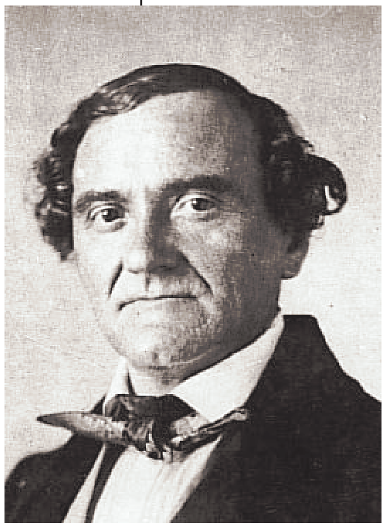
3. Money

„Die Phönizier haben das Geld erfunden - aber warum so wenig?“

Johann Nepomuk Nestroy
(1801-1862)



Chapter 3 - Contents

3. Money (Friedman & Schwartz equation for money demand – *multiple regression*)

3.1 multiple regression

3.2 added variable plots

3.3 the *F*-test

3.4 weighted (generalized) OLS-estimation

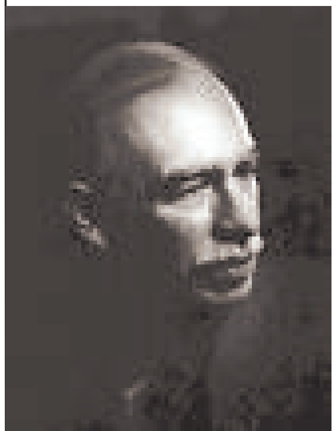
3.5 dummy Variables

Exercise (*Ericsson & Hendry, 1998*)

The equation for money demand

Cornerstone of the IS-LM model

empirical investigation whether the demand for money depends upon the interest rate or not, respectively whether the interest elasticity of money demand is high or low.



John Maynard Keynes

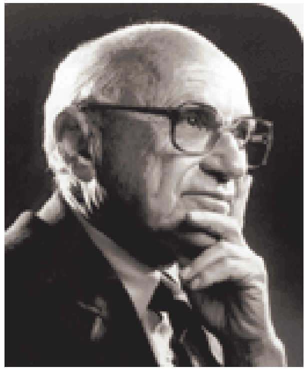
(1883-1946)

http://cepa.newschool.edu/~het/essays/keynes/general.htm

A model for money demand:

$$M/P = f(Y, w; E(r_M), E(r_B), E(r_E), \pi^e; u).$$

Friedman, M., Schwartz, A.J., „Monetary Trends in the United States and the United Kingdom“, University of Chicago Press, Chicago, 1982.



Milton Friedman

(1912-2006)

Anna Schwartz

(1915-)

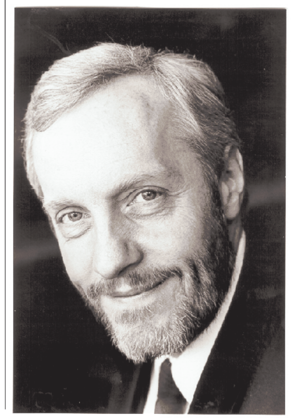
The dataset

UK from 1868 till 1975 but no annual data!

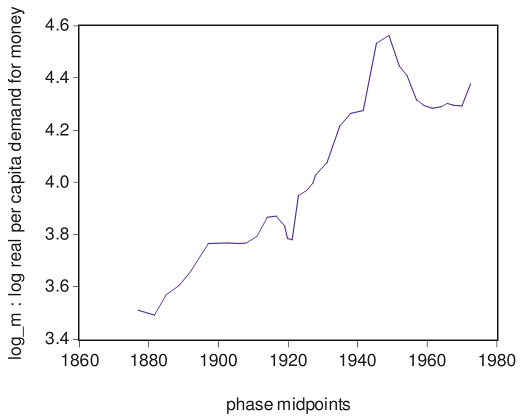
Averages of up and down phases, thus only 36 effective observations.

