v.o.s. (CFP) has in 2006 encountered the subsequent transactions:

- (1) With the intent of a longlasting investment, shares of the Pazourek kamenolom a.s. amounting to CZK 10,000 as well as shares of the Terra nostra pekarna a.s. amounting to CZK 20,000 are bought against money transfer from the company's current account.**
- (2) Because of not being in use anymore, pneumatic post equipment is sold to a museum at its book value of CZK 25,000 against payment in cash.**
- (3) Personal computers (merchandise) invoiced at CZK 75,000 are delivered to CFP. € 25,000 are paid in cash. Another CZK 40,000 are settled by money transfer from the company's current account. The remainder is covered by the use of commercial credit.**
- (4) As of July 1, Johnny B. Good joins the company as third general partner. He deposits his share capital amounting to € 20,000 in cash.**

(5) As of July 1, Peter Cool withdraws CZK 10,000 of his capital against receipt of an equivalent amount from the company's cash.

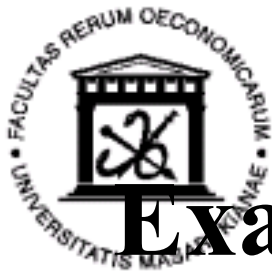
- (6) CFP invoices CZK 40,000 for consultancy services rendered in connection with the foundation of Jemná Čokoláda a.s. and receives the corresponding amount on the company's current account.**
- (7) CFP invoices CZK 10,000 for consultancy services rendered to university graduates at the occasion of a graduate recruitment fair. The organiser of the fair transfers the corresponding amount to the current account of the company.**
- (8) Rent for a part of the company building used by third parties amounting to CZK 20,000 becomes due and is transferred to the company's current account.**
- (9) Personal computers (merchandise) debited with CZK 400,000 are sold for CZK 650,000. CZK 475,000 of the amount due are received on the company's current account. For the coverage of the remaining CZK 175,000, commercial credit is extended to the customer.**

- (10) New personal computers (merchandise) invoiced at CZK 200,000 as well as equipment invoiced at CZK 50,000 are bought against money transfer from the company's current account.**
- (11) On November 1, 2025, a term deposit with a maturity of 180 days amounting to CZK 10,000 is credited from the company's current account.**
- (12) Salaries amounting to CZK 180,000 in total are transferred to the employees' current accounts.**
- (13) Interest due amounting to CZK 35,000 in connection with a bank loan is credited from the company's current account.**
- (14) Redemption due amounting to CZK 50,000 is covered by money transfer from the company's current account.**
- (15) In order to prepare for the closing of the annual accounts, fixed assets are written off as follows: property, plant: CZK 3,000; equipment: CZK 2,000.**

Make up a three-column table and point out by means of this table to what extent these transactions have induced

- *incoming payments (+) or outgoing payments (-),*
- *receipts (+) or expenditures (-),*
- *income (+) or expenses (-).*

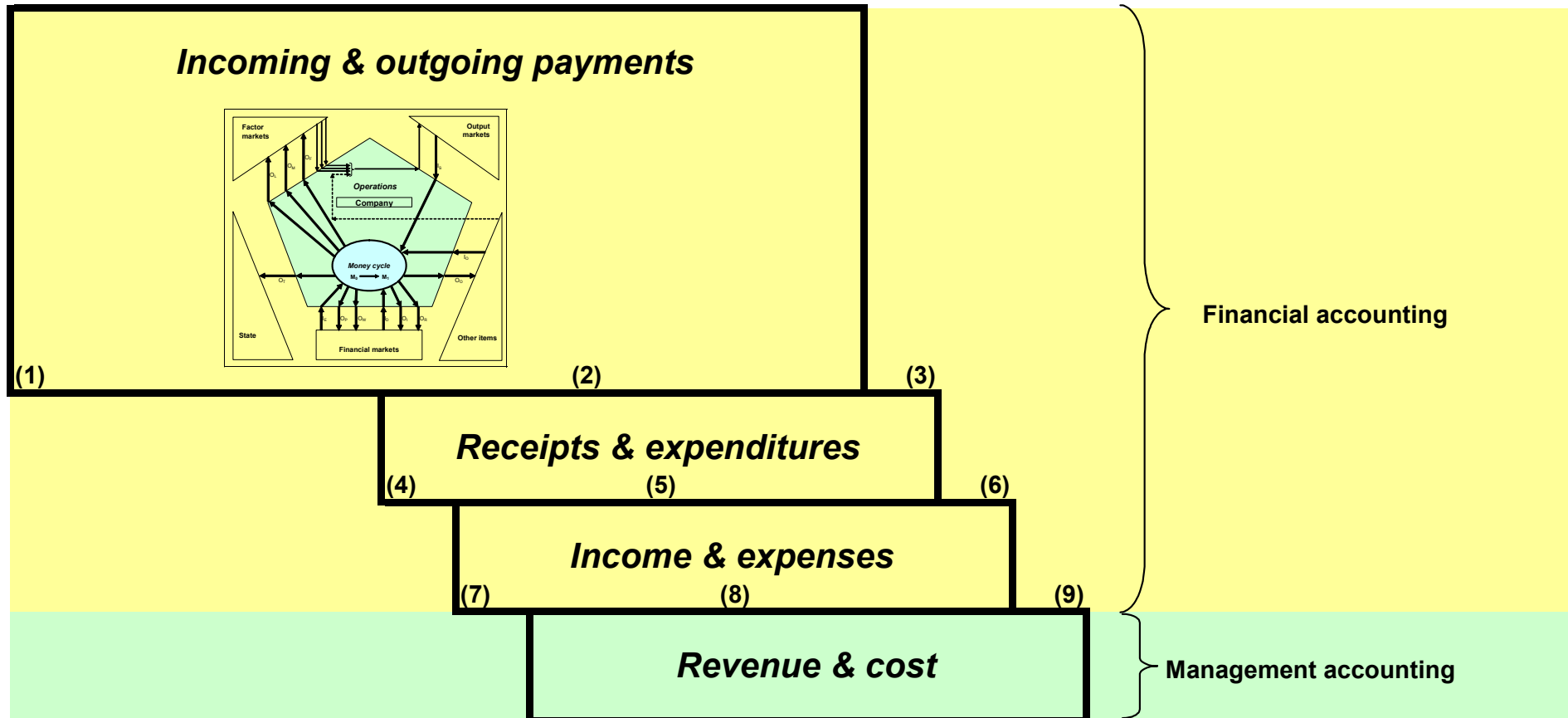
- (ii) *“Translate” the 15 CFP-transactions into the language of the money cycle”*
- (iii) *Make up CFP’s theoretical cash flow statement for the financial year 2025!*
- (iv) *Compare CFP’s theoretical cash flow statement to a cash flow statement according to IAS/IFRS!*



Example for a Cash Flow Statement according to IAS/IFRS (here: Metro Group)

Consolidated cash flow statement (€ million)		
	2006	2005
EBIT	1.983	1.738
Depreciation and amortization of tangible and intangible assets	1.250	1.200
Change in provisions for pensions and other provisions	273	-19
Change in net working capital	1.137	66
Income taxes paid	-543	-499
Elimination of negative difference first-time consolidation	-410	0
Other	-427	-452
Cash flow from operating activities of continuing operations	3.263	2.034
Cash flow from operating activities of discontinued operations	0	150
Total cash flow from operating activities $(+I_S+I_O-O_M-O_L-O_O-O_T)$	3.263	2.184
First-time consolidation	108	19
Company acquisitions	-205	0
Investments in tangible assets (excl. Finance leases)	-1.824	-1.922
Other investments	-268	-253
Company divestments	0	48
Divestment of stores	484	670
Disposals of fixed assets	403	313
Cash flow from investing activities of continuing operations	-1.302	-1.125
Cash flow from investing activities of discontinued operations	0	-43
Total cash flow from investing activities $-O_F+I_F-O_E+I_P+I_W-O_D+I_F+I_R$	-1.302	-1.168
Profit distribution		
- to parent company stockholders	-334	-334
- to other stockholders	-122	-72
Raising of financial liabilities	1.423	935
Redemption/repayment of financial liabilities	-1.571	-1.415
Interest paid	-610	-637
Interest received	169	137
Profit and loss transfers and other financing activities	50	-6
Cash flow from financing activities of continuing operations	-995	-1.392
Cash flow from financing activities of discontinued operations		23
Total cash flow from financing activities $+I_E-O_P-O_W+I_D-O_I-O_R$	-995	-1.369
Total cash flows	966	-353
Exchange rate effects on cash and cash equivalents	-1	13
Overall change in cash and cash equivalents	965	-340
Cash and cash equivalents on January 1 M_0	1.767	2.107
Cash and cash equivalents on December 31 M_1	2.732	1.767

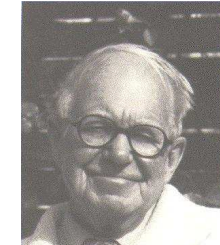
Bar Graph plus Money Cycle



Uncertainty vs. Risk



**FRANK KNIGHT (1885-1972):
Risk, Uncertainty,
and Profit (1921)**



**GEORGE L. S. SHACKLE (1967):
The Years of High Theory
(1926 – 1939)**

**Uncertainty in the
broader sense**

The Century of Decision Theory

**„True” uncertainty,
“unmeasurable uncertainty”
(the distribution is not known)**

**„risk“ proper,
„measurable uncertainty”
(the distribution is known)**

Financial Risks

RISK CAUSED BY TIMELY DEVELOPMENT



(In finance: Risk that is caused by timely differences between consideration and quid pro quo.)

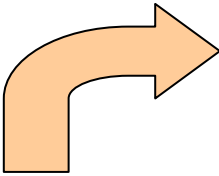
RISK CAUSED BY LACK OF INFORMATION



(But: How much information is possible?
Borderline to risk caused by timely development is difficult to determine.)

Covenants

Examples



*Financial contracts
can be expected to
be more complex.*



**Additional risk
caused by timely
development**

- **Admission to the circle of MIS addressees**
- **Transfer of property rights**
- **Mortgage**
- **Chattel mortgage**
- **Security cession**
- **Guarantee (or surety)**
- **Letter of comfort
(if legally binding)**

Exercise 6-3

Consider the three companies from Brno as presented in the preceding slides.

Explore the information available for regulations that could serve as covenants in financial contracts!

Emission Finance

⇒ **Financial contracts that comply with *all* of the three subsequent criteria:**

- ✓ **certified as a security**
- ✓ **fungible (homogeneous, ISIN)**
- ✓ **sufficient free float**

If at least one of these additional criteria is not fulfilled, it is STANDARD FINANCE.

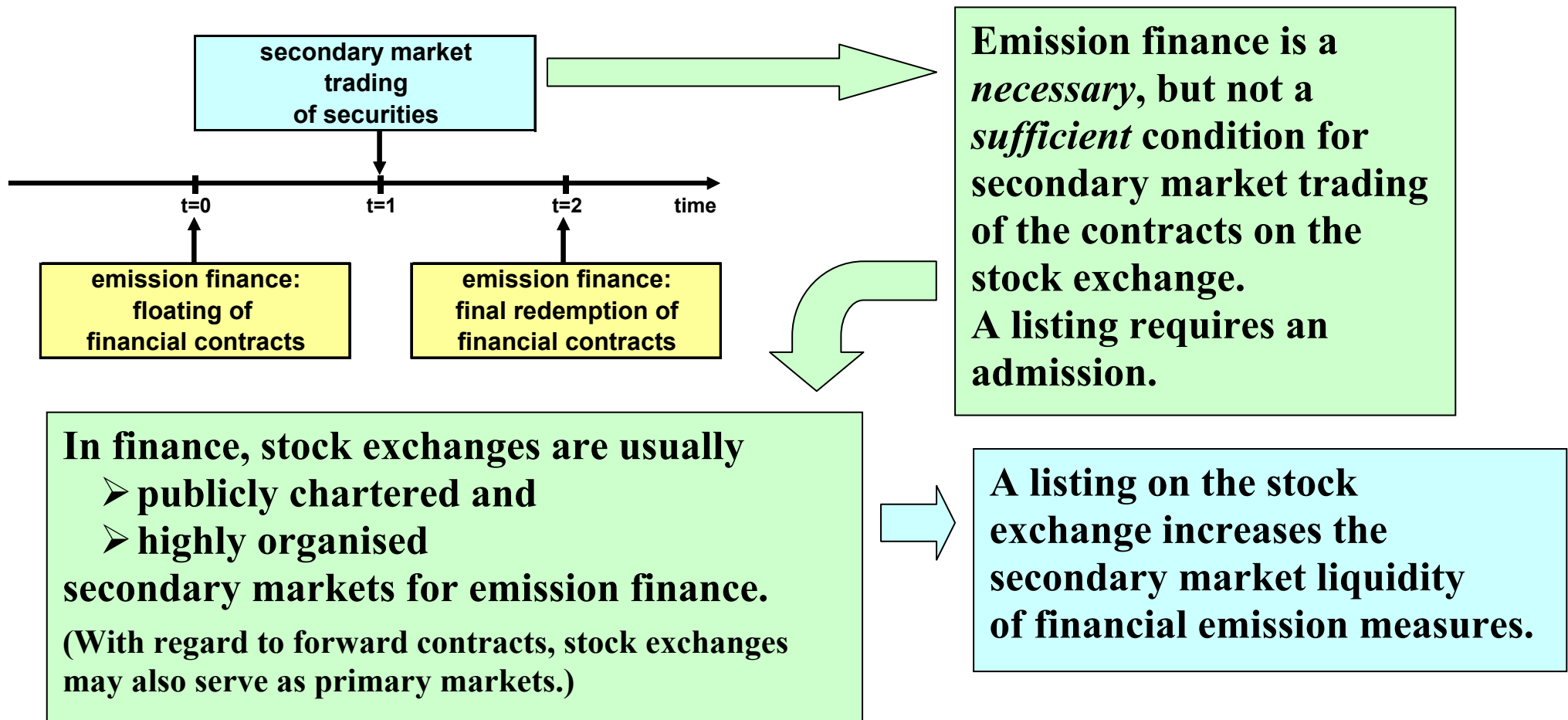
Exercise 6-4a

Consider the subsequent financial instruments:

