Business Management in the Czech Republic

Multistage decision-making

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Structure of this lecture

- Basic introduction to decision making
- Multi-stage decision making, decision trees
  - Expected value
  - Dependent Uncertainties
  - Utility functions
  - Expected utility
  - Certainty equivalent
- Value of perfect information
- Value of imperfect information

What is decision making?

- the cognitive process leading to the selection of a course of action among variations
- can be an action or an opinion

Why we need decision analysis?

- the probability of a right decision in a managerial environment,
  which is full of uncertainty, where all element that influence the result are given only as numbers of probability or are not given at all.

Decision making in business and management

- SWOT Analysis
- Buyer decision processes
- Corporate finance:
  - The investment decision
  - The financing decision
- Cost-benefit analysis
- **Decision trees**
- Linear programming
- Min-max criterion
- Model (economics)
- Monte Carlo method
- Strategic planning process

Multistage decision-making

- Has more than one stage – consequential decisions
- Occurs usually in the environment of uncertainty or risk
- Has to follow the objectives of decision-maker
- Can be illustrated by a decision tree

Xanadu Traders, part 1

Xanadu Traders –
a privately held U.S. metals broker

George Xanadu
owner of Xanadu Traders

Molyzirconium ore

Zeldavia

Decision problem

Opportunity: purchase of 1 million kg of molyzirconium

Purchase: $5 per kg **Costs:** $5 000 000

Sale: $8 per kg **Earnings:** $8 000 000

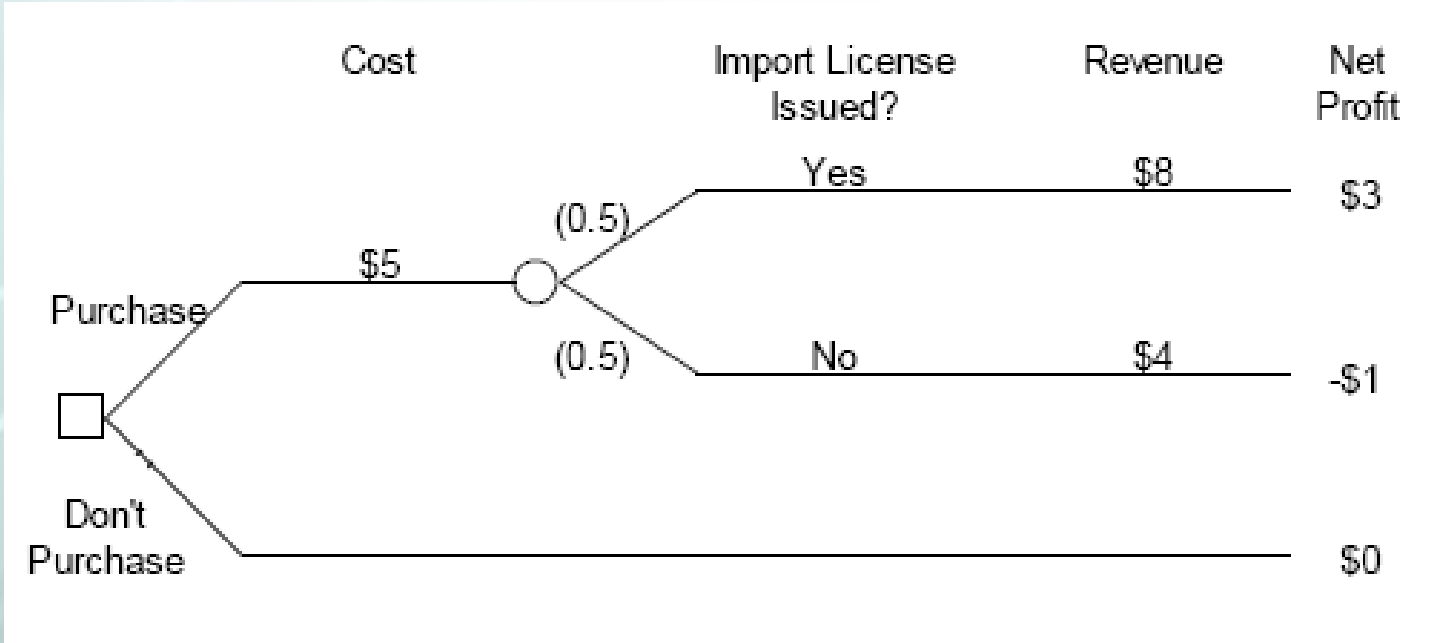
Profit: $3 000 000

- government may ban import of molyzirconium from Zeldavia
- Xanadu Traders will need import license
- there is 50% chance to obtain import license from government

Cancellation Fee: $1 000 000

Buy or not to buy molyzirconium from Zeldavia?

Decision tree n.1



- decision tree
- root mode
- decision node
- decision alternatives

- chance nodes
- probability
- end point

Theory intermission n.1
Expected value (EV)

Bayes rule:

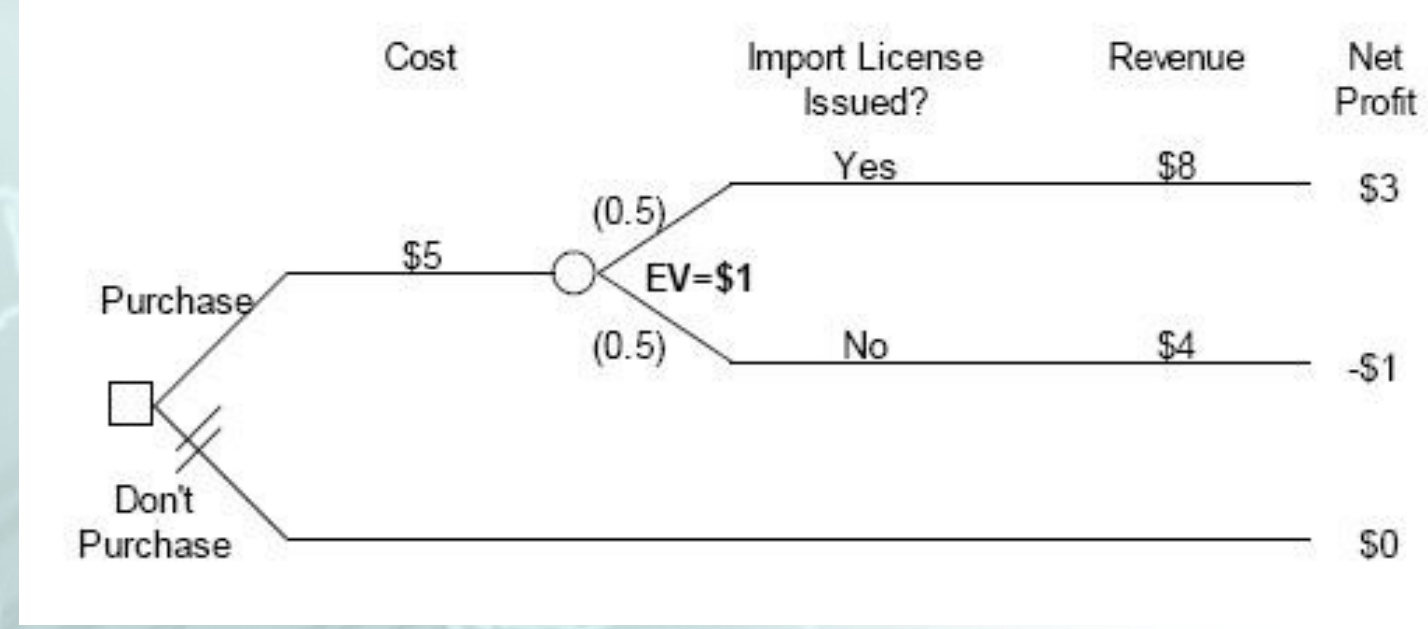
$$\overline{C}_i = \sum_{j=1}^{n} C_{ij} \cdot S_j$$