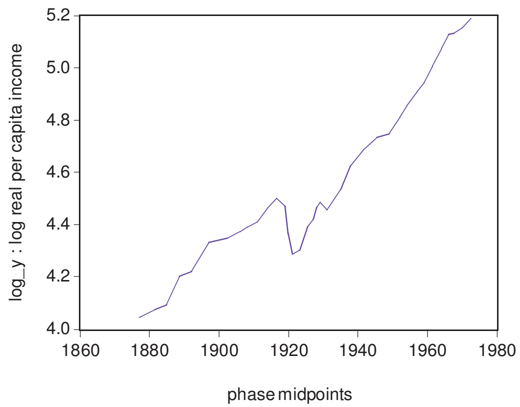
Extended until 1993 in:

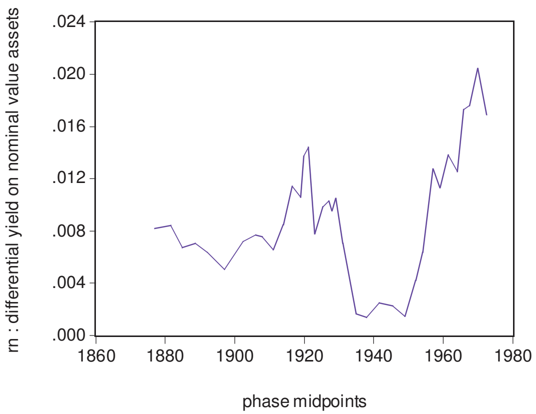
Ericsson, N.R., Hendry, D.F., Prestwich, K.M., “Friedman and Schwartz (1982) revisited: Assessing annual and phase-average models of money demand in the United Kingdom”, *Empirical Economics*, 23, 401-415, 1998.