- shares in the Cool, Fit & Partners v.o.s.
- shares in the Brněnské Marcipánové a Nugátové Kontor s.r.o.
- shares in the Jemná Čokoláda a.s.
- bonds issued by the Brněnské Marcipánové a Nugátové Kontor s.r.o.
- participation certificates issued by the Cool, Fit & Partners v.o.s.
- bills of exchange

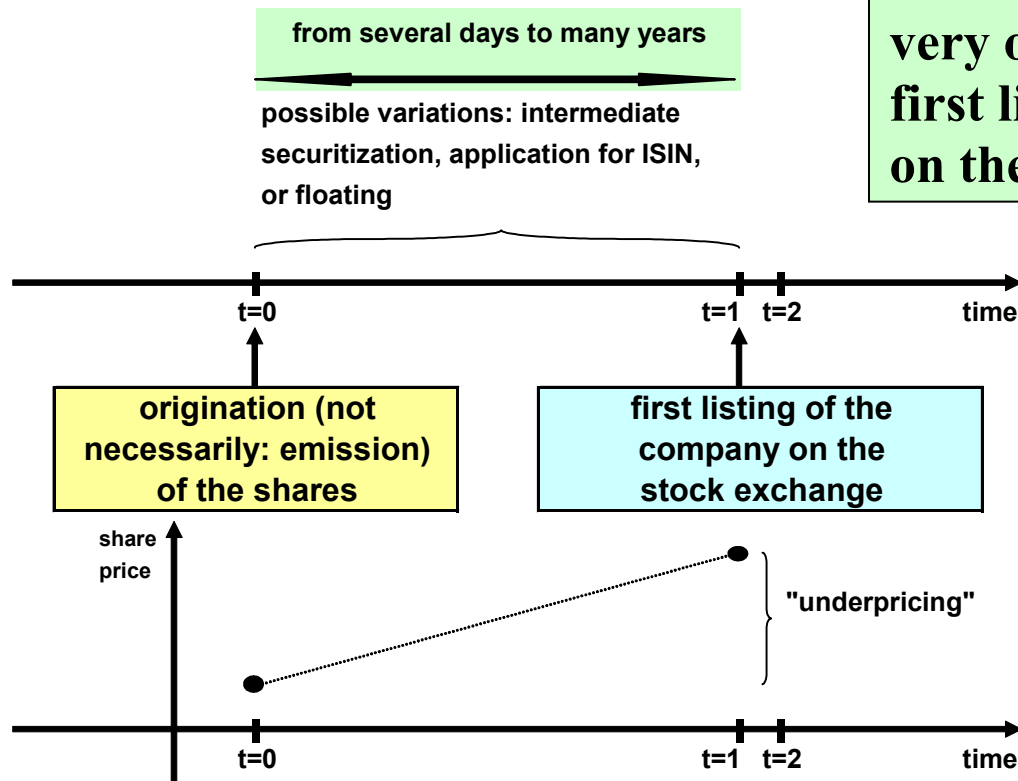
Which of these instruments could basically be used for emission finance?

Secondary Market Liquidity: The Stock Exchange



Initial Public Offering (IPO; Going Public)

An Initial Public Offering („IPO“ in short, very often also called “Going Public”) is the first listing of the shares of a certain company on the stock exchange.



Possible: (i) delay between origination and securitization etc.; (ii) delay between securitization etc. and first listing

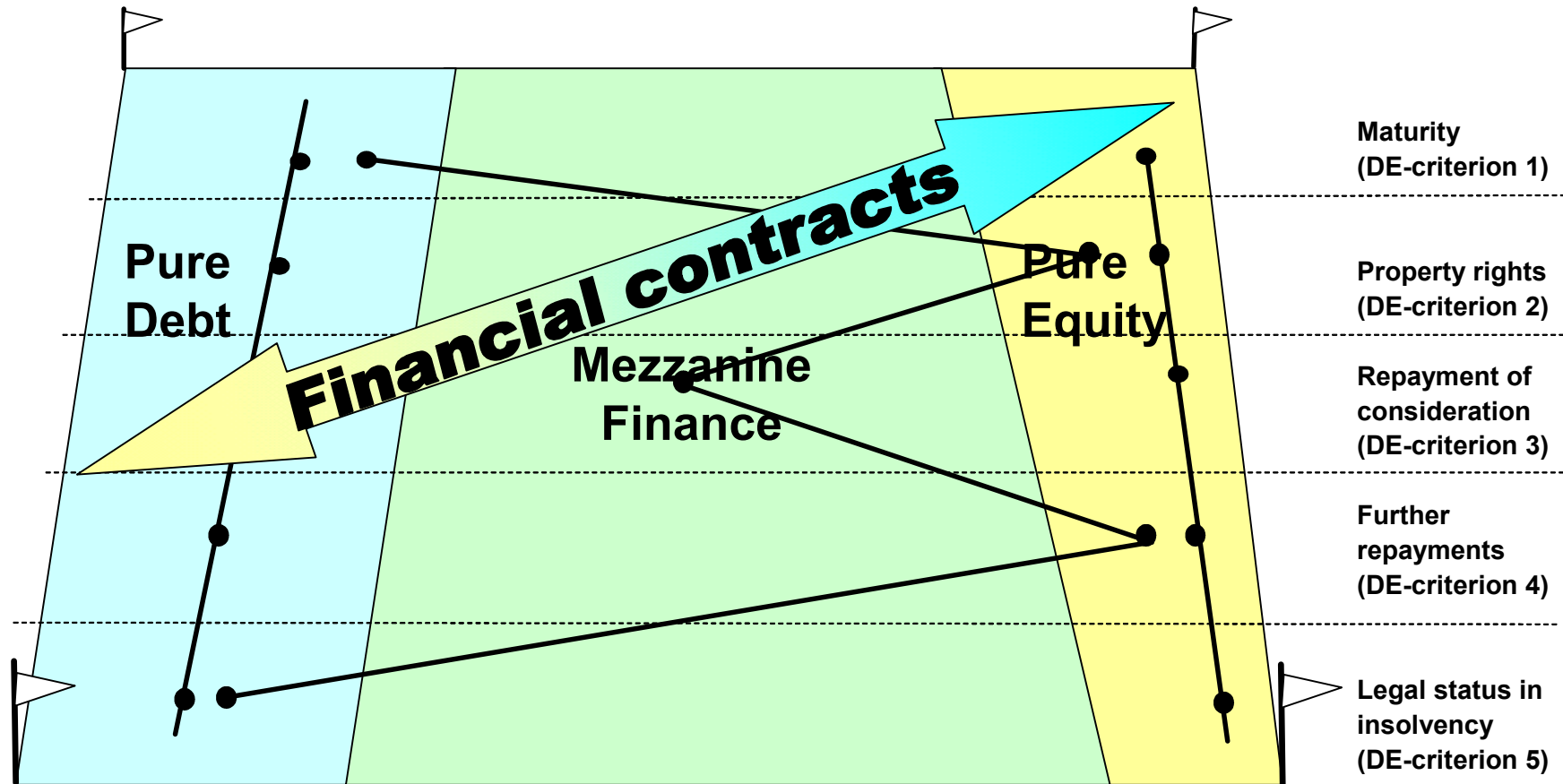
Difference between first quotation and emission price: „underpricing“ (common scenario)

Exercise 6-5

After the formation of the Jemná Čokoláda a.s. on July 1, 2025, its sole shareholder, the Všeobecné potraviny a.s., has induced its subsidiary to securitize its shares and apply for an International Securities Identification Number. After successful completion, the floating of 40% of the share capital was put into practice on February 1, 2030. The Initial Public Offering took place on February 4, 2030.

Analyze the financial effects of this IPO on the Jemná Čokoláda a.s.!

Debt and Equity



Exercise 6-7

- i) *Make up a debt-equity-profile of preference shares according to the five aforementioned criteria!*
- ii) *Subdivide the subsequent financial contracts into debt and equity according to the legal status in insolvency:*
 - preference share
 - ordinary share
 - partner in a v.o.s.
 - silent partner
 - participation certificate

Exercise 6-8

For the short fiscal year 2025, the Jemná Čokoláda a.s. reports the subsequent data:

	incoming payment (+) outgoing payment (-)	income (+) expenses (-)
(1) sales	+1,500	+1,500
(2) materials	-800	-800
(3) wages	-200	-200
(4) taxes	-100	-100
(5) issuance of participation certificates	+10	±0
(6) admission of a silent partner	+1	±0
(7) issuance of preference shares	+11	±0
(8) interest	-50	-50
(9) profit distribution	-340	±0
(10) depreciation and write-off	±0	-60
(11) appreciation	±0	+20
(12) formation of provisions for bad debts	±0	-30
Total	32	280

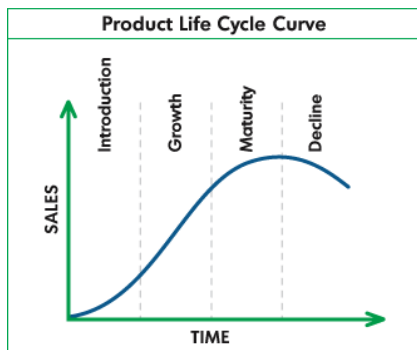
-
- i) Make up Jemná Čokoláda's theoretical cash flow statement for the short fiscal year 2025!*
- ii) Classify the amount of money that Jemná Čokoláda has in the short fiscal year 2025 generated by financial activities by means of a two-column two-rows matrix with the two rows standing for standard and emission finance, respectively, and the two rows for equity and debt, respectively!*

Entrepreneurship, Equity and the Life-Cycle of a Company

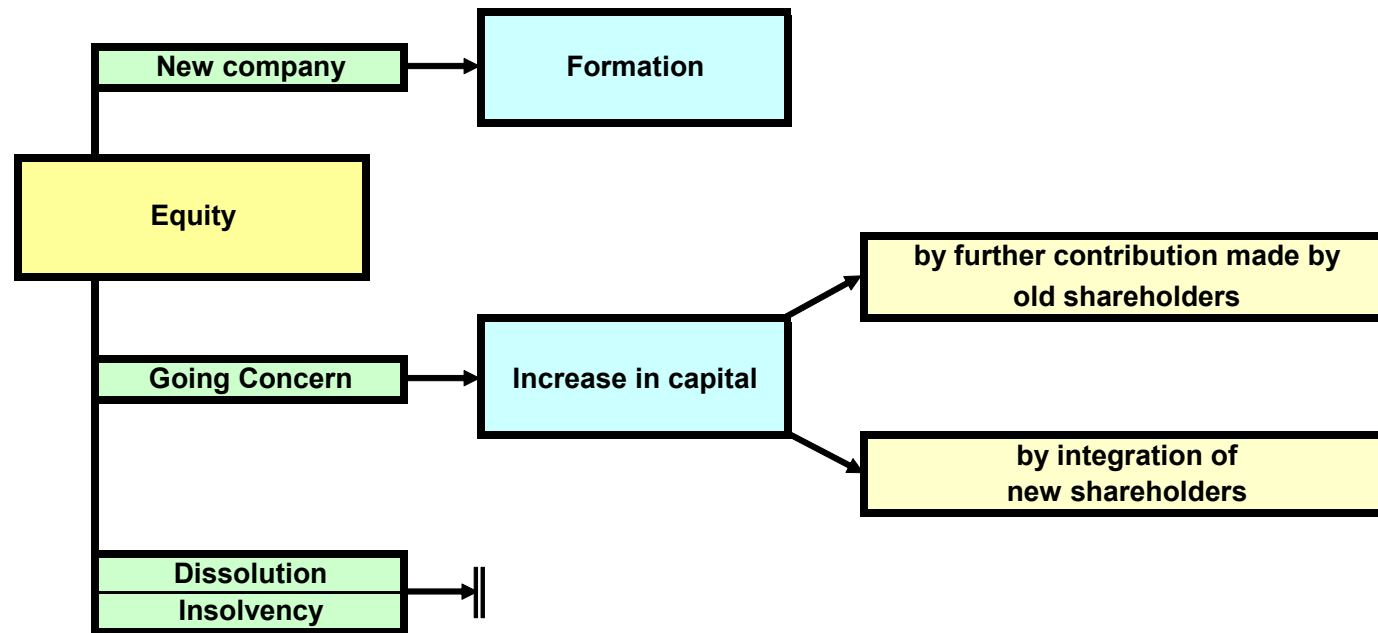
(Fig. 6-6)



Joseph A. Schumpeter,
February 08, 1883 (Třešť)
January 08, 1950 (Taconic)



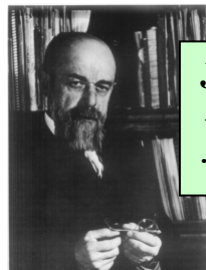
Joel Dean, 1906-1979:
Managerial Economics



Some More Famous Scientists Born in Bohemia or Moravia



Karel Engliš, Economist,
★ August 17, 1880 (Hrabyně)
† June 15, 1961 (Hrabyně)

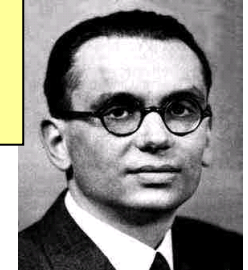


Josef Macek, Economist,
★ September 13, 1887 (Krumpach u Zábřehu)
† February 19, 1972 (Vancouver)



Ota Šik, Economist,
★ September 11, 1919 (Plzeň)
† August 22, 2004 (Sankt Gallen)