- Cij …Possible outcome
  - Sj … Probability
- Ci … Expected value

- Situations involving profits („more is better“) – highest expected value is the best
- Situations involving costs („less is better“) – lowest expected value is the best

Decision tree n.2
Expected value

EV = 0,5 x $3 + 0,5 x (-$1) = $1 milion



Expected value is higher for „purchase“ alternative.

Xanadu Traders will buy molyzirconium ore.

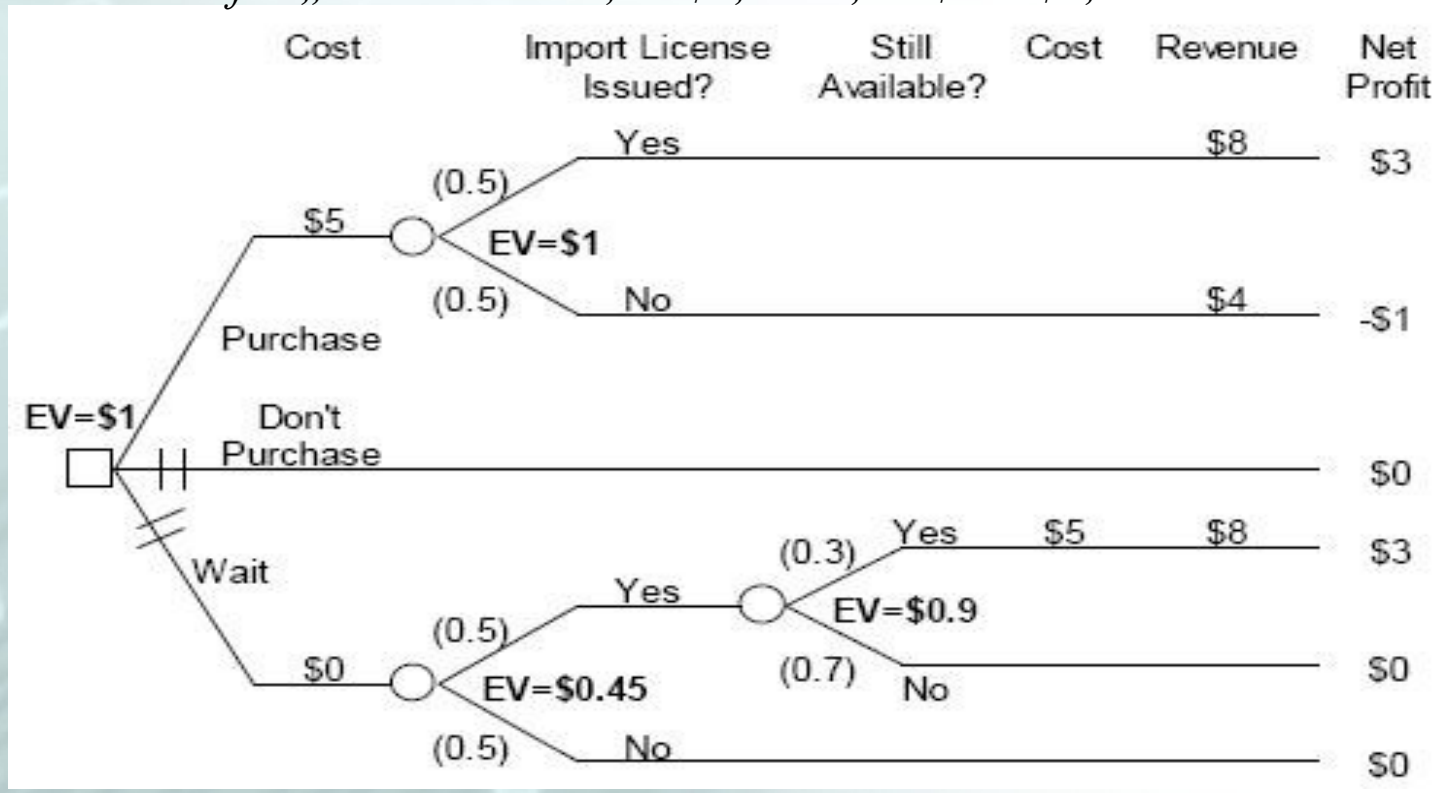
Xanadu Traders, part 2

- Zeldavia has offered molyzirconium ore also to other companies
  - Price of import licence $0
- According to George Xanadu, there is 70% probability that somebody else will take Zeldavia‘s offer

- Question: Is it worth to wait for import licence or not?