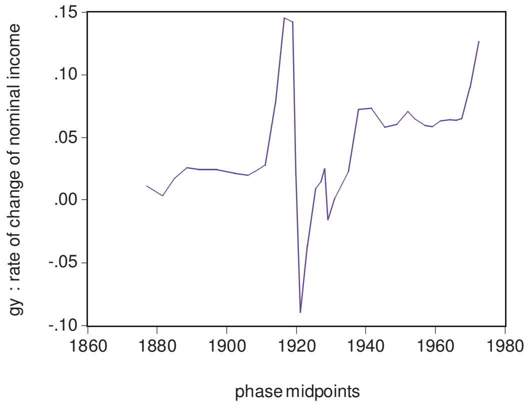


3.1 Multiple regression (pp7)

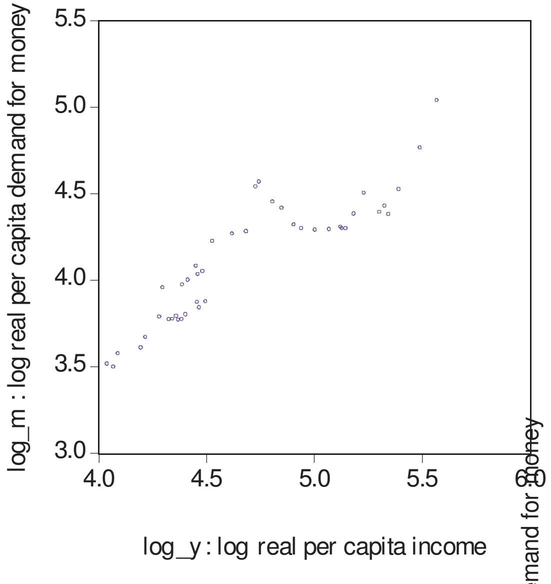


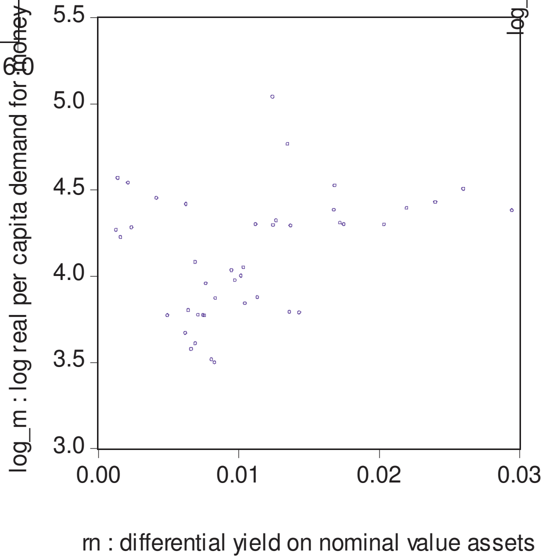


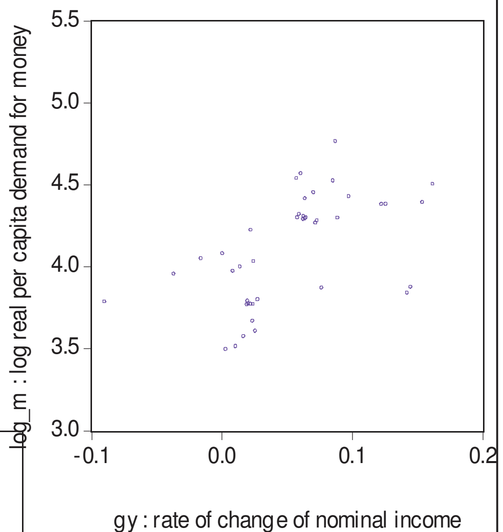




Scatterplots







Outputs

Dependent Variable: LOG_M

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.362372	0.371001	0.976743	0.3356
LOG_Y	0.801767	0.080932	9.906715	0.0000

R-squared	0.742703	Mean dependent var	4.028569
Adjusted R-squared	0.735135	S.D. dependent var	0.305842
S.E. of regression	0.157402	Akaike info criterion	-0.806079
Sum squared resid	0.842360	Schwarz criterion	-0.718106
Log likelihood	16.50942	F-statistic	98.14301
Durbin-Watson stat	0.205516	Prob(F-statistic)	0.000000

Dependent Variable: LOG_M

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.002880	0.112355	35.62709	0.0000
RN	2.834092	11.00699	0.257481	0.7984

R-squared	0.001946	Mean dependent var	4.028569
Adjusted R-squared	-0.027408	S.D. dependent var	0.305842
S.E. of regression	0.310005	Akaike info criterion	0.549497
Sum squared resid	3.267510	Schwarz criterion	0.637471
Log likelihood	-7.890954	F-statistic	0.066297
Durbin-Watson stat	0.063624	Prob(F-statistic)	0.798359

Dependent Variable: LOG_M

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.909157	0.063412	61.64683	0.0000
GY	2.853887	1.025829	2.782031	0.0088

R-squared	0.185428	Mean dependent var	4.028569
Adjusted R-squared	0.161470	S.D. dependent var	0.305842
S.E. of regression	0.280064	Akaike info criterion	0.346353
Sum squared resid	2.666813	Schwarz criterion	0.434327
Log likelihood	-4.234359	F-statistic	7.739699
Durbin-Watson stat	0.156595	Prob(F-statistic)	0.008750

Multiple regression

Dependent Variable: LOG_M
Method: Least Squares
Sample: 1 36
Included observations: 36

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.161671	0.437417	0.369604	0.7140
LOG_Y	0.851301	0.099034	8.596079	0.0000
GY	-0.616611	0.705491	-0.874017	0.3884

R-squared	0.748524	Mean dependent var	4.028569
Adjusted R-squared	0.733283	S.D. dependent var	0.305842
S.E. of regression	0.157951	Akaike info criterion	-0.773408
Sum squared resid	0.823301	Schwarz criterion	-0.641448
Log likelihood	16.92135	F-statistic	49.11271
Durbin-Watson stat	0.224265	Prob(F-statistic)	0.000000