Kurt Gödel, Logician,
★ April 28, 1906 (Brno)
† January 14, 1978 (Princeton)



Gregor Mendel, Biologist,
★ July 22, 1822 (Vražné-Hynčice)
† January 06, 1884 (Brno)

**Three economists
to rediscover!?**

Examples for financial measures with debt character

- ✓ credit on current account
- ✓ raising of a mortgage loan
- ✓ integration of a silent partner
- ✓ issuance of corporate bonds
- ✓ issuance of commercial paper
(certificates of deposit)

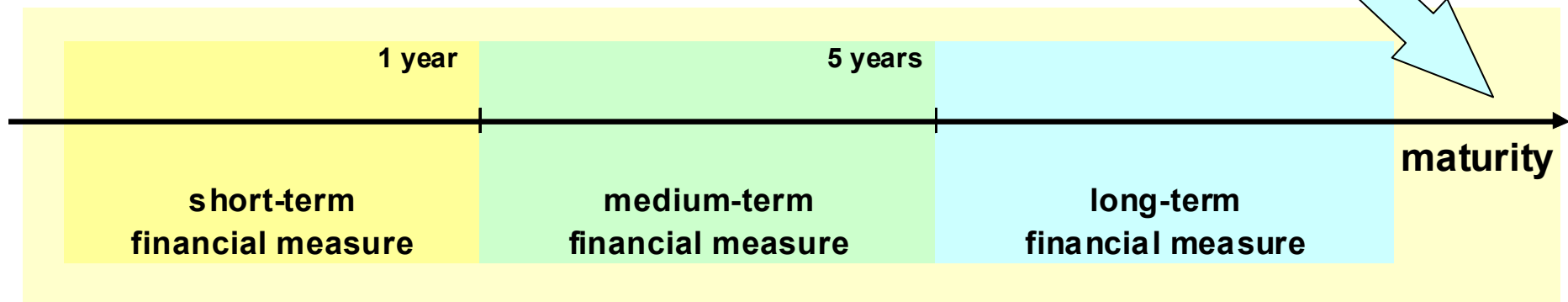


participation certificates

The Maturity of Debt

**EQUITY:
long-term
by nature**

„MATURITY“ refers to:
1) remaining term (term to maturity)
or
2) notice period,
respectively



Exercise 6-12

The Vitaggi s.r.o., a sister company of the Jemná Čokoláda within the Všeobecné potraviny's group of companies producing packet soup, reports the subsequent data for the fiscal year 2025 (CZK mio.):

	income (+) expenses (-)	incoming payment (+) outgoing payment (-)
(1) sales	+1,600	+1,600
(2) materials	-950	-950
(3) wages	-200	-200
(4) taxes	-100	-100
(5) issuance of participation certificates with a term of 6 years	±0	+4
(6) emission of corporate bonds with a maturity of 8 years	±0	+3
(7) absorption of the 2025 increase in capital by the parent company Všeobecné potraviny a.s.	±0	+8

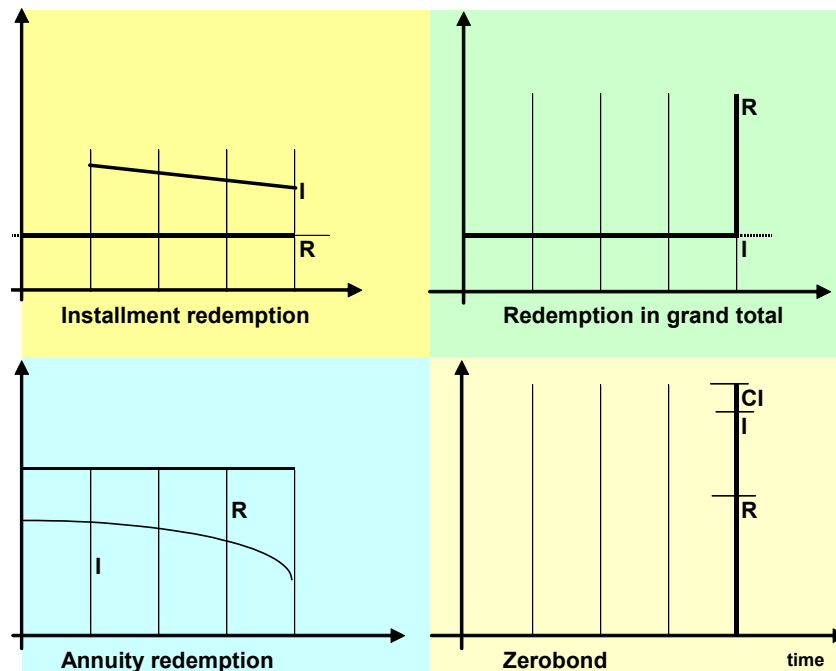
(8) raising of a bank credit on October 1, 2025, with a maturity of 6 months	±0	+5
(9) interest	-50	-50
(10) profit distribution	±0	-210
(11) depreciation and write-off	-55	±0
(12) appreciation	+20	±0
(13) formation of provisions for bad debts	-25	±0

Classify the amount of money that Vitaggi has in the fiscal year 2025 generated with the aid of debt contracts by means of a one-column three-rows table with the three rows standing for short-term, medium-term and long-term finance, respectively!

The Repayment of Debt

(Fig. 6-10)

DE-criterion 3



→ *Different methods for the calculation / approximation of the internal rate of return (ex- 9-13)*

DE-criterion 4

Fixed income debt

Roll-over credit (standard finance)

Floating rate notes (emission finance)

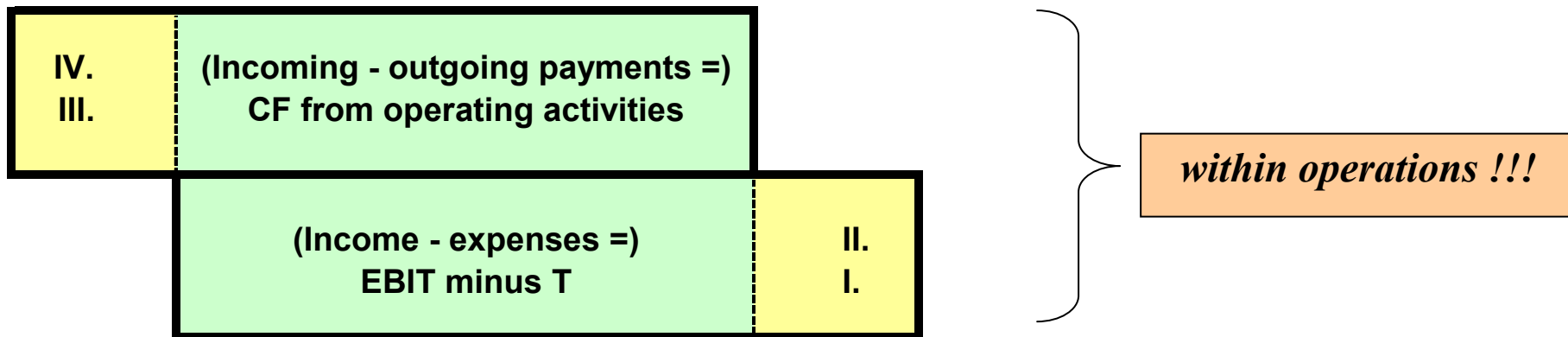
Exercise 7-2

Make up a source income statement for the Brněnské Marcipánové a Nugátové Kontor by restructuring the 2025 income statement according to the following form!

Source income statement (form)		
	2025	2024
Revenues		
Changes in inventories of finished goods and work in progress		
Production for own fixed assets capitalized		
Cost of purchased materials and services		
Personnel expenses		
Depreciation and amortization on tangible and intangible assets		
	Core operating profit	
Other operating income		
Other operating expenses		
	Other operating profit	
	Operating profit ("EBIT")	
Income from participations		
Income from other financial assets		
Other interest income		
Depreciation and amortization on financial assets and financial current assets		
Interest expenses		
	Financial profit ("I")	
Extraordinary income		
Extraordinary expenses		
	Extraordinary items	
	Earnings before tax ("EBT")	
Income tax		
Other taxes		
	Earnings after tax ("EAT")	

Schmalenbach's Bar Graph revisited

(Fig. 7-1)



	EBIT minus T	
+	expenses that do not affect cash & cash equivalents (correction type I)	within operations!!!
-	income that does not affect cash & cash equivalents (correction type II)	
-	non-expense applications of cash & cash equivalents (correction type III)	
+	non-expense originations of cash & cash equivalents (correction type IV)	
=	Cash flow from operating activities	

Exercise 7-3

Starting point is exercise 4-4. As you can easily check, this is CFP's source income statement for the fiscal year 2025.

Bridge the gap between EBIT minus T and cash flow from operating activities by making up a calculation considering for the four different types of corrections!

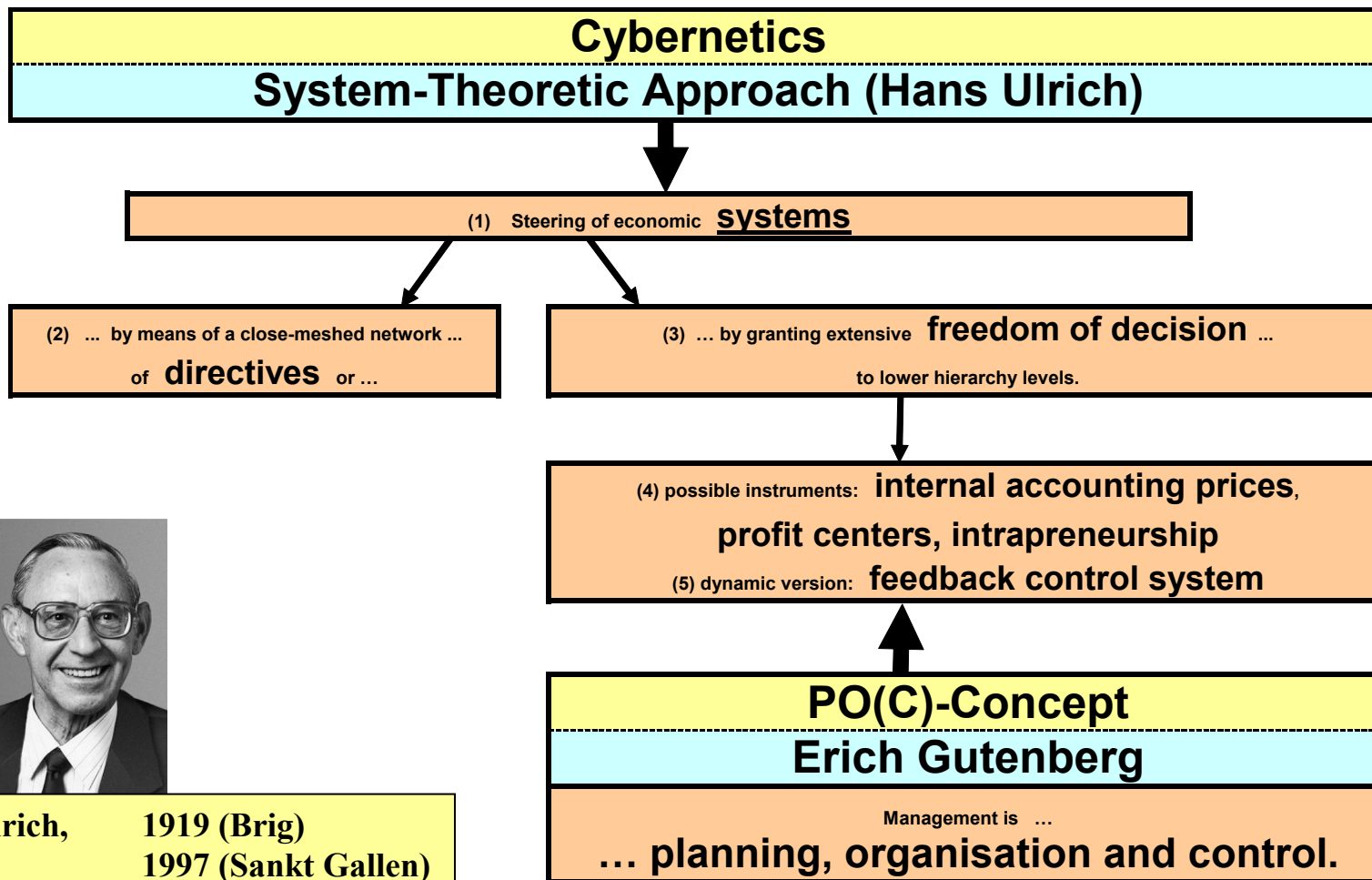
Source income statement for CFP v.o.s., Brno, for the time period from January 01, 2025, to December 31, 2025 (TCZK)		2025
Revenues		700
Changes in inventories of finished goods and work in progress		0
Production for own fixed assets capitalized		0
Cost of purchased materials and services		400
Personnel expenses		180
Depreciation and amortization on tangible and intangible assets		5
	Core operating profit	115
Other operating income		20
Other operating expenses		0
	Other operating profit	20
	Operating profit ("EBIT")	135
Income from participations		0
Income from other financial assets		0
Other interest income		0
Depreciation and amortization on financial assets and financial current assets		0
Interest expenses		35
	Financial profit ("I")	-35
	Earnings before tax ("EBT")	100
Income tax	} ("T")	0
Other taxes		0
	Earnings after tax ("EAT")	100

Exercise 7-1

After receipt of a corresponding purchase order by fax as of September 01, 2026, the Brněnské Marcipánové a Nugátové Kontor still on the same day delivers 3 kg of crude marzipan amounting to CZK 450 to Jemná Čokoláda. As the Kontor grants September 30, 2026, as time of payment, Jemná Čokoláda pays the amount due only at the end of the month of September.