Decision tree n.3
Dependent Uncertainties

- *EV for „Still available“: EV = 0,3 x $3 + 0,7 x $0 = $0,9*
- *EV for „Wait“: EV = 0,5 x $0,9 + 0,5 x $0 = $0,45*



- *EV „Wait“ < EV „Purchase“*

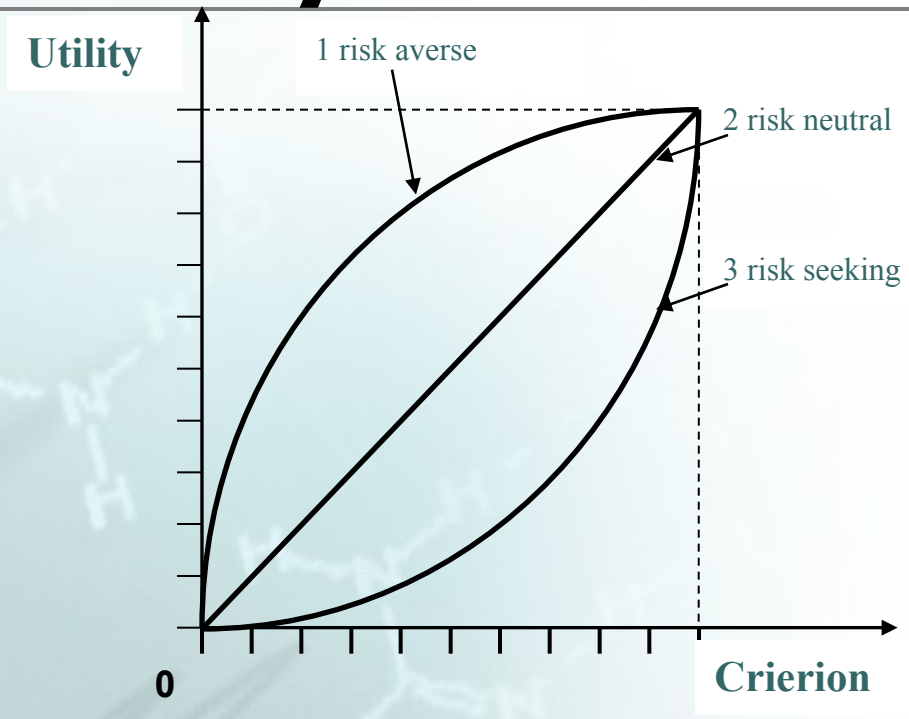
Xanadu will not wait for the import licence

Theory intermission n.2
Utility functions

- **Risk attitude**
  - risk averse
  - risk seeking
  - risk neutral

- **Certainty equivalent (CE)**
  - counts with the risk involved in decision making
  - may be different than Expected Value (EV)
- **If:**
  - CE > EV – risk seeking
  - EV > CE – risk averse
  - EV = CE – risk neutral

Theory intermission n.2
Utility functions



Exponential utility function

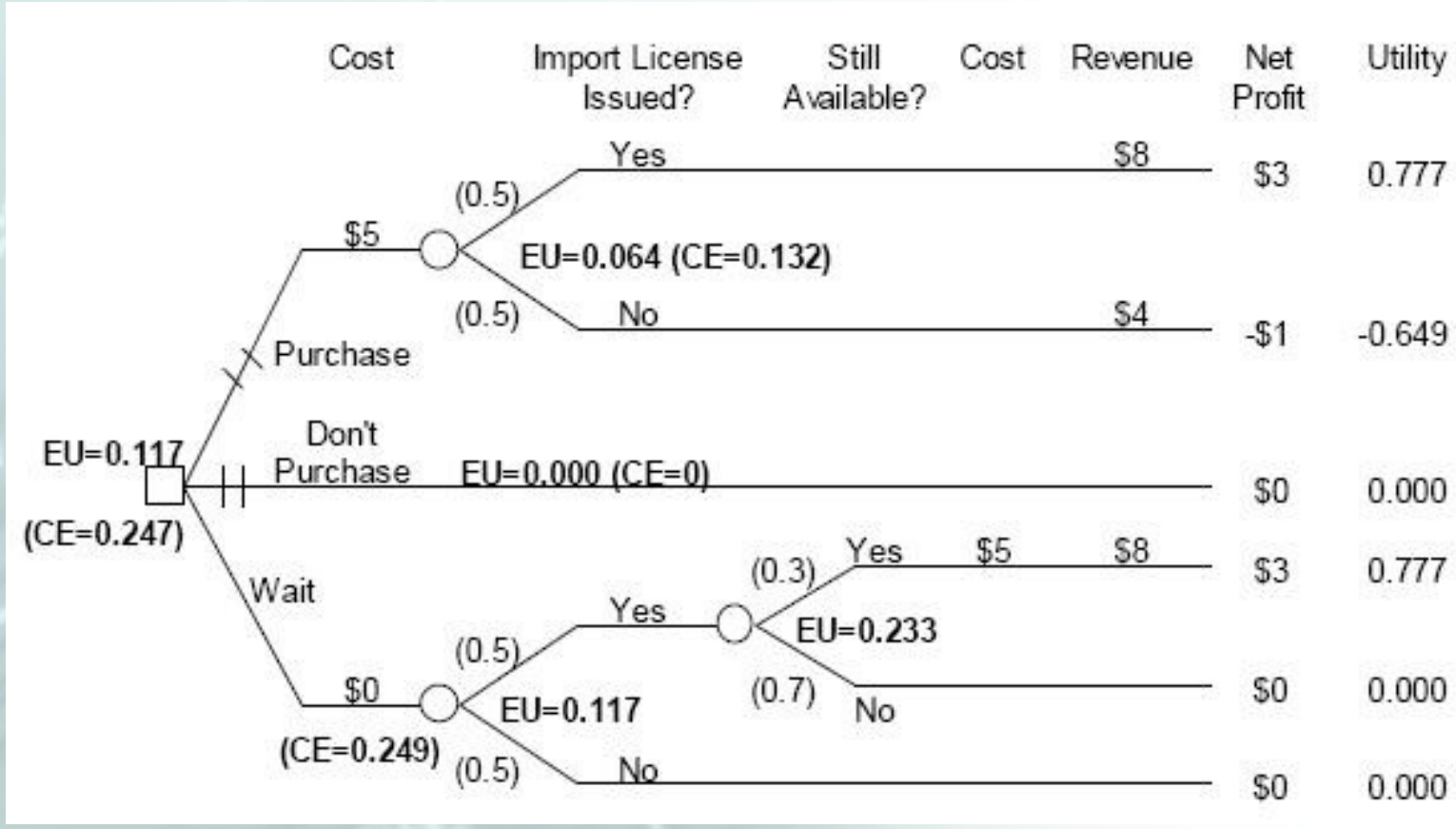
- $u(x) = 1 - e^{-x/R}$, R>0, in decisions involving profits
- $u(x) = 1 - e^{x/R}$, R>0, in decision involving costs
  - u(x) utility function,
  - x is evaluation measure
  - R is risk tolerance

Xanadu Traders, part 3

- G. Xanadu: I would be just willing to accept a deal with a fifty-fifty chance of making \$2,000,000 or losing \$1,000,000. However, if the upside were \$2,100,000 and the downside were \$1,050,000, I would not take the deal.
  - $r_0 =$ \$2,000,000
- Rule: If the profit alternative r_0 comes up with the same probability as the lose alternative $r_0/2$, then CE = 0 and R is approximately equal to r_0.
  - Utility function of Mr. Xanadu is $u(x) = 1 - e^{-x/2}$