Added variable

It follows from $\hat{y} = X_{0,1}\hat{\beta}_{0,1} + x_2\hat{\beta}_2$

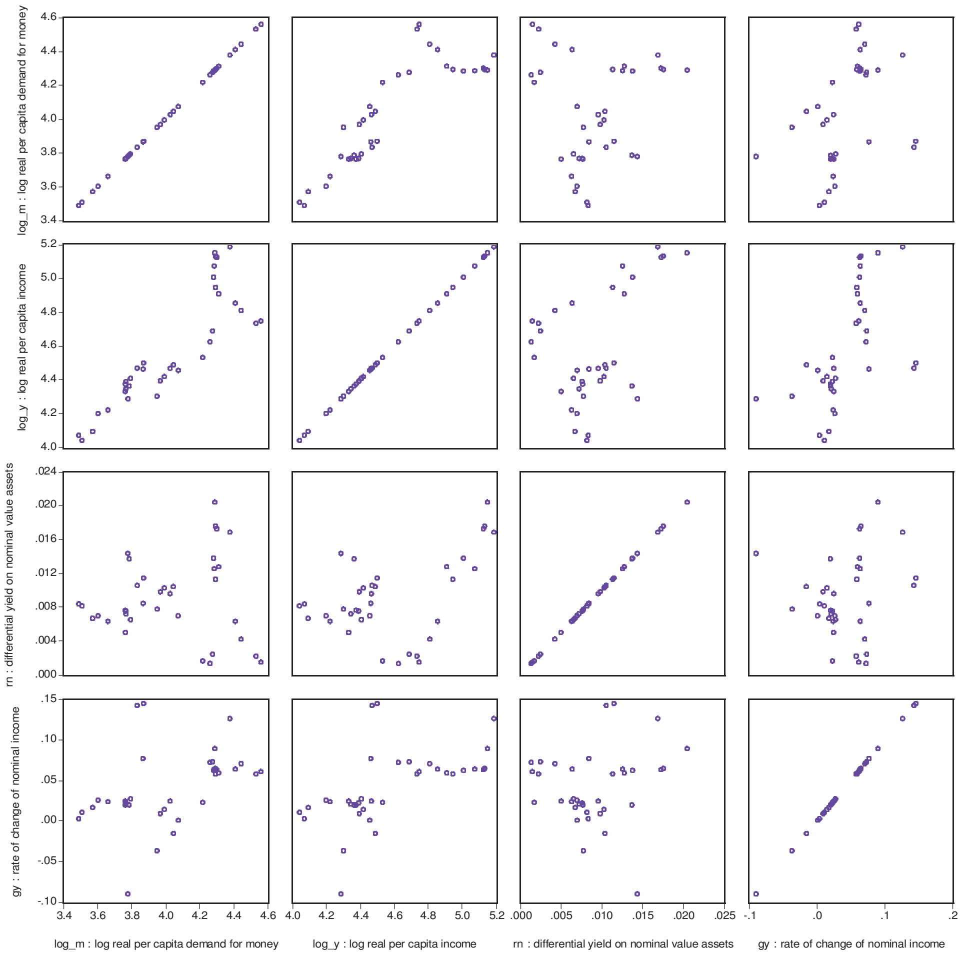
immediately $X_{0,1}'\hat{y} = X_{0,1}'X_{0,1}\hat{\beta}_{0,1} + X_{0,1}'x_2\hat{\beta}_2$

and thus

$$\hat{\beta}_{0,1} = \tilde{\beta}_{0,1} - (X_{0,1}'X_{0,1})^{-1} X_{0,1}'x_2\hat{\beta}_2 .$$

Here $\tilde{\beta}_{0,1}$ stands for the estimator from the simple regression and the difference vanishes (under appropriate scaling) either, when the regressors are uncorrelated ($x_1'x_2 = 0$) or when the second regressor has no influence ($\hat{\beta}_2=0$).