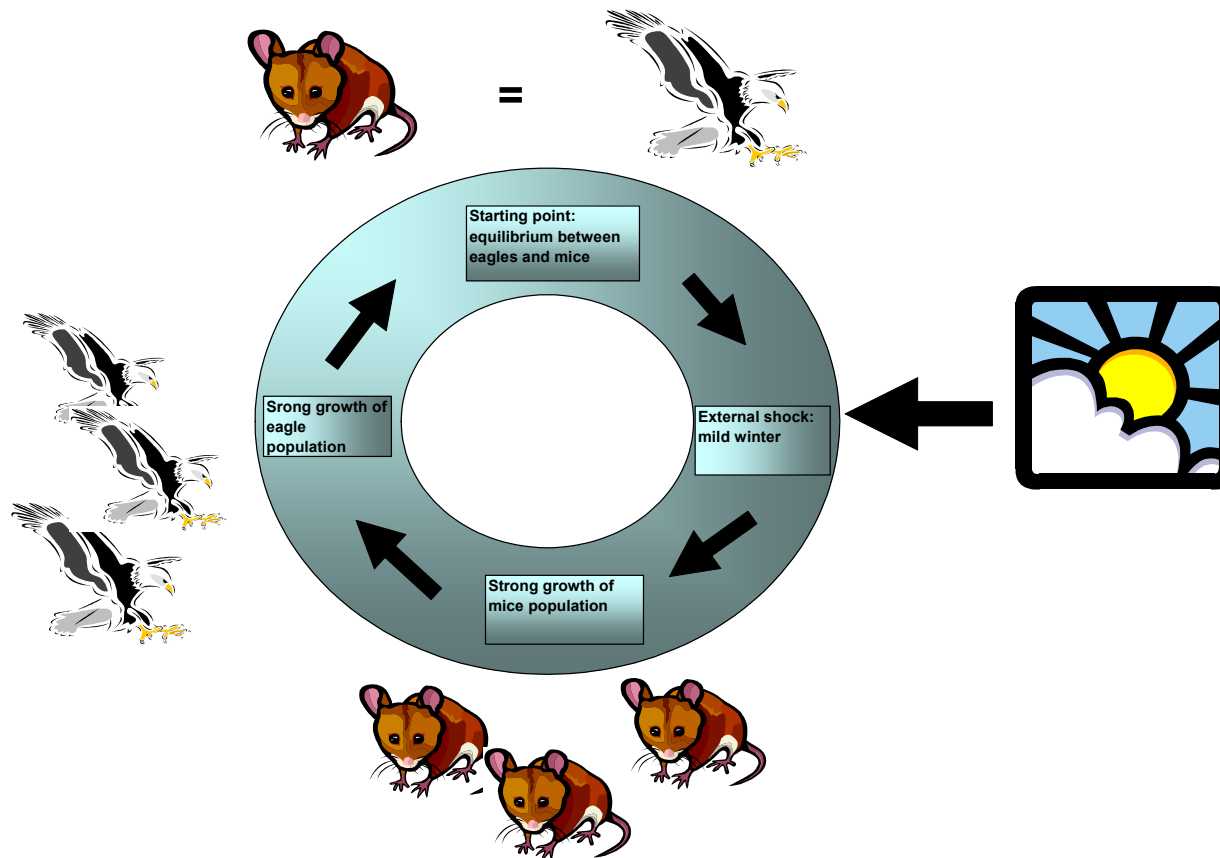
Assume Jemná Čokoláda's perspective and translate the transactions between buyer and supplier into the symbolism known to you from exercise 2-2! To this end, first register the transactions on a "gross"-level by means of two exchange contracts and then on a "net"-level by only one contract!

Management



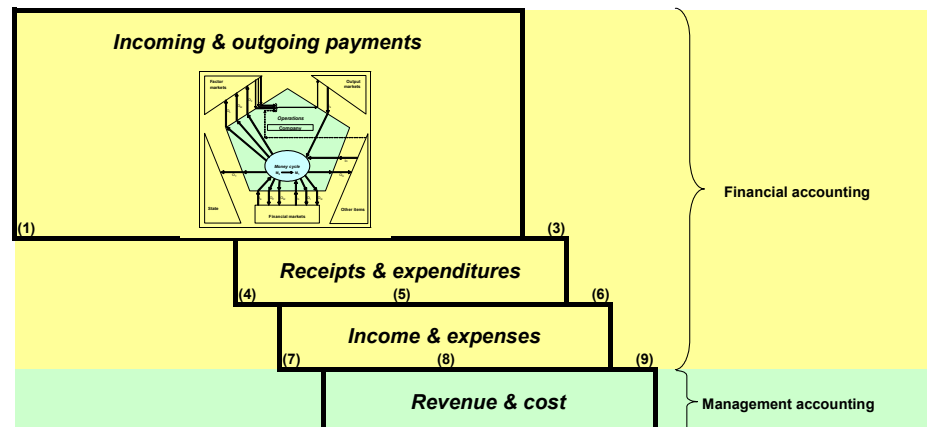
Hans Ulrich, 1919 (Brig)
1997 (Sankt Gallen)

Feedback Control Systems: An Example



Treasury Management (Financial Management)

- ✓ Steering of the money cycle
- ✓ Management of the cash flow statement



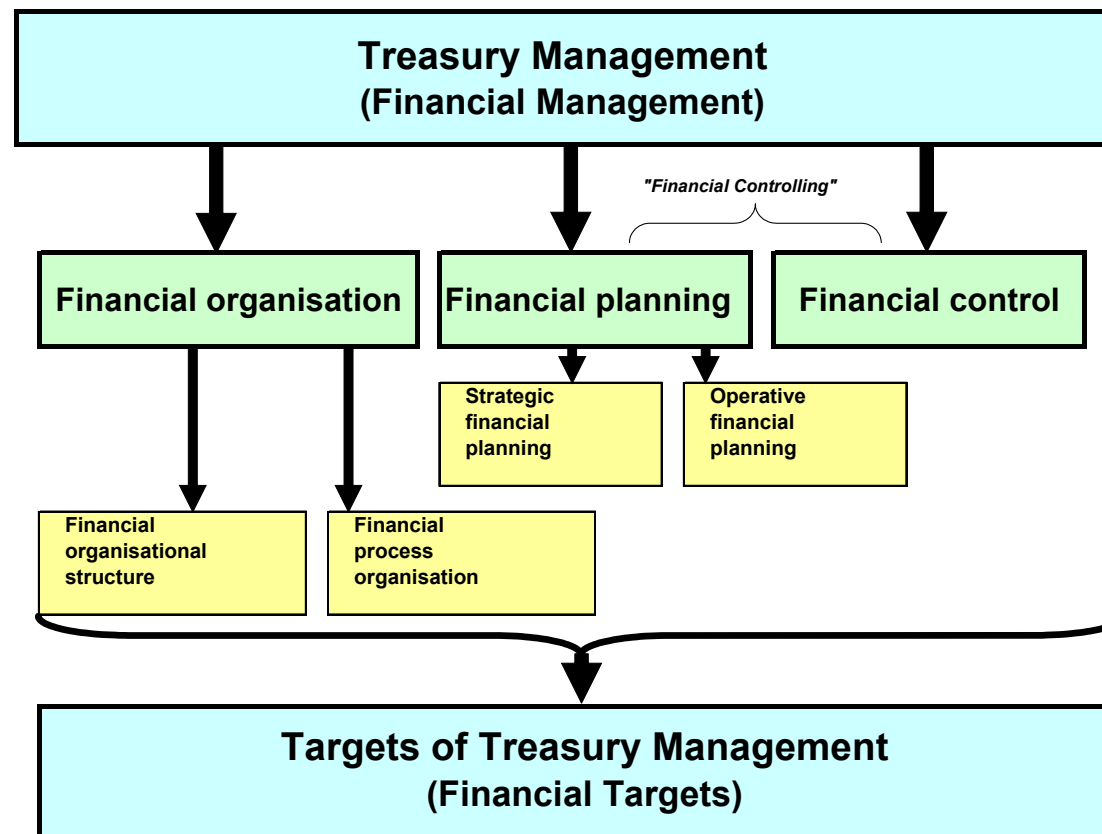
Consolidated cash flow statement (€ million)		
	2006	2005
EBIT	1.983	1.738
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Cash flow from operating activities of continuing operations	3.263	2.034
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Total cash flow from operating activities $(+I_S+I_G-O_M-O_L-O_G-O_T)$	3.263	2.184
First-time consolidation	108	19
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Redemption/repayment of financial liabilities	-1.571	-1.415
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Total cash flows	966	-353
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Overall change in cash and cash equivalents	965	-340
Cash and cash equivalents on January 1 M_0	1.767	2.107
Cash and cash equivalents on December 31 M_1	2.732	1.767

Objectives of Treasury Management

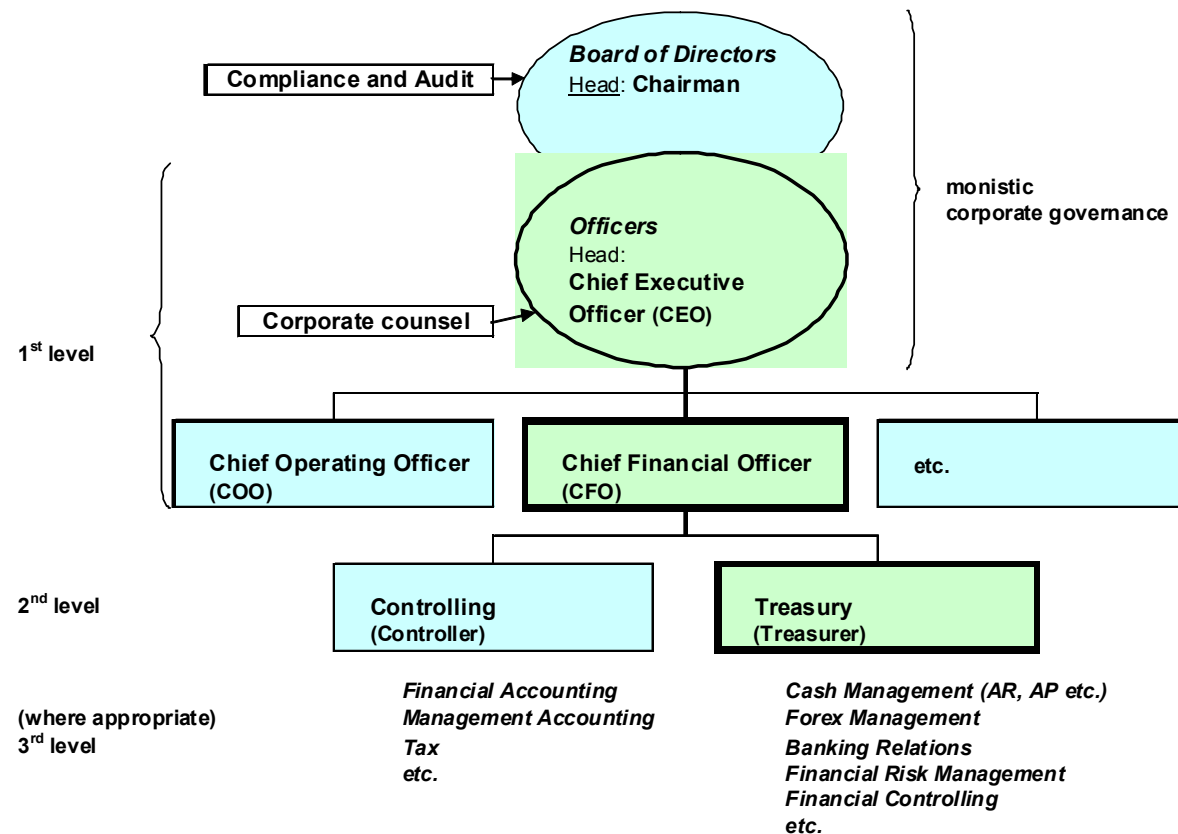
- ✓ Assuring the company's ability to pay
(⇒ sufficient liquidity)
- ✓ Little annoyance or (even better) strong
support of operations
(⇒ no affluent liquidity)
- ✓ Great contribution to the company's rentability
(⇒ efficient use of liquidity reserves)
- ✓ Efficient risk management
(⇒ implementation of hedging, insurance
contracts, derivatives etc.)
- ✓ Little restriction of entrepreneurial freedom
(⇒ avoidance of too many covenants)