Decision tree n.4
Expected utility

- $u(3) = 1 - e^{-3/2} = 0{,}777$
- Expected utility: EU=0,5x(0,777)+0,5x(-0,649)=0,064



Theory intermission n.3
Certainty equivalent for an Exponential Utility Function

- CE = - R x ln(1 - EU), in decisions involving profits
- CE = R x ln(1 - EU), in decisions involving costs
  - CE – certainty equivalent
  - EU - expected utility
  - R - risk tolerance
  - ln -natural logarithm

Certainty equivalent for „Purchase“ alternative:

- CE = –2 x ln (1 - 0,064) = $0,132 millions
- the alternative with the greatest certainty equivalent is most preferred for situations where more of an evaluation measure is preferred to less

The “wait” alternative is now most preferred

Alternative	EV	CE	Diference
Purchase	**1,000**	**0,132**	**0,868**
Don‘t purchase	**0,000**	**0,000**	**0,000**
Wait	**0,450**	**0,249**	**0,201**

Theory intermission n.4
The value of information

- **Perfect information** removes all uncertainty about the outcomes for the decision alternatives.
- No source of information (**imperfect information**) can be worth more than the value of perfect information.

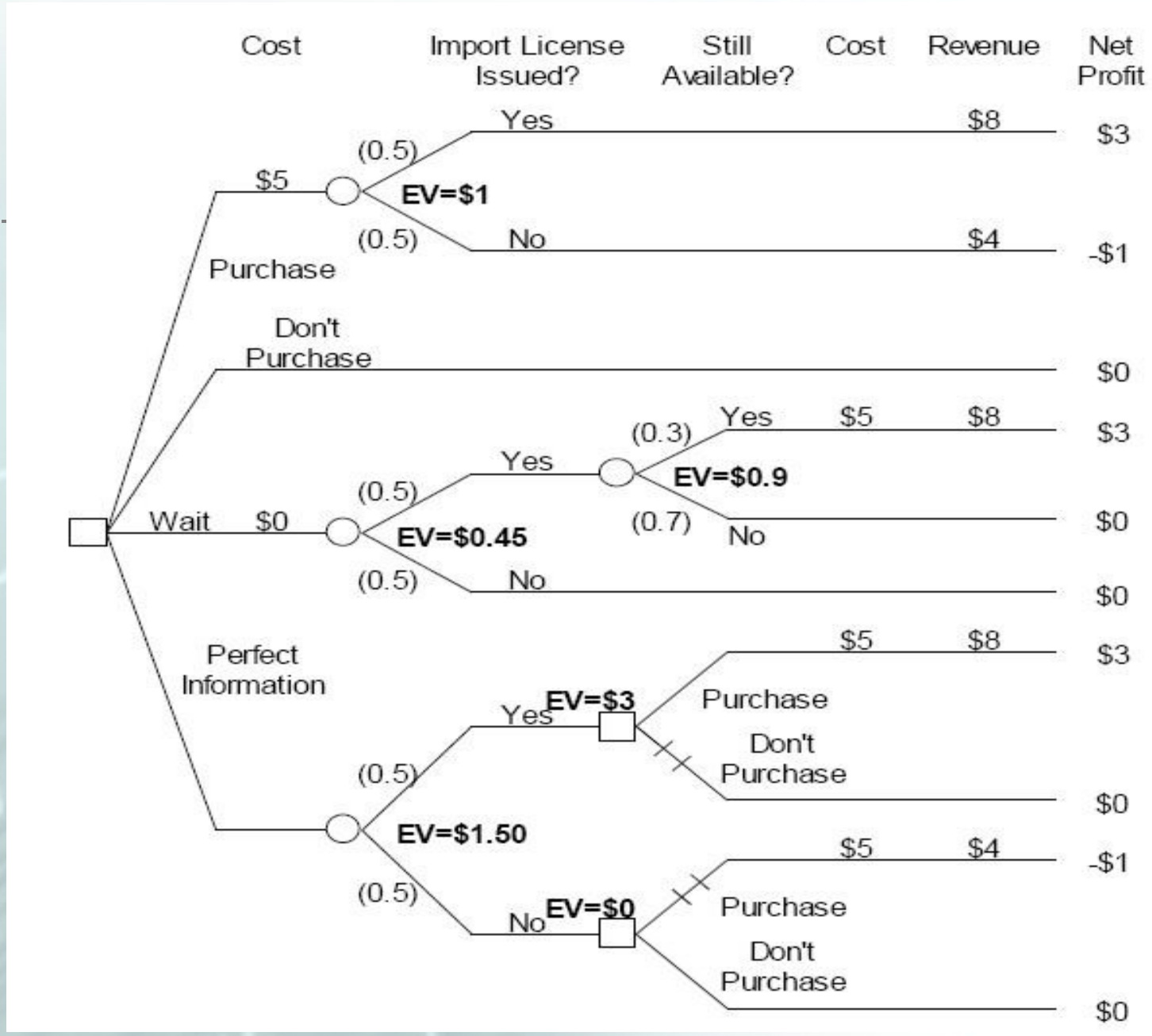
Xanadu Traders, part 4

Question: How much money would it be worth to obtain perfect information about issuance of the import license?

- There exists a source of perfect information in the government that would let George Xanadu know if the import license would be issued.

- probability of receiving the licence is still 0,5
- perfect information alternative has an expected value: EV = $1.5 million
- the value of perfect information is $1.5 - $1.0 = $0.5 million
- since $0.5 million is the value of perfect information, this places an upper limit on how much it is worth paying for any information about whether the license will be issued.