Scatterplot matrix

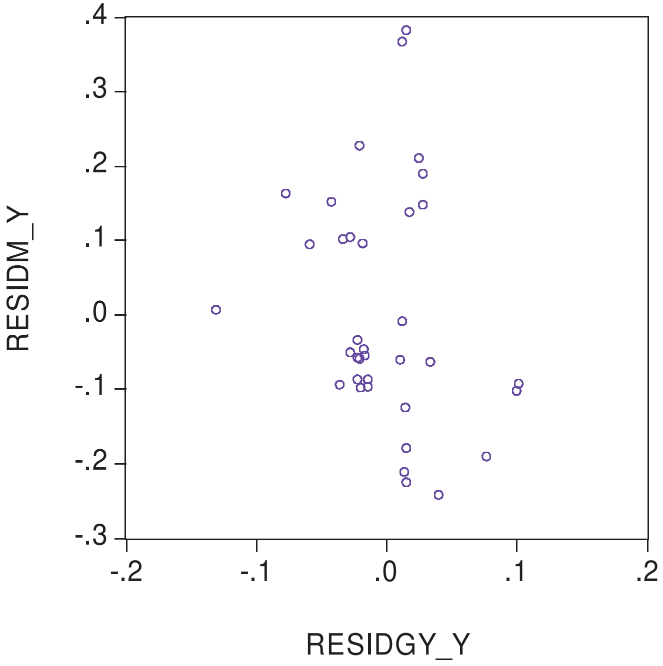


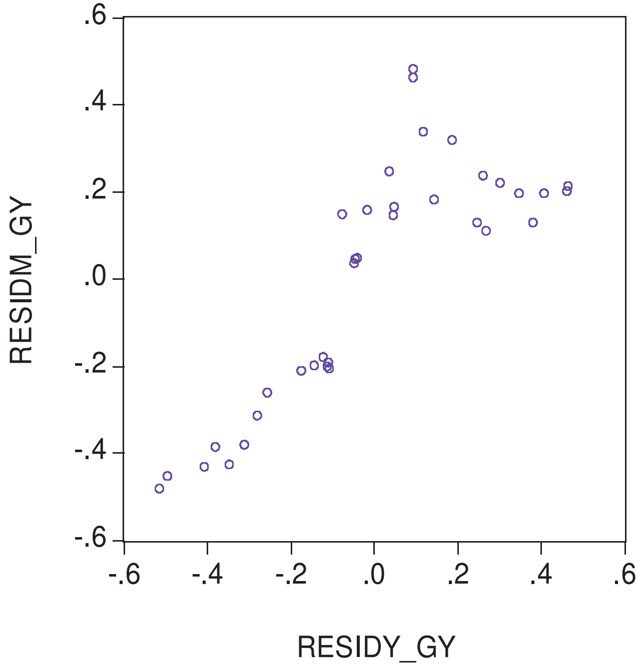
3.2 Added variable plots

The estimator for the „interesting" parameter can now be written as

$$\hat{\beta}_2 = [x_2'(I-H_{0,1})x_2]^{-1}x_2'(I-H_{0,1})\hat{\varepsilon}_{0,1}$$

as a coefficient from a regression (without intercept) of the residuals $\hat{\varepsilon}_{0,1}=(I-H_{0,1})y$ from a regression of y on $X_{0,1}$ on the residuals $(I-H_{0,1})x_2$ the regression of x_2 on $X_{0,1}$.


.4
.3
.2
.1
.0
-.1
-.2
-.3
RESIDM_Y
-.2
-.1
.0
.1
.2
RESIDGY_Y


.6
.4
.2
.0
-.2
-.4
-.6
RESIDM_GY
-.6
-.4
-.2
.0
.2
.4
.6
RESIDY_GY

Opposite Effect!

Dependent Variable: LOG_M
Method: Least Squares
Sample: 1 36
Included observations: 36

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.625109	0.281156	-2.223351	0.0334
GY	-1.137629	0.428107	-2.657347	0.0122
LOG_Y	1.086674	0.066687	16.29503	0.0000
RN	-29.53291	3.816182	-7.738863	0.0000

R-squared	0.912425	Mean dependent var	4.028569
Adjusted R-squared	0.904215	S.D. dependent var	0.305842
S.E. of regression	0.094655	Akaike info criterion	-1.772709
Sum squared resid	0.286709	Schwarz criterion	-1.596762
Log likelihood	35.90876	F-statistic	111.1344
Durbin-Watson stat	0.488062	Prob(F-statistic)	0.000000

3.3 the F-test (pp27)

Testprocedure for checking hypotheses of the form $\beta_i = \ldots = \beta_j = 0$, that is eliminating simultaneously a number of potential variables.