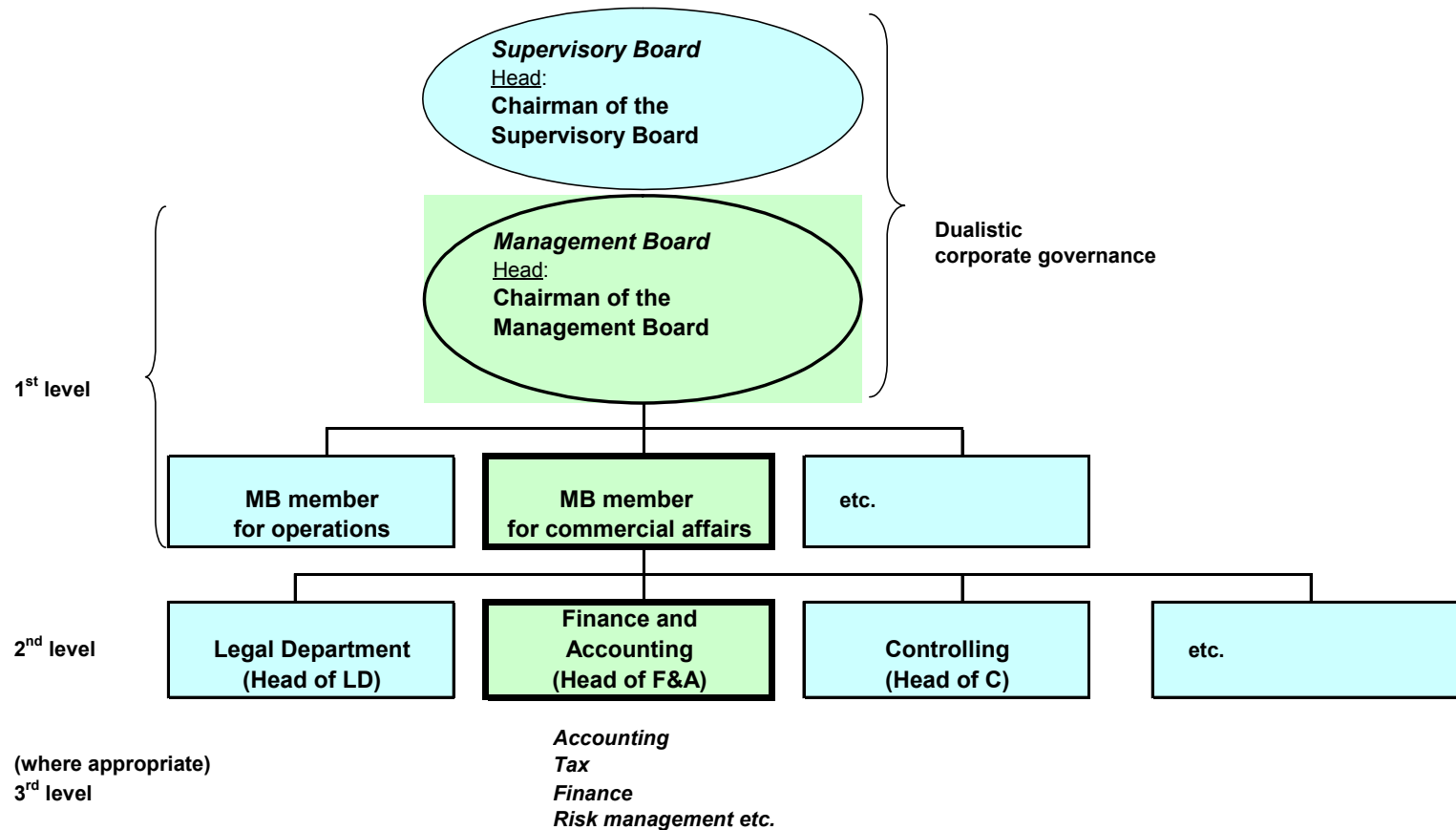
The POC-Structure of Treasury Management



Financial Organisational Structure: The US Model (here: Inc.)

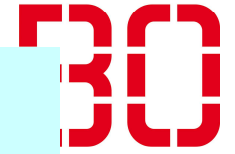


Financial Organisational Structure: The European Model (here: AG)



Questions of Strategic Financial Planning

- ✓ How many main banking relationships?
Which banks?
- ✓ Securitization and Going public
- ✓ (Debt:) Fix–floating mix
- ✓ Target rating
- ✓ Three year budget



etc.

Exercise 13-1

The Inovativny obchod a.s. from Brno is listed on the stock exchange and with 21,000 employees and 150 locations one of the leading department store companies in Europe. Within the management board, Mrs. Alice Babičková is in charge of the entire commercial affairs of the company. Among other organisational units, the treasury department directed by Mr. František Kohut belongs to her area of responsibility.

i) Is the corporate governance of the company monistic or dualistic? Does the organisation of the financial sphere of Inovativny obchod follow the European or the US-Model?

The accumulated lines of credit of the company amount to CZK 7,000,000. As of December 31, 2025, the company reports the subsequent data (consolidated balance sheet, consolidated income statement, consolidated cash flow statement).

Consolidated balance sheet, Inovativný obchod a.s., Brno, as of December 31, 2025

Assets	25, CZK mio.	%	24, CZK mio.	%	Liabilities	25, CZK mio.	%	24, CZK mio.	%
Fixed assets					Equity				
<i>Intangible assets</i>	20,0	1,4%	18,0	1,3%	<i>Capital stock</i>	240,0	16,9%	240,0	17,2%
<i>Tangible assets</i>	315,0	22,2%	300,0	21,5%	<i>Additional paid-in capital</i>	20,3	1,4%	20,3	1,5%
<i>Financial assets</i>	125,0	8,8%	115,0	8,2%	<i>Reserves from retained earnings</i>	103,3	7,3%	97,3	7,0%
	460,0	32,4%	433,0	31,0%	<i>Net profit</i>	59,1	4,2%	86,0	6,2%
						422,7	29,8%	443,6	31,8%
Current assets					Provisions				
<i>Inventories</i>	810,3	57,1%	805,2	57,7%	<i>Provisions for pensions & similar commitments</i>	198,5	14,0%	154,2	11,1%
<i>Accounts receivable</i>	10,9	0,8%	12,3	0,9%	<i>Other provisions</i>	125,9	8,9%	55,2	4,0%
<i>Other receivables</i>	62,4	4,4%	69,5	5,0%		324,4	22,8%	209,4	15,0%
<i>Cash, cash equivalents etc.</i>	70,8	5,0%	70,8	5,1%	Liabilities				
1. Cash, cheques	4,1	0,3%	6,2	0,4%	<i>Financial liabilities</i>	257,0	18,1%	261,2	18,7%
2. Bank deposits	61,4	4,3%	59,8	4,3%	<i>Accounts payable</i>	256,3	18,0%	310,1	22,2%
3. Securities	5,3	0,4%	4,8	0,3%	<i>Other liabilities</i>	155,6	11,0%	165,8	11,9%
	954,4	67,2%	957,8	68,7%		668,9	47,1%	737,1	52,8%
Deferred tax assets	2,2	0,2%	1,8	0,1%	Deferred tax liabilities	1,3	0,1%	1,9	0,1%
Prepaid expenses & deferred charges	3,4	0,2%	2,6	0,2%	Prepayments & deferred income	2,7	0,2%	3,2	0,2%
	1420,0	100,0%	1395,2	100,0%		1420,0	100,0%	1395,2	100,0%

Consolidated cash flow statement, Inovativny obchod a.s., Brno, 2025, CZK mio.		
	2025	2024
EBIT	153,4	221,7
Depreciation and amortization of tangible and intangible assets	89,6	84,3
Change in provisions for pensions and other provisions	50,0	45,0
Change in net working capital	-52,4	-60,3
Income taxes paid	10,3	40,4
Other	-20,3	-60,3
Cash flow from operating activities	230,6	270,8
Investments in tangible assets (excl. Finance leases)	-100,0	-105,9
Other investments	-16,2	-15,8
Company acquisitions & divestments	-10,2	-30,8
Cash flow from investing activities	-126,4	-152,5
Profit distributions		
- to parent company stockholders	-80,0	-80,0
- to other stockholders	-1,0	-1,0
Raising of financial liabilities	301,2	350,4
Redemption/repayment of financial liabilities	-281,6	-299,1
Interest paid	-70,5	-87,4
Interest received	25,1	24,3
Profit & loss transfers and other financing activities	2,1	-10,5
Cash flow from financing activities	-104,7	-103,3
Total cash flows	-0,5	15,0
Cash and cash equivalents as of January 1	66,0	51,0
Cash and cash equivalents as of December 31	65,5	66,0

Consolidated income statement, Inovativny obchod a.s., Brno, CZK mio.										
	2028B	%	% change	2027B	%	% change	2026B	%	% change	2025A
Gross sales	4813,0	115,1%	2,2%	4710,0	115,1%	2,4%	4600,0	115,1%	2,7%	4477,4
Sales tax	633,0	15,1%	2,2%	619,5	15,1%	2,4%	605,0	15,1%	3,0%	587,4
Net sales	4180,0	100,0%	2,2%	4090,5	100,0%	2,4%	3995,0	100,0%	2,7%	3890,0
Cost of sales	2435,0	58,3%	2,3%	2380,0	58,2%	2,1%	2330,0	58,3%	2,2%	2279,5
Gross profit on sales	1745,0	41,7%	2,0%	1710,5	41,8%	2,7%	1665,0	41,7%	3,4%	1610,5
Selling expenses	1705,0	40,8%	2,1%	1670,0	40,8%	1,8%	1640,0	41,1%	2,6%	1598,8
General administrative expenses	114,0	2,7%	3,6%	110,0	2,7%	2,8%	107,0	2,7%	5,8%	101,1
Other operating income	298,0	7,1%	1,0%	295,0	7,2%	0,7%	293,0	7,3%	0,4%	291,8
Other operating expenses	35,0	0,8%	-22,2%	45,0	1,1%	12,5%	40,0	1,0%	2,8%	38,9
EBITA	189,0	4,5%	4,7%	180,5	4,4%	5,6%	171,0	4,3%	4,7%	163,4
Amortization good will	10,0	0,2%	0,0%	10,0	0,2%	0,0%	10,0	0,3%	0,0%	10,0
EBIT	179,0	4,3%	5,0%	170,5	4,2%	5,9%	161,0	4,0%	5,0%	153,4
Result from associated companies	-1,0	0,0%	50,0%	-1,5	0,0%	40,0%	-2,1	-0,1%	-5,0%	-2,0
Interest result	-49,0	-1,2%	-4,1%	-47,0	-1,1%	-4,3%	-45,0	-1,1%	0,0%	-45,0
Other financial result	1,3	0,0%	0,0%	1,3	0,0%	-7,1%	1,4	0,0%	0,0%	1,4
Financial profit	-48,7	-1,2%	-3,1%	-47,2	-1,2%	-3,2%	-45,7	-1,1%	-0,2%	-45,6
EBT	130,3	3,1%	5,7%	123,3	3,0%	6,9%	115,3	2,9%	7,1%	107,7
Income taxes	51,5	1,2%	5,7%	48,7	1,2%	6,9%	45,5	1,1%	6,9%	42,6
Net profit for the period	78,8	1,9%	5,7%	74,6	1,8%	6,9%	69,8	1,7%	7,0%	65,2
Allocable to minorities	9,1	0,2%	3,4%	8,8	0,2%	25,7%	7,0	0,2%	14,8%	6,1
Net profit	69,7	1,7%	6,0%	65,8	1,6%	4,8%	62,8	1,6%	6,2%	59,1

ii) Make yourself familiar with the data by elaborating the three immediately recognisable links between the three calculations!

As can be scrutinized here, Mr. Kohut's treasury department has made up the internal budgeting and reporting form similar to the cash flow statement according to IAS/IFRS:

Consolidated Cash Flow Statement (Budget), Inovativny obchod a.s., Brno, CZK mio.			
	2028	2027	2026
Net sales			
Costs of goods sold			
Wages			
Overheads			
Other incoming payments from operations			
Other outgoing payments from operations			
Income taxes			
Cash flow operations			
Cash flow investment			
Profit distribution			
Increase in capital			
Debt finance			
Redemption			
Interest result			
Other			
Cash flow finance			
Total cash flows			
Cash & cash equivalents as of January 1			
Cash & cash equivalents as of December 31			

iii) Does the treasury department for internal purposes calculate the cash flow from operating activities in a direct or in an indirect manner?



For budgeting purposes 2026-2028, the treasury department deems the subsequent projections to be valid:

- ⇒ incoming payments from net sales will be like in the consolidated budget income statement
- ⇒ outgoing payments for cogs will be like in the consolidated budget income statement
- ⇒ outgoing payments for wages per capita will equal CZK 30,000 in 2026; from then onwards yearly increase of 2.3%
- ⇒ outgoing payments for overheads will amount to 49% of consolidated total assets in 2026; from then onwards yearly increase of 2.0%
- ⇒ other incoming payments from operations amount to 10% of other operating income in the consolidated budget income statement
- ⇒ other outgoing payments from operations amount to 60% of other operating expenses in the consolidated budget income statement

income taxes paid will equal the expenses for income taxes in the consolidated budget income statement

- ⇒ profit distribution remains on the 2025 level visible in the consolidated cash flow statement
- ⇒ cash flow investment: CZK 230 mio. in 2026, CZK 240 mio. in 2027, CZK 250 mio. in 2028
- ⇒ debt finance and redemption, respectively, will in 2026, 2027 and 2028 equal the amounts of the year 2025 visible in the consolidated cash flow statement rounded to CZK 10 mio.
- ⇒ interest results on the payment level will equal the interest results in the consolidated budget income statement
- ⇒ other financial items will equal the total of result from associated companies and other financial result in the consolidated budget income statement



*iv) Make up a first draft of the strategic treasury budget of Inovativny ob-
chod by using the internal form and taking into consideration the afore-
mentioned projections! Then comment on your result!*

Series of Payment

Cash effectiveness

e

+

Timely difference

$t = 0, 1, \dots, \bar{t}$

=

Series of payment

$e_t ; t = 0, 1, \dots, \bar{t}$

Investment Projects

An investment measure (sometimes called “investment” in short) is a series of payment

- **that begins with an outgoing payment**
- **and features at least one change in sign**

Classification scheme can be transferred mutatis mutandis to “financing measures” and “financing projects”.

Types:

- **Real investments**
- **Financial investments**

An investment project is an investment that fulfils a third criterion:

- **its realisation is still subject to decision.**

Exercise 5-1

The Jemná Čokoláda a.s. wants to acquire a machine for the production of chocolate bars. The price of the machine due for immediate payment in $t=0$ amounts to CZK 100,000. The machine would for three years allow the production of 5,000 chocolate bars per year. Each bar could be sold for instant incoming payment of CZK 20. On the other hand, ingredients (secret recipe!) and other production factors would require outgoing payments of CZK 10 per bar. At the end of the physical life of the machine, disassembly cost would induce outgoing payments of CZK 40,000.

Assume Jemná Čokoláda's point of view and determine the series of payment of the project "chocolate bar machine"!

Financing projects

A financing measure is a series of payment

- that begins with an incoming payment
- and features at least one change in sign

A financing project is a financing measure that fulfils a third criterion:

- its realisation is still subject to decision.