Decision tree n.5
Value of perfect information


Cost
Import License Issued?
Still Available?
Cost
Revenue
Net Profit
Yes
$8
$3
(0.5)
$5
EV=$1
(0.5)
No
$4
-$1
Purchase
Don't Purchase
$0
(0.3)
Yes
$5
$8
$3
Yes
EV=$0.9
(0.5)
Wait
$0
EV=$0.45
(0.7)
No
$0
(0.5)
No
$0
Perfect Information
$5
$8
$3
Yes
EV=$3
Purchase
Don't Purchase
(0.5)
$0
EV=$1.50
$5
$4
-$1
(0.5)
No
EV=$0
Purchase
Don't Purchase
$0

Xanadu Traders, part 5

The value of imperfect information

- Consultant John Lofton
  - often makes mistakes

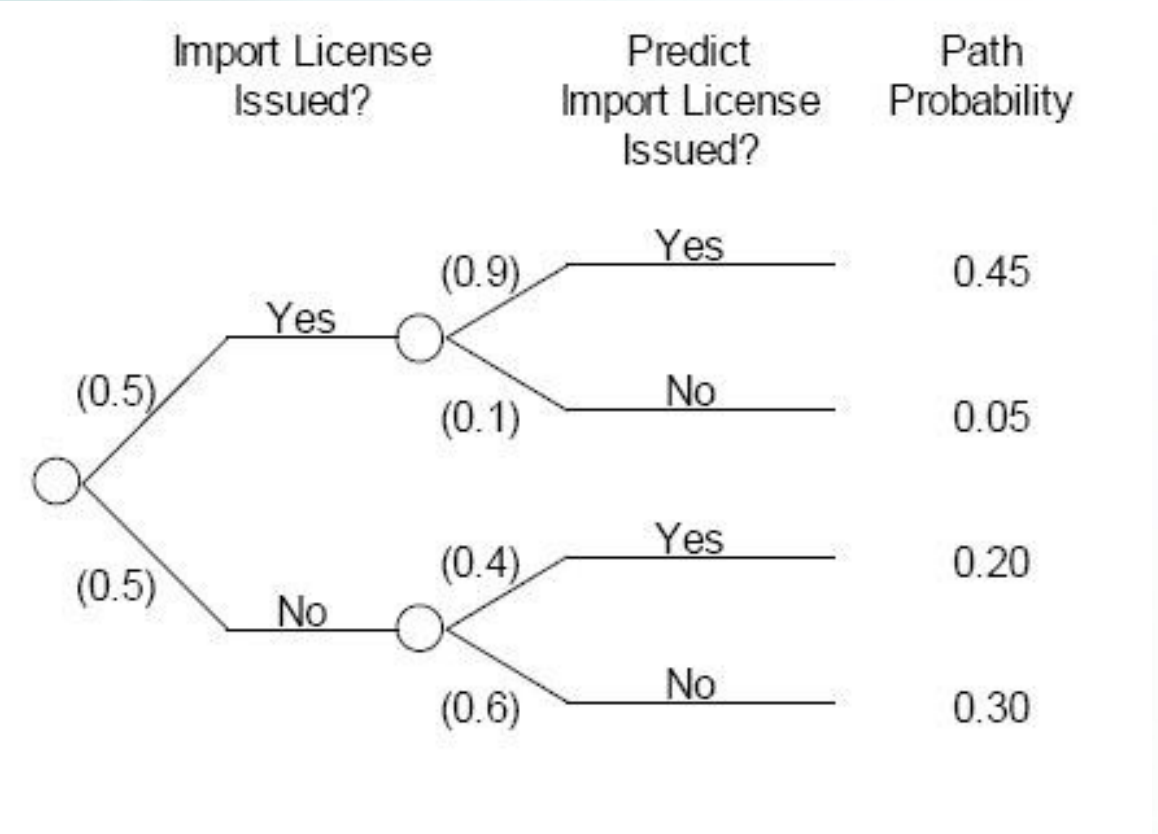
- In cases, where the license was issued, he had truth in 90% of time
- In cases,where the license wasn‘t issued, he had truth 60% of time
- Loftons fee: $10 000

Should Xanadu hire Lofton, and if so, what is the maximum amount that he should pay Lofton for his services?

Decision tree n.6

Value of imperfect information

- Unfortunately, as often happens in real problems, the information presented about Lofton's accuracy in his predictions is not in a form that directly provides the required probabilities

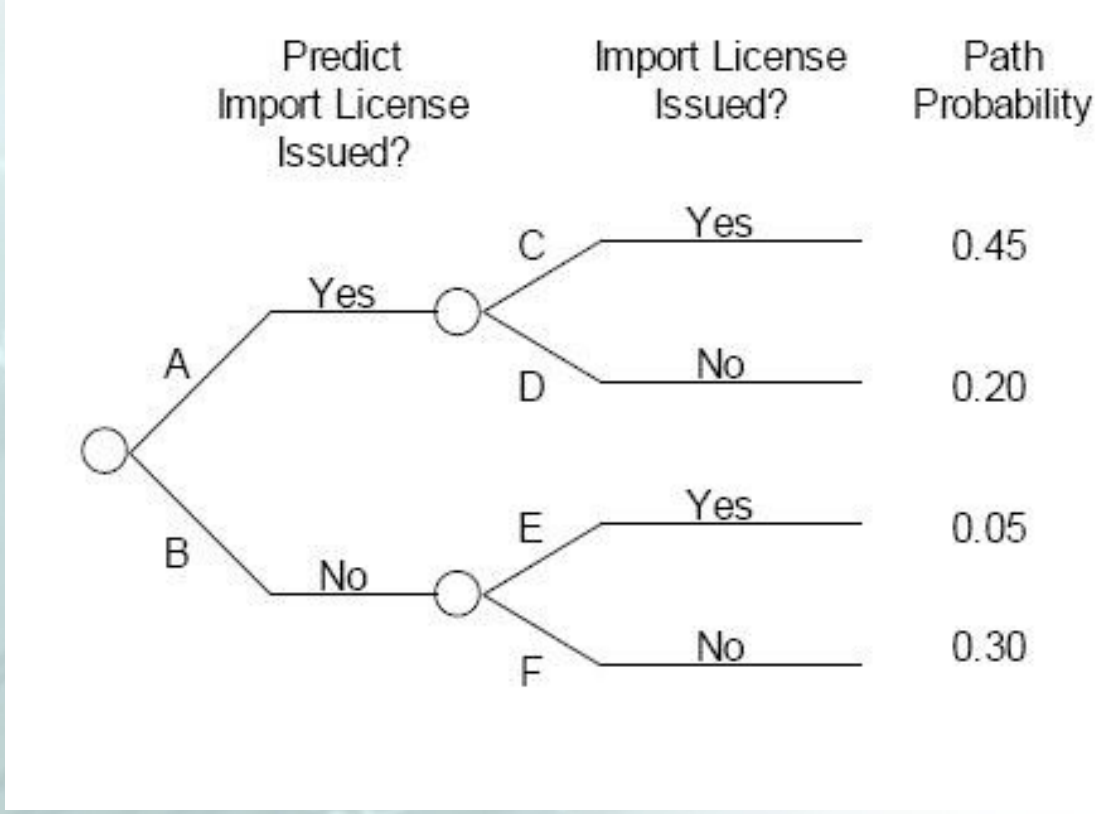


- A **path probability** is the probability of a particular sequence of branches from the root node to a specified endpoint in a probability tree

Decision tree n.7

Value of imperfect information

- **Tree flipping** is the process of calculating the probabilities for a probability tree with the order of the chance nodes reversed.