F-statistics, general form:

$$F = (SSR_R - SSR)/SSR \cdot (T-k-1)/g$$

Sir Ronald Aylmer Fisher, FRS (17 February 1890 – 29 July 1962)

Overall F-test: $SSR_R = SST$

Relationship to the coefficient of determination (p28):

$$R^2 = SSE/SST = SSE/(SSE+SSR)$$

$$1\text{-}R^2 = (SSE+SSR)/(SSE+SSR) – SSE/(SSE+SSR)$$

$$= SSR/(SSE+SSR)$$

$$R^2/(1\text{-}R^2) = SSE/SSR.$$

$F = SSE/SSR\cdot (T\text{-}k\text{-}1)/k$ also $F = R^2/(1\text{-}R^2)\cdot (T\text{-}k\text{-}1)/k.$

3.4 weighted LS-estimation (pp88)

Heteroskedasticity: nonconstant variances of the noise/observations

From $\bar{x}_i = (x_{i,1}/2 + \sum_{j=2,ni} x_{i,j} + x_{i,ni+1}/2)/ni$ we yield for the variance:

$$\mathrm{Var}(\bar{x}_i) = (\mathrm{Var}(x_i)/4 + \sum_{j=2,ni} \mathrm{Var}(x_i) + \mathrm{Var}(x_i)/4)/n_i^2$$
$$= (n_i-1/2)\,\mathrm{Var}(x_i)/n_i^2 = (2n_i-1)/2n_i^2\,\mathrm{Var}(x_i).$$

„Standardization" of the observations through proportionality factors!

Target $\sum_t v_t(y_t - \hat{y}_t)^2$ is minimized!

Dependent Variable: LOG_M/V0
Method: Least Squares
Sample: 1 36
Included observations: 36

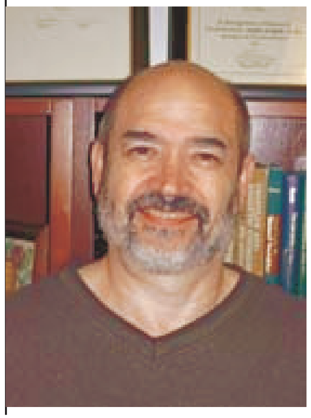
Variable	Coefficient	Std. Error	t-Statistic	Prob.
1/V0	-0.763842	0.281027	-2.718039	0.0105
LOG_Y/V0	1.114142	0.066509	16.75166	0.0000
RN/V0	-30.05410	3.790253	-7.929312	0.0000
GY/V0	-0.996446	0.470155	-2.119399	0.0419

R-squared	0.987624	Mean dependent var	7.333740
Adjusted R-squared	0.986464	S.D. dependent var	1.530226
S.E. of regression	0.178032	Akaike info criterion	-0.509268
Sum squared resid	1.014252	Schwarz criterion	-0.333322
Log likelihood	13.16683	F-statistic	851.2442
Durbin-Watson stat	0.673531	Prob(F-statistic)	0.000000

Heteroskedasticity-correction of White (p94)

$$\mathrm{Var}(\hat{\beta}_{OLS} \backslash X) = \sigma^2 (X'X)^{-1} X' V X (X'X)^{-1}$$
$$\neq \sigma^2 (X'X)^{-1}$$

Workaround: replace the diagonal of V by the squares of standardized OLS-residuals, namely $\hat{V}_{tt} = T\hat{\varepsilon}_t^{\,2} / \sum \hat{\varepsilon}_t^{\,2}$.



Halbert L. White, Jr.

White test for heroskedasticity (p99)

Additional regression:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 x_{1t} + \ldots + \alpha_m x_{mt} + \alpha_{m+1} x_{1t}^2 + \ldots + \alpha_{2m} x_{mt}^2 + \alpha_{2m+1} x_{t1} x_{t2} + \ldots + \varpi_t$$

Check via overall F-test.

Breusch-Pagan-Godfrey-test (p99)

Nullhypothesis: Homoskedasticity

Alternative: functional dependence of the variance from variable Z.

Example:

$y = X\beta + u,\ \mathrm{Var}\{u\} = \sigma^2 h(Z\alpha)$

Here Z represents the weighting variable!

Other variants: Harvey, Glejser, etc.

Goldfeld-Quandt-test (p98)

Nullhypothesis: Homoskedasticity

Alternative: Two regimes with σ_1^2 and σ_2^2 as Variance of noise; variable Z indicates which regime applies.