⇒ *Most of the concepts developed subsequently for investment projects can be transferred mutatis mutandis to financing projects.*

Exercise 6-1

The financial contract known from exercise 4-1 is again taken into consideration.

From whose perspective is the signing of this contract a financing project, from Candice's or Quentin's?

Exercise 5-2

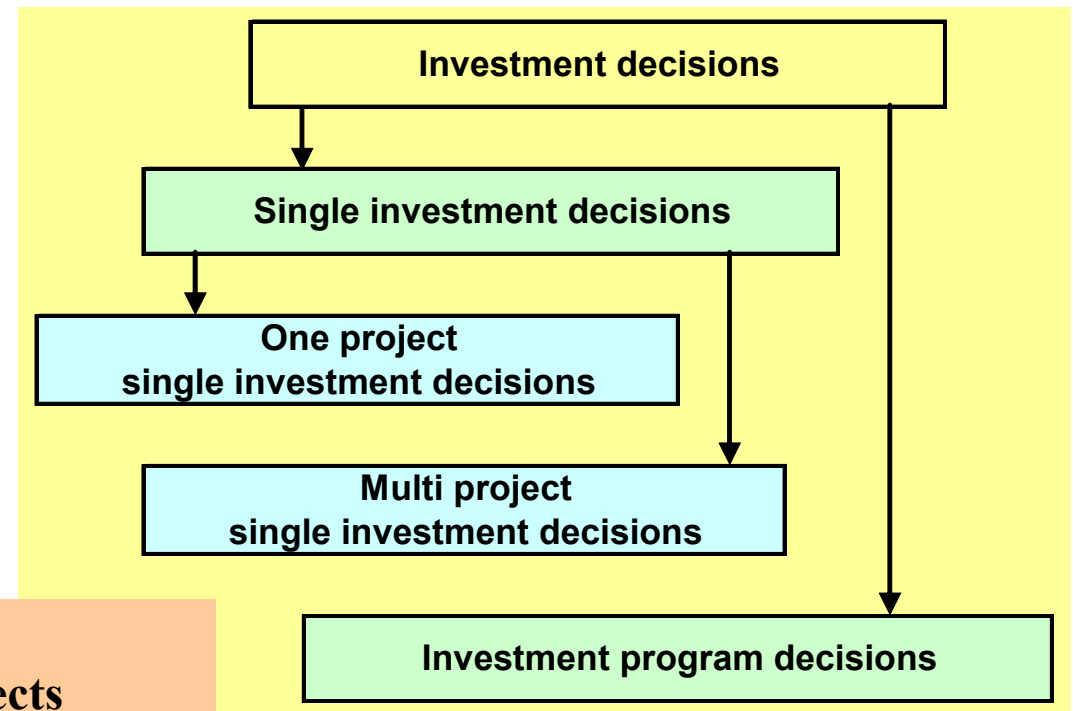
The Jemná Čokoláda a.s. has entered the stage of a more intense scrutiny of the chocolate bar machine known from exercise 5-1. To this end, it is to be compared with the status quo.

Determine the series of payment of the status quo for this scenario!

Investment Decisions

(Fig. 5-1)

An investment decision is a selection from a catalogue of investment projects and the status quo.



Typical of TREASURY MANAGEMENT:
Selection from a catalogue of investment projects
or
from a catalogue of financing projects,
but not
from a catalogue of investment *and* financing projects.

Investment calculus

Investment calculus offers procedures for investment decisions (i. e.: for the comparison of certain series of payment).

(Economic calculus is the more general concept and offers procedures for investment decisions *and* financing decisions.)

⇒ **6 D's**

- Complete account
- Dominance
- Net present value
- Annual equivalent
- Internal rate of return
- Payback period

“D” like...

...Dynamic Investment Calculus...

DYNAMIC INVESTMENT CALCULUS

- (1) starts off from incoming and outgoing payments (upmost level of Schmalenbach’s bar graph) and**
- (2) takes the timely differences between different payments explicitly into consideration.**

(In other words: Dynamic investment calculus is based on series of payment.)

If one of the two aforementioned criteria is not fulfilled, it is STATIC INVESTMENT CALCULUS.

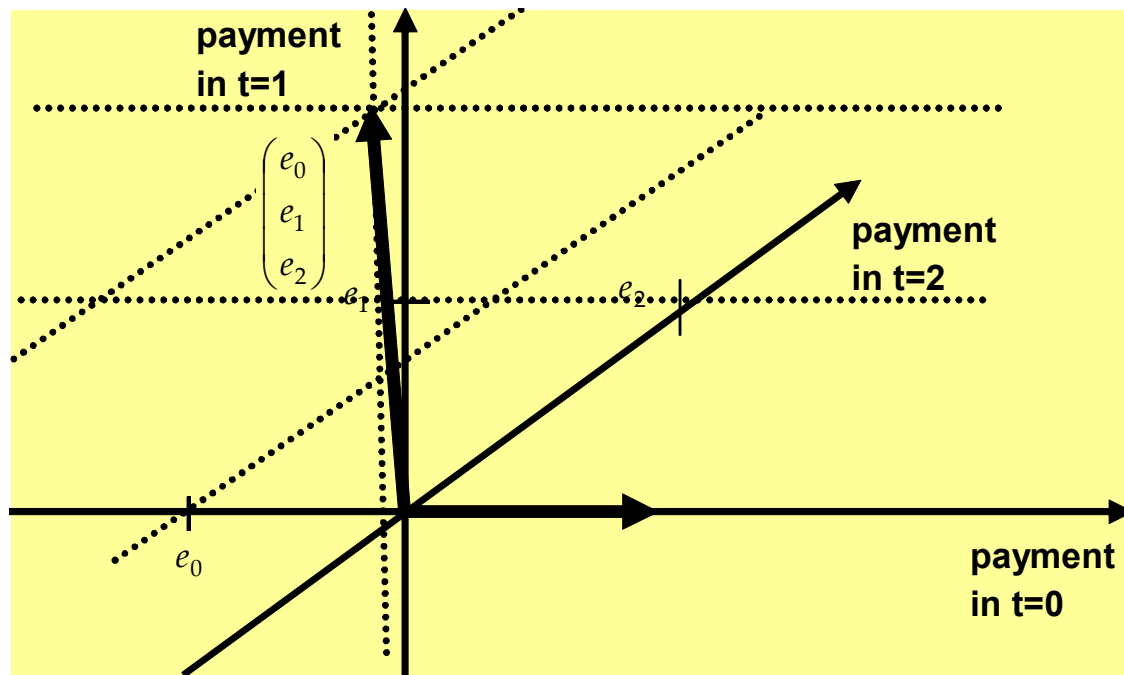
Scalars and Vectors, Norm of a Vector

SCALAR:

onedimensional figure
in vector space

VECTOR:

multidimensional figure
in vector space



Mathematically speaking, investment calculus requires the comparison of vectors, i. e. of multidimensional figures in the vector space.

Norm of a vector ε :

$$\|\varepsilon\| = \sqrt{e_0^2 + e_1^2 + \dots + e_i^2}$$

Exercise 8-1

The chocolate bar machine known from exercises 5-1 and 5-2 is again taken into consideration.

Determine the norm of the series of payment of this investment project and interpret on your result from an economic point of view!

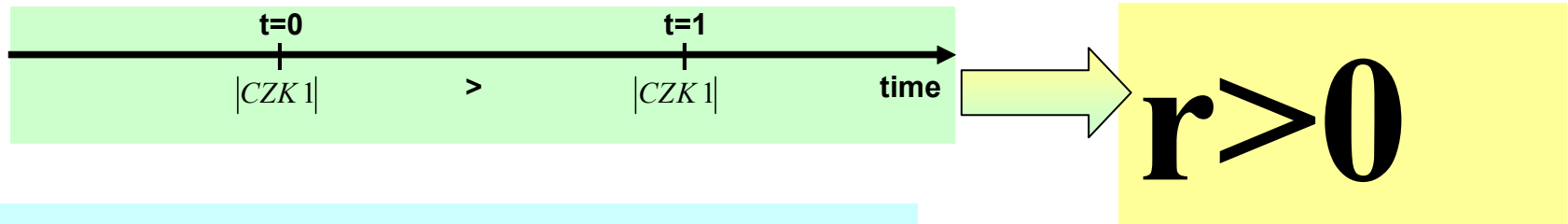
Our Catalogue of Assumptions for Investment Calculus

1. Payments do only occur at points in time like $t=0$ and $t=1$ and not inbetween (discrete time)
2. Payments relating to a period of time (like interest) are realised at the end of the relevant period and not at its beginning
(payments in arrear, no payments in advance)
3. Deterministic payments and interest rates
(no stochastic)



4. Timely invariant interest rates
(homogenous term structure of interest rates)
5. No taxes

The Relevance of the Interest Rate



Usual scenario: $r_B > r_L$; borrowing rate exceeds lending rate (a market imperfection)

*It is better to receive CZK 1 today than tomorrow.
It is worse to be obliged to pay CZK 1 today than tomorrow.*

After transformation to a fixed point in time by means of the interest rate, the absolute value of CZK 1 paid in the present is greater than the absolute value of CZK 1 paid in the future.

Complete Account of an Investment Project (preliminary definition)

Explicit clearing of a series of payment by means of a simple account complying with the following conventions:

- i. Except for the final point in time, the account may never be overdrafted. To this end, possible deficits



have to covered by a separate credit at borrowing
rate r_B .

(Please turn over.)

- ii. Except for the final point in time, the account may never display a positive balance. To this end, possible surpluses have to be invested in a term money at lending rate r_L .
- iii. Credits and term moneys each have a duration of one period.
- iv. At the final point in time, no credits may be raised and no term money investments are possible anymore. The final balance of the account is represented by the symbol FW_p , i. e. the final wealth generated by the project in consideration



v. For the time being, the initial wealth of the holder of the account (or the decision taking entity, respectively) equals zero: $IW_s = 0$

Exercise 8-2

The chocolate bar machine known from exercises 5-1, 5-2 and 8-1 is again taken into consideration. The borrowing rate is $r_B = 0.05$ per period, the lending rate $r_L = 0.01$.

Determine by means of a complete account complying with the aforementioned conventions i. to v. the final wealth FW_P of the project (and thus the increase in final wealth compared to the status quo!)

The Objective of Investment Calculus

(Or more general: The Objective of
Economic Action in a Dynamic Context)



Final Wealth Maximization

Complete Account: Decision Rule

- a) For ONE PROJECT SINGLE INVESTMENT DECISIONS, the project taken into consideration is favourable *if it generates an increase in final wealth compared to the status quo.*
- b) For MULTI PROJECT SINGLE INVESTMENT DECISIONS, the project *that maximizes the increase in final wealth compared to the status quo* is favourable. (If none of the projects taken into consideration generates an increase, at all, the status quo is favourable.)

Equivalent Initial Wealth of a Project

Fictitious back-calculation of the final wealth caused by a project that complies with the following two conventions:

- i. After the point in time of decision ($t = 0$), for $\bar{t} \geq 1$ no further payments occur in $t = 1, \dots, \bar{t}$. This means that in $t = 0$ there is one single investment of a term money or one single raising of credit, respectively.
- ii. If $FW_P > 0$, the equivalent initial wealth IW_P can only be generated by means of an investment in a term money in $t = 0$; r_L is the relevant interest rate then. For $FW_P < 0$, the equivalent

initial wealth IW_P can only go back to an initial credit raising;
in this case, r_B is the relevant interest rate.

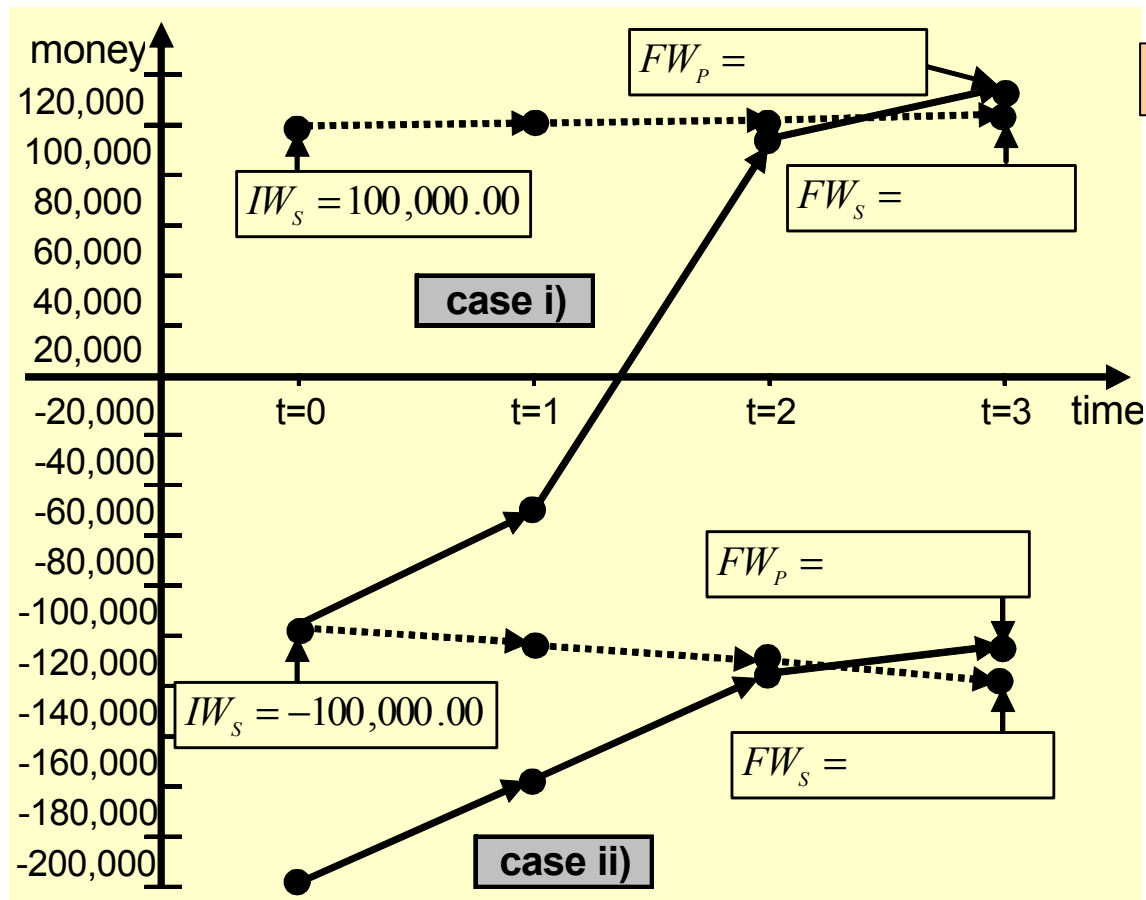
Exercise 8-4a

The chocolate bar machine known from exercises 5-1, 5-2, 8-1 and 8-2 is again taken into consideration. Like before, the borrowing rate is $r_B = 0.05$ per period, the lending rate $r_L = 0.01$.