- **Condition: the paths from the root node to the endpoints have to be the same in the Figure 6 and Figure 7 trees - they are arranged in a different order – „backwards“.**

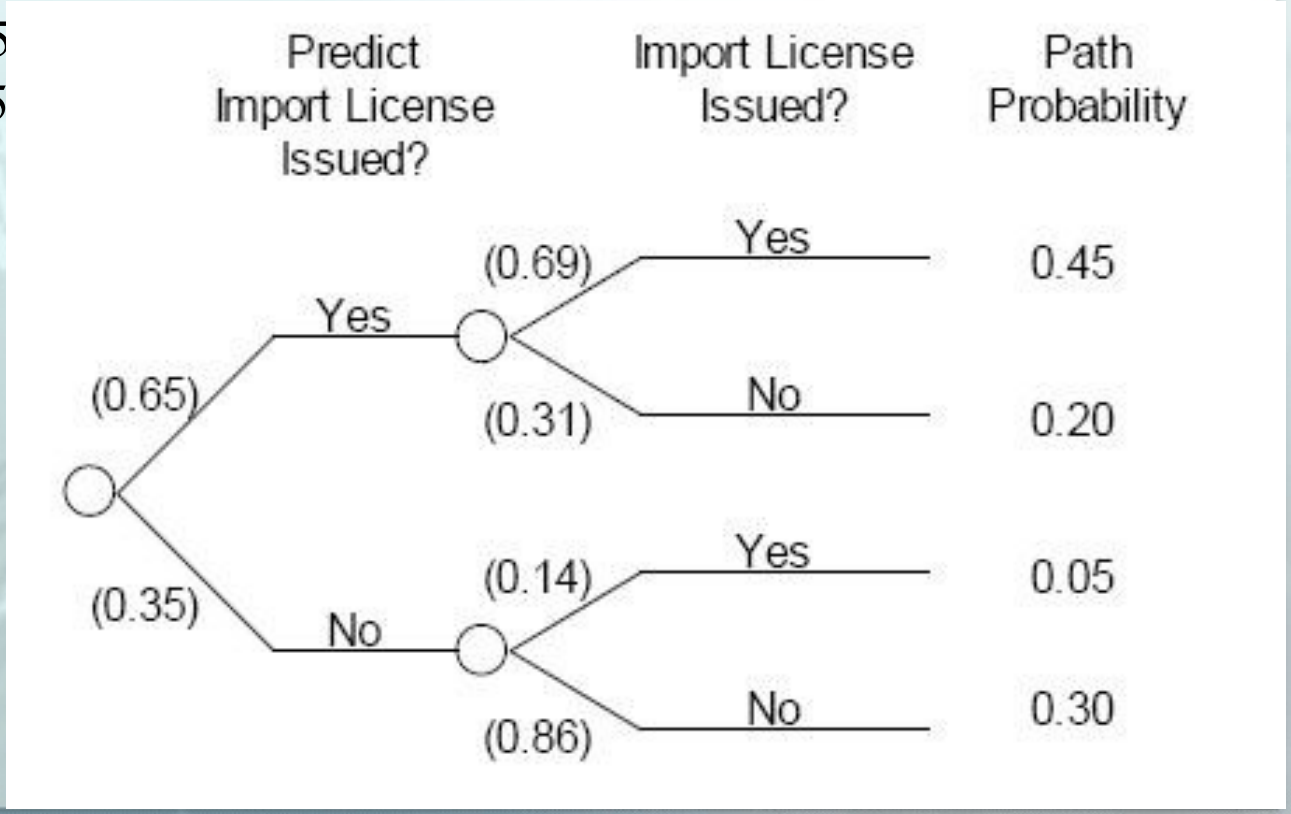
Decision tree n.8

Value of imperfect information

- Probability A is the probability of a „yes“ prediction regarding license approval

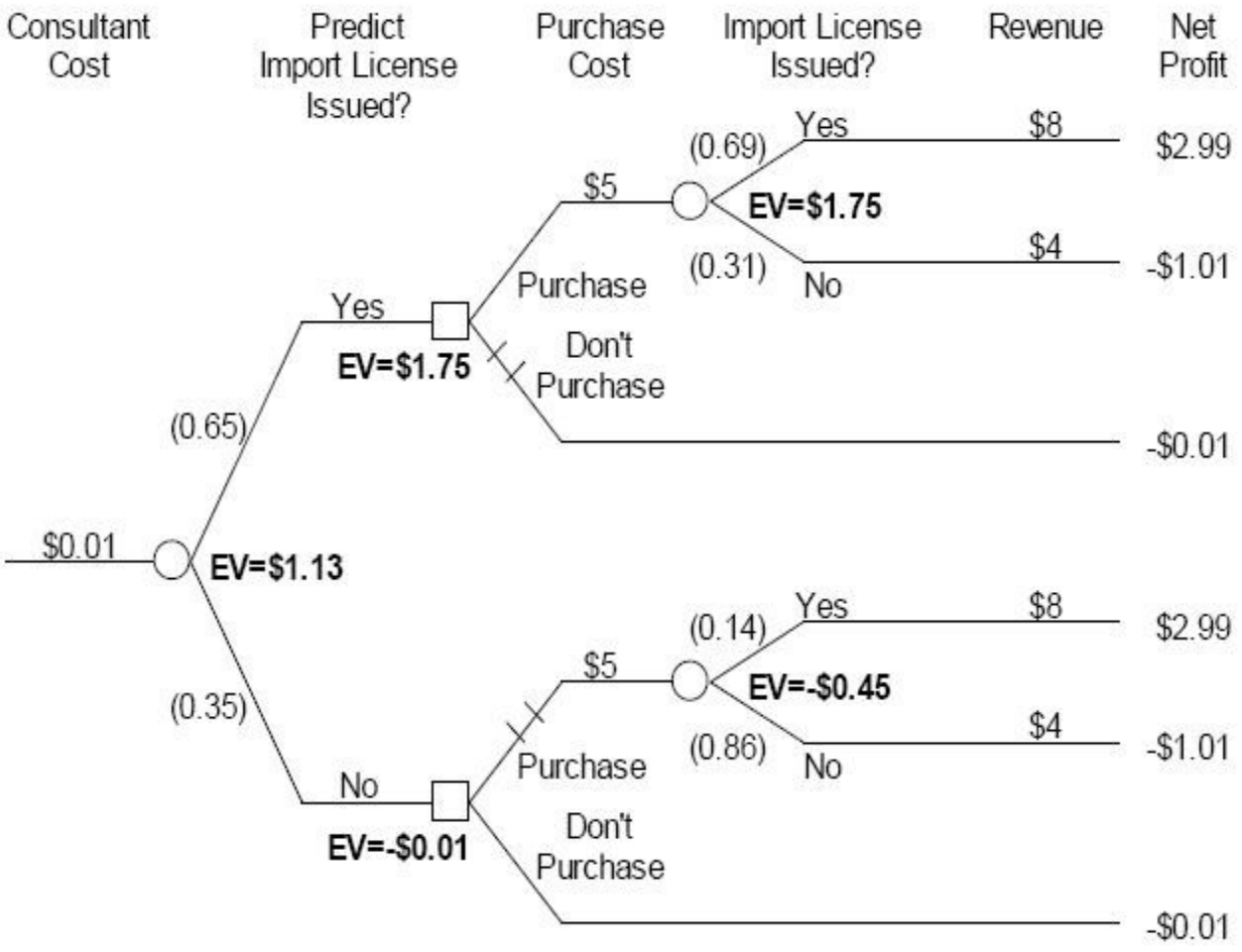
- A= 0,45 + 0,20 = 0,65
  B= 0,30 + 0,05 = 0,35