Example:

$y_1 = X_1\beta_1 + u_1$, $\mathrm{Var}\{u_1\} = \sigma_1^2 I_{T1}$ (regime 1)

$y_2 = X_2\beta_2 + u_2$, $\mathrm{Var}\{u_2\} = \sigma_2^2 I_{T2}$ (regime 2)

Nullhypothesis: $\sigma_1^2 = \sigma_2^2$

F-test: $$F = \frac{S_1}{S_2}\frac{n_1 - k}{n_2 - k}$$

S_i: sum of squared residuals for i-th regime.

Goldfeld-Quandt-test, cont. consists of the following steps:

- Sort observations ascending with Z.
- Remove $2c$ observations from the middle of the sorted series.
- Separate OLS-fits for the first T_1 and the last T_2 observations [typically $T_1 = T_2 = (n\text{-c})/2$] and determination of the OLS-estimates b_i and the residual sum of squares S_i ($i = 1,2$).
- Calculation of the test statistics *F*; it is under H_0 approximately *F*-distributed with T_2-c-k and T_1-c-k degrees of freedom

$$F = \frac{S_1}{S_2} \frac{T_1 - c - k}{T_2 - c - k}$$

3.5 Dummy variables (p12)

Originally: with 0 or 1 coded regressors that indicate the existence of a certain condition.

OLS with dummies

Dependent Variable: LOG_M
Method: Least Squares
Sample: 1 36
Included observations: 36

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.121560	0.188028	-0.646497	0.5227
LOG_Y	0.924542	0.047119	19.62137	0.0000
RN	-14.74058	3.134206	-4.703130	0.0001
GY	-0.371056	0.286295	-1.296060	0.2045
S	0.198453	0.027521	7.210842	0.0000

R-squared	0.967290	Mean dependent var	4.028569
Adjusted R-squared	0.963069	S.D. dependent var	0.305842
S.E. of regression	0.058775	Akaike info criterion	-2.701961
Sum squared resid	0.107089	Schwarz criterion	-2.482028
Log likelihood	53.63531	F-statistic	229.1804
Durbin-Watson stat	1.249423	Prob(F-statistic)	0.000000

WLS with dummies

Dependent Variable: LOG_M
Method: Least Squares
Sample: 1 36
Included observations: 36
Weighting series: 1/W0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.021083	0.193736	0.108825	0.9141
LOG_Y	0.882881	0.048982	18.02473	0.0000
RN	-11.21374	3.285429	-3.413172	0.0019
W	0.013782	0.005821	2.367800	0.0245
S	0.205901	0.027388	7.517951	0.0000
GY	-0.212149	0.291022	-0.728979	0.4717

Weighted Statistics

R-squared	0.996072	Mean dependent var	4.017565
Adjusted R-squared	0.995417	S.D. dependent var	0.838288
S.E. of regression	0.056751	Akaike info criterion	-2.749292
Sum squared resid	0.096619	Schwarz criterion	-2.485372
Log likelihood	55.48726	F-statistic	1521.371
Durbin-Watson stat	1.503403	Prob(F-statistic)	0.000000

Unweighted Statistics

R-squared	0.966056	Mean dependent var	4.028569
Adjusted R-squared	0.960399	S.D. dependent var	0.305842
S.E. of regression	0.060863	Sum squared resid	0.111128
Durbin-Watson stat	1.325585		

Dummy-variables for seasons

For the seasons define:

$Q_{it} = 1,$ i-th quarter

$Q_{it} = 0,$ else

Spring-dummy Q_{1t} has value 1 in every first quarter; analogously the summer-dummy ($i = 2$), etc.

Attention: For every period ($t = 1,\ldots, n$) holds

$$Q_{1t} + Q_{2t} + Q_{3t} + Q_{4t} = 1$$

Dummy variable trap ! (p80)

Homework 3

- Construct added variable plots for all relevant variants of regressors in the correctly weighted model and give interpretations.
- Ericsson et al. (1998) have reestimated the model on an extended database till 1993. Replicate their results with phase-average data and discuss differences to the previous estimation.
- Perform a Goldfeld-Quandt test for heteroscedasticity for the above.
- What are the particular difficulties in comparing weighted least squares Regressions?