Determine by means of a complete account following the aforementioned conditions i. and ii. the initial wealth IW_p that is equivalent with the final wealth FW_p calculated in exercise 8-2!

Non-Zero Initial Wealth

(Fig. 8-3)

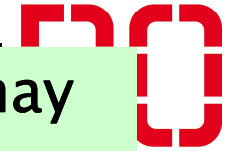


In general, the increase in final wealth generated by an investment project is not independent from the level of the initial wealth of the decision taking entity.

Complete Account of an Investment Project (final definition)

Explicit clearing of a series of payment by means of a simple account complying with the following conventions:

- i. Except for the final point in time, the account may never be overdrafted. To this end, possible deficits have to be covered by a separate credit at borrowing rate r_B .



- ii. Except for the final point in time, the account may never display a positive balance either. To this end, possible surpluses have to be invested in a term money at lending rate r_L .
- iii. Credits and term moneys each have a duration of one period.
- iv. At the final point in time, no credits may be raised and no term money investments are possible anymore. The final balance of the account is represented by the symbol FW_P , i. e. the final wealth generated by the project in consideration



Compared to the preliminary definition of the complete account, the decision rule remains unchanged. (See separate slide.)

Exercise 8-5

The chocolate bar machine known from exercises 5-1, 5-2, 8-1, 8-2 and 8-4a is again taken in consideration. In contrast to exercise 8-2, the initial wealth of the decision-taking entity IW_s is different from zero and amounts to i) CZK 100,000 and ii) CZK -100,000.

Determine the increase in final wealth caused by the machine in case i) and in case ii), respectively, transfer your results to figure 8-3 and comment on them!



Dominance: Definition and Decision Rule

Let ε and ε' be non-identical series of payments (or, speaking differently, vectors of dimension $\bar{t} + 1$).

ε DOMINATES ε' if and only if in pairwise comparison every element e_t of ε is greater or equal the corresponding element e'_t of ε' . Formally:

$$(D) \quad e_t \geq e'_t \quad \forall t = 0, 1, \dots, \bar{t}$$

If a series of payments DOMINATES another one, it is also PREFERABLE to the other one in the sense of final value maximization.

Exercise 8-6

In addition to the well established brands hazelnut and brittle, the Jemná Čokoláda a.s. is currently having marzipan chocolate in contemplation. An investment in the marzipan project would have the following consequences (all data in CZK and as change compared to the status quo):

$t=0$

- (1) Acquisitions amounting to 10,000,000 (immediately payment effective)**
- (2) Employee Lissi, who has always dreamt of marzipan chocolate when she was working at the conveyor belt and proposed the idea**

to the staff suggestion scheme, is to receive an immediately payment effective premium amounting to 10,000.

$t=1, 2, \dots, 10$ (figures per year)

- (3) Incoming payments caused by increased sale of chocolate amounting to 3,000,000**
- (4) Additional outgoing payments for (a) crude chocolate amounting to 1,000,000, (b) marzipan amounting to 300,000, (c) wages amounting to 200,000**
- (5) Lissi would be much more content with her work.**
- (6) Writeoffs of the new machinery in the financial accounting amounting to 600,000**
- (7) Writeoffs of the new machinery and the good will in the management accounting amounting to 400,000**
- (8) The supplier Brněnské Marcipánové a Nugátové Kontor would encounter a payment effective surplus amounting to 260,000.**

i) Determine the series of payment of project marzipan!

Being deeply impressed by Lissi's idea, the marketing department of the Jemná Čokoláda a.s. has instantaneously calculated the projects nougat, walnut, strawberry yoghurt, raisin nut and peanut. The following series' of payment are presented to the board of directors:

brand	t=0	t=1, 2, ..., 10 (per year)
Nougat	-10,010,000	1,400,000
Walnut	-10,010,000	1,300,000
Strawberry yoghurt	-10,300,000	1,500,000
Raisin nut	-5,000,000	600,000
Peanut	-2,000,000	300,000

ii) *Preselect efficiently those chocolate brands that are under no circumstances consistent with the objective of final wealth maximization!*

Implicit Consideration of Accompanying Financial Activities

Discounting

$$g_0 = g_{\bar{t}} \cdot (1 + r_L)^{-\bar{t}} \quad \text{if} \quad g_{\bar{t}} > 0$$

$$g_0 = g_{\bar{t}} \cdot (1 + r_B)^{-\bar{t}} \quad \text{if} \quad g_{\bar{t}} < 0$$

Compounding

$$g_{\bar{t}} = g_0 \cdot (1 + r_L)^{\bar{t}} \quad \text{if} \quad g_0 > 0$$

$$g_{\bar{t}} = g_0 \cdot (1 + r_B)^{\bar{t}} \quad \text{if} \quad g_0 < 0$$

Exercise 8-7

The initial wealth of the decision taking entity be zero, i. e.: $IV_S = 0$. Consider an investment project with the following series of payment:

$(-980.00, 1,100.00, -10.00)$

As it is shown in the draft (margin number 161, table 8-5), the final wealth of the project for $r_L = 0.01$ and $r_B = 0.05$ per period is 71.81, i. e. $FV_P = 71.81$.

Check whether it is possible to calculate the final wealth of the project by means of compounding! To this end, apply (a) the lending rate, (b) the borrowing rate and (c) a more subtle strategy!

Deviation Analysis for Exercise 8-7 (and thus for More Complex Series of Payment)

Final wealth of the project

$$\begin{aligned} FV_P &= [e_0 \cdot (1+r_B) + e_1] \cdot (1+r_L) + e_2 \\ &= e_0 \cdot (1+r_B) \cdot (1+r_L) + e_1 \cdot (1+r_L) + e_2 \end{aligned}$$

What interest rate is relevant? The answer to this question in general requires a complete account!!!

Equivalent initial wealth of the project

$$IW_P = \frac{FW_P}{(1+r_L)^2} = \frac{e_0 \cdot (1+r_B)}{1+r_L} + \frac{e_1}{1+r_L} + \frac{e_2}{(1+r_L)^2}$$

Exercise 8-8

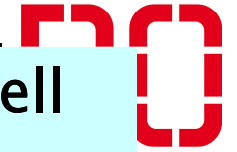
The investment project known from exercise 8-7 is again taken into consideration. The initial wealth of the decision taking entity remains at zero (i. e.: $IW_S = 0$), lending rate and borrowing rate remain at $r_L = 0.01$ and $r_B = 0.05$ per period, respectively.

Check whether it is basically possible to calculate the final wealth and the equivalent initial wealth of the project implicitly on the basis of the preceding deviation analysis!

Perfect Financial Markets (PERFIMA)

Perfect financial markets comply in particular with the following three criteria:

- i. There are no quantitative restrictions on accompanying financial activities (ABSENCE OF RATIONING).
- ii. The same interest rates are relevant for the raising of credit and the investment in term money (BORROWING RATE EQUALS LENDING RATE).



iii. The one-period investment in term money as well as the one-period raising of credit are possible.
(FINEST TIMELY SCALING OF FINANCIAL CONTRACTS).

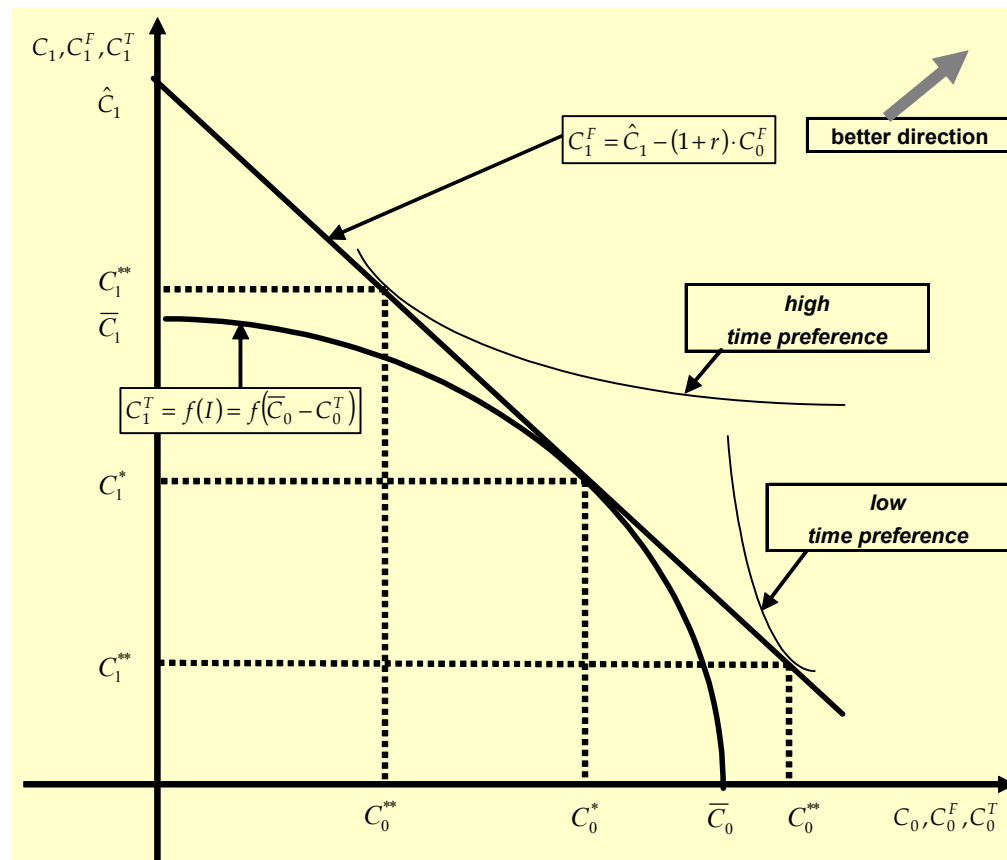
PERFIMA

⇒ Fisher-Separation

(Fig 8-4)



Irving Fisher, 1867-1947



On perfect financial markets, the real investment decision (“firms”) is independent of the time preference and the intertemporal consumption decision (“households”). The opposite does not hold true.

PERFIMA \Rightarrow Implicit Calculus

Even of More Complex Series of Payment

Final wealth

$$\begin{aligned}
 FV_P &= e_0 \cdot (1+r_B) \cdot (1+r_L) + e_1 \cdot (1+r_L) + e_2 \\
 &\stackrel{r_B=r_L=r}{=} e_0 \cdot (1+r)^2 + e_1 \cdot (1+r) + e_2 \\
 &= \sum_{t=1}^{\bar{t}} e_t \cdot (1+r)^{\bar{t}-t} \equiv FW
 \end{aligned}$$

Net present value

$$\begin{aligned}
 IW_P &= \frac{e_0 \cdot (1+r_B)}{1+r_L} + \frac{e_1}{1+r_L} + \frac{e_2}{(1+r_L)^2} \\
 &\stackrel{r_B=r_L=r}{=} e_0 + e_1 \cdot (1+r)^{-1} + e_2 \cdot (1+r)^{-2} = \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{-t} \equiv NPV
 \end{aligned}$$

Net Present Value and Final Wealth Maximization

Final wealth maximization: The difference between the final wealth of the project and the final wealth in the status quo case has to be maximized. For arbitrary initial wealth of the decision taking unit IW_S , the final wealth of the project FW_P may in spite of the aforementioned problems be formulated in the following manner:

$$FW_P = \sum_{t=0}^{\bar{t}} \hat{e}_t \cdot \prod_{\tau=0}^{\bar{t}-t} (1 + r_{B,L}(\tau))$$

where

$$\hat{e}_t = \begin{cases} e_0 + IW_S & \text{for } t = 0 \\ e_t & \text{for } t = 1, 2, \dots, \bar{t} \end{cases}$$

Continuing on this approach, the final wealth in the status quo case is given by:

$$FW_S = IW_S \cdot \prod_{t=0}^{\bar{t}} (1 + r_{B,L}(t))$$

As we have reduced the timely accrual of the initial wealth in the status quo case on $t=0$, this more subtle terminology is apparently redundant in the last equation. (In this case, the borrowing rate *or* the lending rate has to be applied, but not a combination of them. On the other hand, the terminology allows to prove the result even for whole timely sequences of initial wealth.)