- C = 0,45/A=0,69
  D=0,2/A=0,31
  E=0,05/B=0,14
  F = 0,30/B = 0,86



Decision tree n.8

Value of imperfect information



Alternative without Lofton EV = $1 mil

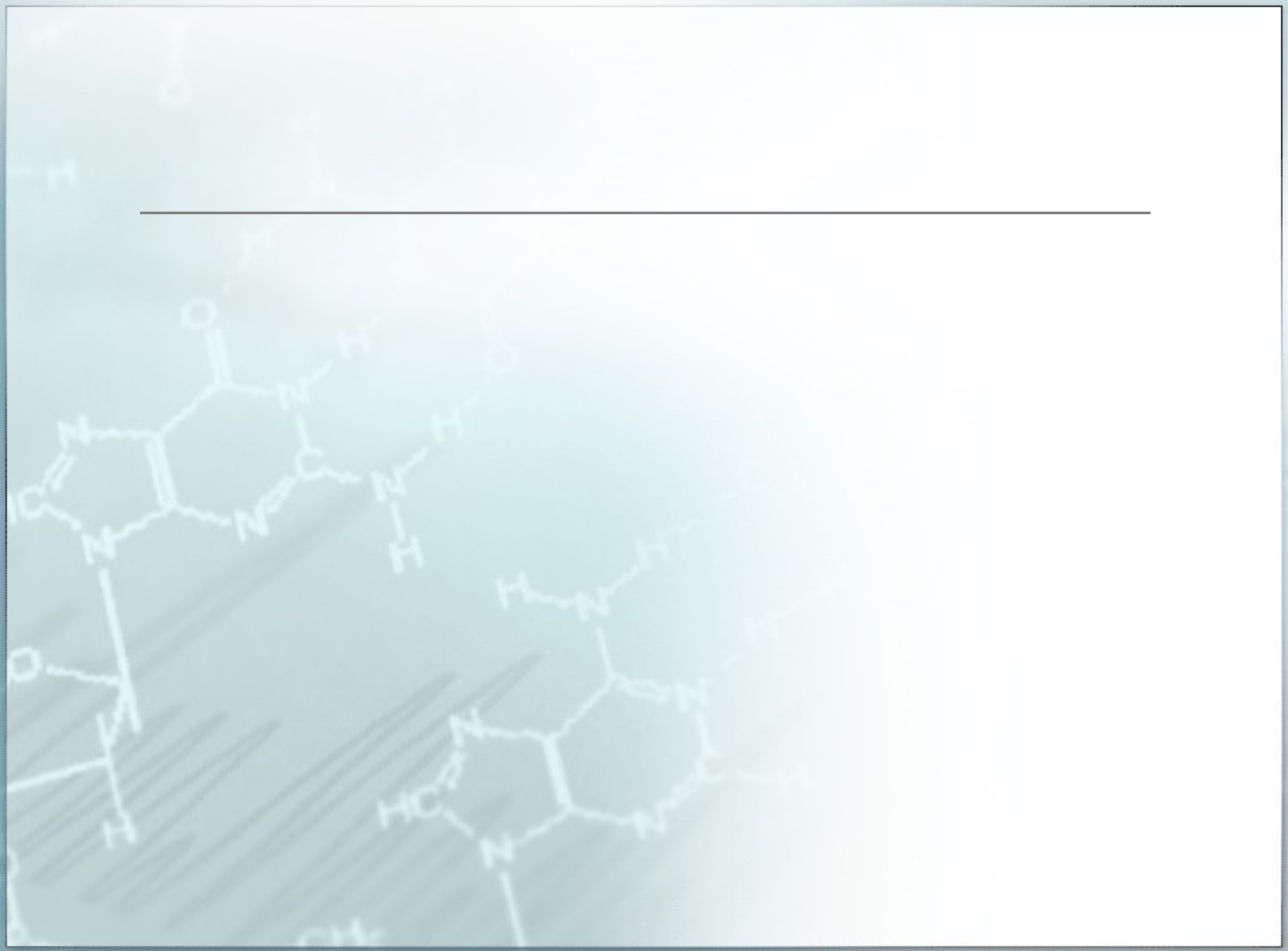
- Alternative with Lofton EV= $1.13 mil

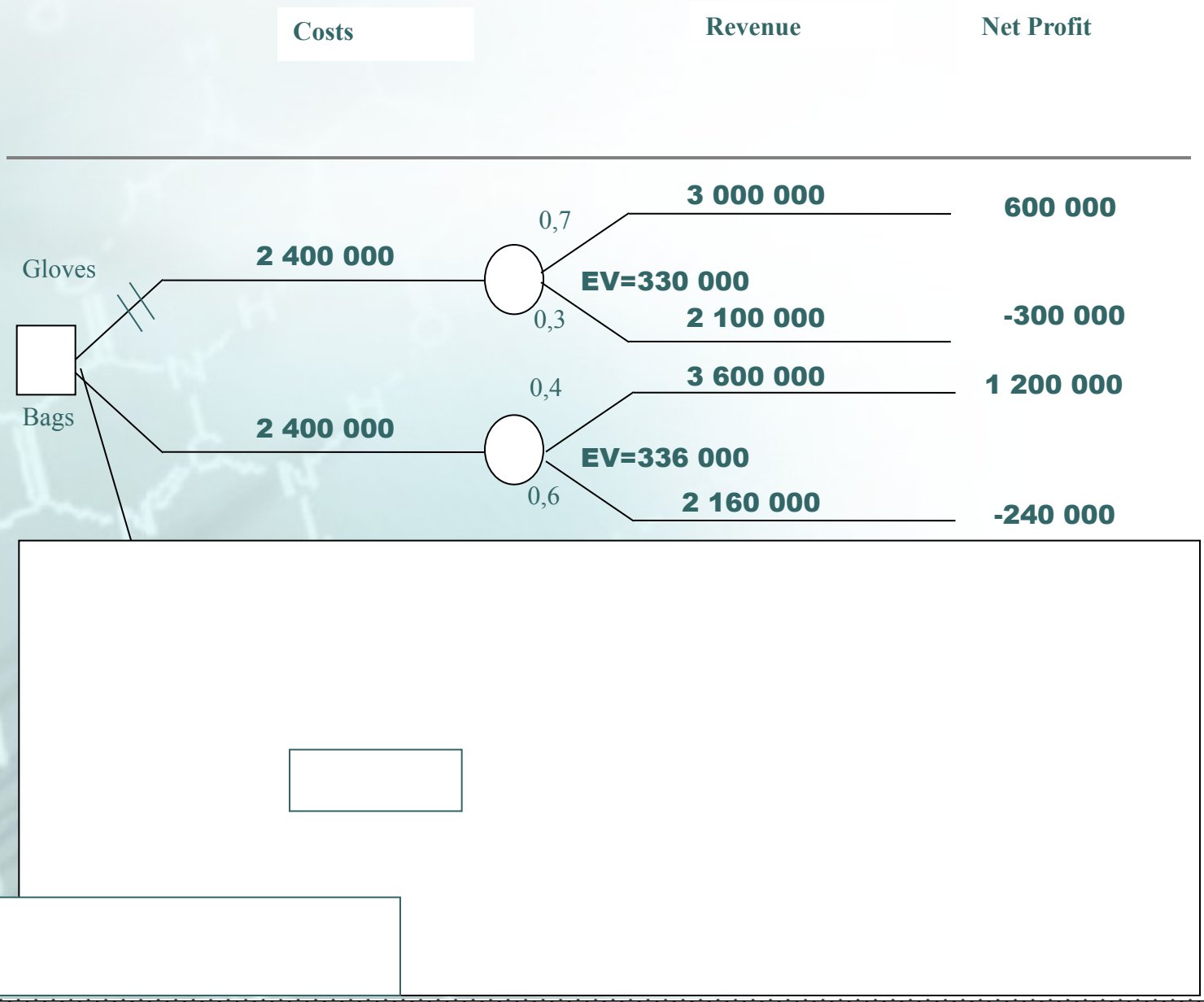
$1.13 – $1 = $0,13 mil

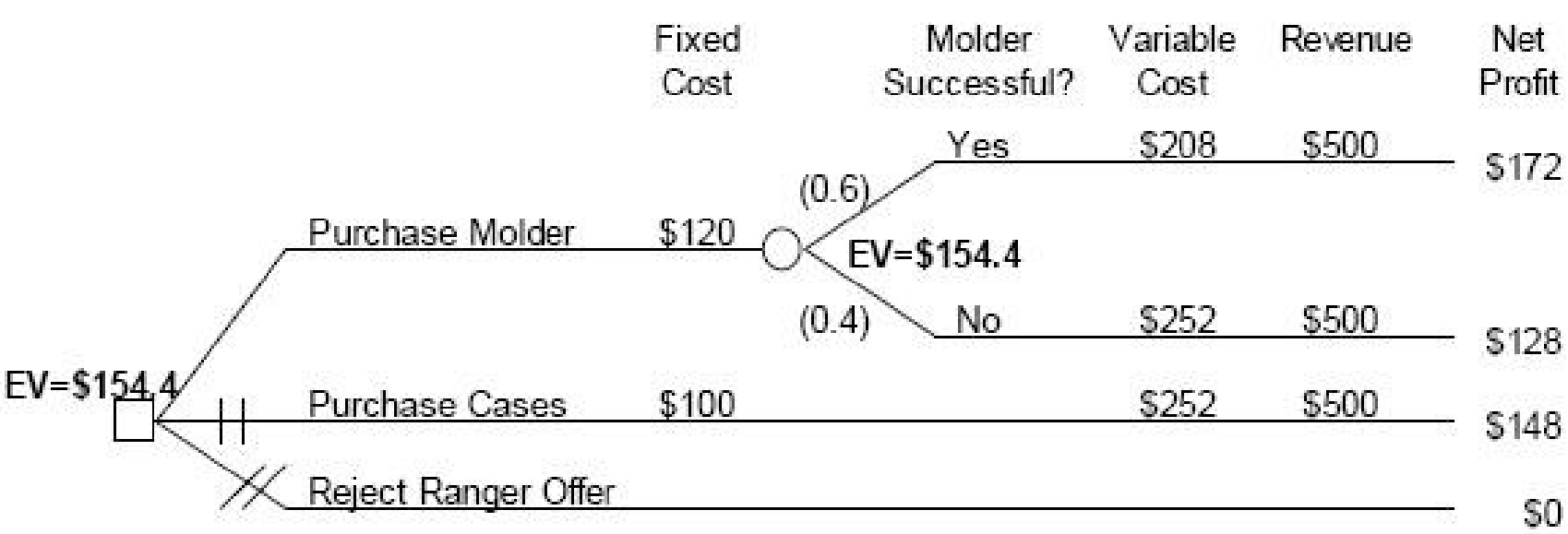
It is worth it to hire Lofton as long as he costs less than **$130 000.**

Conclusion

- Advantage of using decision trees:
  - Clarity
  - Possibility of showing the whole process with all incoming factors
- Disadvantage of using decision trees:
  - Unable to show more complex or complicated decisions




Costs
Revenue
Net Profit
Gloves
2 400 000
0,7
3 000 000
600 000
EV=330 000
0,3
2 100 000
-300 000
Bags
2 400 000
0,4
3 600 000
1 200 000
EV=336 000
0,6
2 160 000
-240 000


Fixed Cost
Molder Successful?
Variable Cost
Revenue
Net Profit
Yes
$208
$500
$172
(0.6)
Purchase Molder
$120
EV=$154.4
(0.4)
No
$252
$500
$128
EV=$154.4
Purchase Cases
$100
$252
$500
$148
Reject Ranger Offer
$0