Now: PERFIMA

$$\begin{aligned}
 FW_P &= \sum_{t=0}^{\bar{t}} \hat{e}_t \cdot \prod_{\tau=0}^{\bar{t}-t} (1 + r_{B,L}(\tau)) \\
 &\stackrel{r_B=r_L=r}{=} \sum_{t=0}^{\bar{t}} \hat{e}_t \cdot (1+r)^{\bar{t}-t}
 \end{aligned}$$

As this expression is more simple now, a back-transformation from \hat{e}_t to e_t is possible for the final wealth of the project:

$$FW_P = IW_S \cdot (1+r)^{\bar{t}} + \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{\bar{t}-t}$$

Accordingly, the final wealth in the case of the status quo may for PERFIMA be formulated like follows:

$$\begin{aligned}FW_S &= IW_S \cdot \prod_{t=0}^{\bar{t}} (1 + r_{B,L}(t)) \\ &\stackrel{r_B=r_L=r}{=} IW_S \cdot (1+r)^{\bar{t}}\end{aligned}$$

Now let us take the difference of both expressions:

$$\begin{aligned}FW_P - FW_S &= IW_S \cdot (1+r)^{\bar{t}} + \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{\bar{t}-t} - IW_S \cdot (1+r)^{\bar{t}} \\ &= \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{\bar{t}-t} = FV\end{aligned}$$

The resulting expression was already defined as the final value FV of a project. If we now multiply by $(1+r)^{-\bar{t}}$, we obtain the following expression which is exactly the net present value of a project:

$$NPV = \sum_{t=0}^{\bar{i}} e_t \cdot (1+r)^{-t} = FV \cdot (1+r)^{-\bar{i}} = (FW_P - FW_S) \cdot (1+r)^{-\bar{i}}$$

In other words:

- (i) The PERFIMA-assumption is crucial for the NPV-concept.**
- (ii) The NPV-concept is in full accordance with the objective of final wealth maximization.**
- (iii) The NPV is completely independent of the initial wealth of the decision-taking entity in the status quo case.**

Exercise 9-1

The initial wealth in the case of the status quo be zero, i. e.: $IW_s = 0$. The chocolate bar machine known from exercises 5-1, 8-1 and 8-2 is again taken into consideration. The financial market has become perfect now and the interest rates for borrowing and lending both equal 4% now, i. e.: $r = 0.04$.

- i) Calculate the final value as well as the net present value of the project!*



ii) Now check the preceding statement as to which the net present value equals the increase in final wealth discounted to the present!



Explicit Definition of the Net Present Value and Corresponding Decision Rule

Simon Stevin, 1548/49-1620
(the concept of discounting)

The net present value of a series of payment $e_0, e_1, \dots, e_{\bar{t}}$ equals the sum of all its discounted future payments and the initial payment, i. e.:

$$(NPV1) \quad K = \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{-t} = \sum_{t=0}^{\bar{t}} e_t \cdot q^{-t}$$



Gottfried Wilhelm Leibniz,
1646-1716 (dto.)

- a) For one project single investment decisions, the project considered is **PREFERABLE** if and only if it has a **POSITIVE NET PRESENT VALUE**.
- b) For multi project single investment decisions, the project featuring the **MAXIMUM NET PRESENT VALUE** is **PREFERABLE** if the latter one is **POSITIVE**.
(If no project features a positive net present value, at all, the status quo is preferable.)

Special Computational Methods for the Net Present Value

Annuity

$$(NPV 2) \quad K^{Annuity} = e_0 + e \cdot \frac{1 - q^{-\bar{t}}}{q - 1} = e_0 + e \cdot Q(\bar{t}, r)$$

(Q : annuity factor)

Perpetuity

$$(NPV 3) \quad K^{Perpetuity} = e_0 + \frac{e}{r}$$

($1/r$: multiplier)

Exercise 9-2

Determine efficiently the net present values of those new chocolate brands known from exercise 8-6 that cannot be eliminated by means of the dominance criterion! To this end, start off from an interest rate amounting to 5% per period, i. e.: $r = 0.05$!

Annual Equivalent: Definition and Decision Rule

The ANNUAL EQUIVALENT \bar{e} of a project featuring the series of payment $e_0, e_1, \dots, e_{\bar{t}}$ is a constant payment occurring in $t = 1, 2, \dots, \bar{t}$ the net present value of which equals the net present value of the project:

$$(AE1) \quad \bar{e} = \frac{1}{Q(r, \bar{t})} \cdot K(r)$$

- a) For one project single investment decisions, the project considered is **PREFERABLE** if it has a **POSITIVE ANNUAL EQUIVALENT**.
- b) For multi project single investment decisions, the annual equivalent criterion is **ON PRINCIPLE NOT APPLICABLE** (inconsistent with the objective of final wealth maximization)

Exercise 9-4

Determine the annual equivalent of the chocolate bar machine by going back to the results of exercises 5-1 and 9-1 and starting off from an unchanged interest rate amounting to 4% per period, i. e.: $r = 0.04$!

Special Computational Methods for the Annual Equivalent

Annuity

$$(AE2) \quad \bar{e}^{Annuity} = e + \frac{e_0}{Q(r, \bar{t})}$$

(1/Q: reciprocal of the annuity factor)

Perpetuity

$$(AE3) \quad \bar{e}^{Perpetuity} = e + r \cdot e_0$$

Exercise 9-5

Interest rate per period: 5%, i. e.: $r = 0.05$

- i) *Determine efficiently the annual equivalent of project marzipan (which is known from exercises 8-6 and 9-2)!*
- ii) *Would the annual equivalent criterion in the context of exercise 8-6 be apt to select the optimal chocolate brand?*
- iii) *Now assume project marzipan had an infinite duration and determine once more its annual equivalent!*

Exercise 9-6

Starting point is the chocolate bar machine known from exercise 5-1.

- i) Make up a table of values for the project by calculating the net present values for the following interest rates: a) 0%; b) 2%; c) 4%; d) 6%; e) 8%; f) 10%; g) 12%; h) 15%; i) 20%! Round up or down to even CZK amounts!***
- ii) Draw the net present value function of the project for positive interest rates!***

The Net Present Value Function

Face value

$$K(r = 0\%) = \sum_{t=0}^{\bar{t}} e_t \cdot (1+0)^{-t} = \sum_{t=0}^{\bar{t}} e_t$$

(the sum of all the elements of the series of payment)

Convergence

$$\lim_{r \rightarrow \infty} K(r) = \lim_{r \rightarrow \infty} \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{-t} = \lim_{r \rightarrow \infty} \left[e_0 + \sum_{t=1}^{\bar{t}} e_t \right] = e_0$$

(converges against the initial payment)

Slope and curvature

$$\frac{\delta K}{\delta r} < 0 \quad ; \quad \frac{\delta^2 K}{\delta r^2} > 0$$

(strictly monotonously decreasing and strictly convex for standard investment projects)

Standard Investment Projects



STANDARD PROJECTS are projects the series of payments of which features exactly one change of sign (be they standard financing projects or standard investment projects).

The series of payment of a STANDARD INVESTMENT PROJECT begins with an outgoing payment $e_0 < 0$ which is then followed by incoming payments only, i. e.:

$e_t \geq 0 \quad \forall t = 1, 2, \dots, \bar{t}$, where at least one of these payments is strictly positive, i. e.: $\exists t : e_t > 0$.

Exercise 9-8

**Start off from the following series of payment
(all elements in CZK):**

(−100,000, 158,900, 20,000, −80,010)

Make up a table of values for the project by calculating the net present values for the following interest rates: a) 0%; b) 2%; c) 4%; d) 6%; e) 8%; f) 10%; g) 12%; h) 15%; i) 20% and rounding to even CZK amounts and then draw the net present value function of the project for positive interest rates!



The Internal Rate of Return: Definition and Decision Rule

The INTERNAL RATE OF RETURN r^* of a project featuring the series of payment $e_0, e_1, \dots, e_{\bar{t}}$ is given by the interest rate that makes the net present value equal to 0, i. e.:

$$(IRR1) \quad K(r^*) = \sum_{t=0}^{\bar{t}} e_t \cdot (1 + r^*)^{-t} = 0$$

- a) For one project single investment decisions, a standard investment project is **PREFERABLE** if ITS INTERNAL RATE OF RETURN EXCEEDS THE MARKET RATE ($r^* > r$). If the internal rate of return is lower than the market rate ($r^* < r$), it is **disadvantageous** compared to the status quo. For non-standard investment projects, the IRR-criterion is on principle **NOT APPLICABLE**.
- b) For multi project single investment decisions, the IRR-criterion is on principle **not applicable**.

The Internal Rate of Return for Standard Investment Projects (SIP's)

Descartes' Rule of Signs
(applied to investment calculus):

The NUMBER OF INTERNAL RATES OF RETURN of a project is either equal to the number of sign changes of its series of payment or less than it by a multiple of 2.



René Descartes, 1596-1650

Conclusion: A standard investment project with a strictly positive face value has exactly one internal rate of return.

Special Computational Methods for the Internal Rate of Return

SIP, series of payment with only 2 elements

$$(IRR2) \quad r^* = \sqrt[\bar{t}]{-\frac{e_{\bar{t}}}{e_0}} - 1$$

SIP, finite annuity part immediately after initial outgoing payment

$$(IRR3) \quad Q(r^*, \bar{t}) = -\frac{e_0}{e} \Rightarrow \text{Table III}$$

SIP, perpetuity immediately after initial outgoing payment

$$(IRR4) \quad r^* = -\frac{e}{e_0}$$

SIP, outgoing payment, then fixed interest, then redemption in grand total

$$(IRR5a) \quad r^* = \frac{i + \frac{z - a}{\bar{t}}}{a}$$

i : interest rate; z : pay back rate; a : payout rate

SIP, outgoing payment, then fixed interest and instalment redemption

$$(IRR5b) \quad r^* = \frac{i + \frac{z - a}{T}}{a} ; \quad T \equiv f + \frac{\hat{t} + 1}{2} ; \quad \hat{t} = \bar{t} - f$$

f : years free of redemption; \hat{t} : years with redemption; T : "medium" term

Base formula for regula falsi

$$(IRR8) \quad \hat{r}_{(1)} = \frac{r_L \cdot K(r_R) - r_R \cdot K(r_L)}{K(r_R) - K(r_L)}$$

L : left of the zero; R : right of the zero

Regula Falsi-Algorithm

- i. Find interest rates r_L and r_R that comply with the conditions (a) $r_L < r_R$ (as close as possible) and (b) $K(r_L) \cdot K(r_R) < 0$.
- ii. Determine $\hat{r}_{(1)}$ by means of formula (IRR8).
- iii. If $K(\hat{r}_{(1)}) = 0$, the procedure ends. Otherwise substitute r_L or r_R , respectively, by $\hat{r}_{(1)}$ so that again (b) $K(r_L) \cdot K(r_R) < 0$ is valid.
- iv. Go back to steps ii. and iii. and apply the rules mutatis mutandis to determine $\hat{r}_{(2)}$ ($\hat{r}_{(3)}, \hat{r}_{(4)}$ and so on).

Exercise 9-9

(All payments in CZK.) Consider an investment project whose series of payment consists of an outgoing payment in $t=0$ amounting to -10,000.00 and an incoming payment in the amount of 11,576.25 in $t=3$.

- i) *Determine the internal rate of return of the project!*
- ii) *Which payment in $t=3$ would instead result in an internal rate of return of 6%?*

Exercise 9-10

Determine efficiently the internal rates of the return of the different chocolate brands known from exercise 8-6!

Exercise 9-11

Approximate the internal rate of return of project marzipan known from exercise 8-6 by assuming its annuity payment would cover an infinite time horizon and compare your result with the one from exercise 9-10!

Exercise 9-12

A fixed income credit contract that is redeemed in grand total after a maturity of $\bar{t} = 2$ years is considered. The interest rate is $i = 0.05$ per period, the payout rate $a = 0.95$ and the payback rate $z = 1.05$.

Approximate the internal rate of return of this financial contract!

Exercise 9-13

A fixed income credit contract with a face value amounting to CZK 100,000 that is paid out in the amount of CZK 94,714.62 is taken into consideration. The interest per year amounts to CZK 5,000. Repayment will be at face value after 5 years.

Determine the internal rate of return of this financial contract for the following repayment patterns: (a) redemption in grand total, (b) instalment redemption (no free years), (c) annuity redemption and (d) zerobond! Then comment on your result!

Exercise 9-14

The chocolate bar machine known from exercises 9-6 and 5-1 is again taken into consideration.

- i) *Make a first linear estimate $\hat{r}_{(1)}$ for its internal rate of return by implementing the regula falsi-algorithm and choosing (intentionally in a suboptimal manner) $r_L = 0.04$ and $r_R = 0.08$!*
- ii) *Now round $\hat{r}_{(1)}$ from part i) to entire percent and make a second linear estimate $\hat{r}_{(2)}$!*

Payback period: Definition and Decision Rule

The PAYBACK PERIOD $[0, t^*]$ of a project featuring the series of payment $e_0, e_1, \dots, e_{\bar{t}}$ is given by the point in time $t^* < \bar{t}$ at which its net present value becomes positive for the first time, i. e.:

$$(PBP1) \quad \sum_{t=0}^{t^*-1} e_t \cdot (1+r)^{-t} < 0 \leq \sum_{t=0}^{t^*} e_t \cdot (1+r)^{-t}$$

- a) For one project single investment decisions, a standard investment project is **PREFERABLE** if it has a payback period. For non-standard investment projects, the PBP-criterion is on principle not applicable.
- b) For multi project single investment decisions, the PBP-criterion is on principle not applicable.

A Special Computational Method for the Payback Period

Annuity

$$(PBP2) \quad Q(t^* - 1, r) < -\frac{e_0}{e} \leq Q(t^*, r)$$

\Rightarrow *table III*

Exercise 9-15

Consider an investment project that is characterized by the subsequent series of payment (all figures in CZK):

$$e_0 = -500, \quad e_1 = 200, \quad e_2 = 100, \quad e_3 = 300; \quad e_4 = 60$$

- i) Start off from a market rate amounting to $r = 0.05$ and determine the payback period of the project!*
- ii) Determine efficiently the payback period of project marzipan known from exercises 8-6, 9-2 and 9-5 for $r = 0.05$!*