# Is the Geometry of Nature Fractal?

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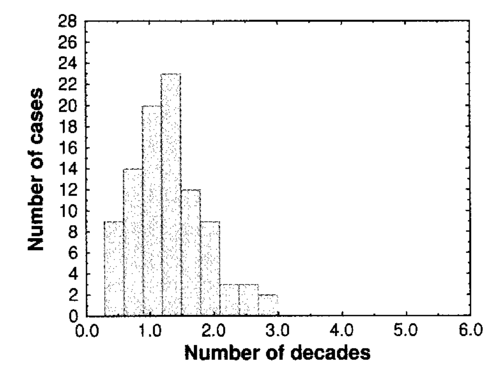
Fractals are beautiful mathematical contructs characterized by a never-ending cascade of similar structural details that are revealed upon magnification on all scales. Over the past two decades, the notion has been intensively put forward that fractal geometry describes well the irregular face of nature. But does it? Consider the recent Perspective in *Science* by Marder (*1*). Marder summarizes a simulation study of fractured silicon nitride by Kalia *et al.* (*2*) that successfully mimics experimental data, and he generally emphasizes the role of fractal geometry in describing physical structures of complex geometry. Specifically, the results of Kalia *et al.* were interpreted as "showing that this mechanism . . . leads to fractal fracture surfaces." However, upon examining Kalia's results [figure 4 in (*2*)], one finds that Marder's statement is based on four exponents, all of which hold over less than one order of magnitude. A fractal object, in the purely mathematical sense, requires infinitely many orders of magnitude of power-law scaling, and a consequent interpretation of experimental results as indicating fractality requires "many" orders of magnitude. In the celebrated fractal Koch curve, which resembles a symmetric snow-flake with many edges, one order of magnitude means that one stops its construction after about two iterations; a two-iterations Koch curve is not a fractal object. Marder, like many others in the scientific community, may have been swayed by the widespread image and belief that fractality has been found over many orders of magnitude in experimental documentation.

We have reason to believe that this is not the case (*3*). In fact, reported experimental fractality in a wide range of physical systems is typically based on a scaling range that spans only 0.5 to 2.0 decades (factors of 10). To assess this, we surveyed all experimental papers reporting fractal analysis of data that appeared over a period of 7 years in all *Physical Review* journals (*Phys. Rev.* A to E and *Phys. Rev. Lett.*, 1990 to 1996). In these papers, an empirical fractal dimension $D$ was calculated from various relations between a property $P$ and the resolution $r$ of the general form

$$P = kr^{f(D)} \qquad (1)$$

where $k$ is the prefactor for the power law and the exponent $f(D)$ is a simple function of $D$. In most cases, fitting the data to Eq. 1 was done through its linear log-log presentation. Typically, the range of linear behavior terminated on both sides either because further data was not accessible or because of crossover bends. A histogram of the number of orders of magnitude used to declare fractality, covering all 96 reports, reveals a clear picture (see figure): The scaling range of experimentally declared fractality is extremely limited, centered around 1.3 orders of magnitude, spanning mainly between 0.5 and 2.0 (*4*). This limited range stands in stark contradiction to the public image of the status of experimental fractals.




**Limited scaling range.** The number of decades (factors of 10) spanned by experimentally derived scaling exponents that led to the labeling of the studied systems as fractal (*4*).

The most acute questions posed by these data are if the limited range is inherent, if these limited-range power-law objects are fractal, and if, in fact, nature is describable in terms of fractality. The question of fractality is actually secondary to the benefits of carrying out a multiple resolution analysis (Eq. 1); these benefits outweigh the perhaps erroneous fractal label.

The existence of cutoffs is inherently associated with experimentation on real physical objects. The lower cutoff is dictated typically by the basic building block unit (such as an atom, molecule, microcrystal, or small aggregate) of the system. The upper cutoff is, at most, of the order of the system size but is usually far below it. It is bounded either by the mechanical strength, by growth rates (which drop sharply with time), by the emergence of background effects (such as nonisotropic fields), or by the depletion of resources. Temporal self-affine trails scale over many orders of magnitude, but this is a completely different issue: the time axis can be extended at will.

Do power laws that are limited in range represent fractals? Is it justified to term them as such (*5*)? Regardless of the question of fractality, a more basic question should be asked: Is this presentation useful? The very existence of so many reports by competent researchers who are well aware of the problematics of declaring fractality for experimental results that span only one order of magnitude suggests that experimentalists seem to gain from the resolution analysis and from the fact that the result of such analysis is often a power law. The usefulness is in the following points: (i) The power law condenses the description of a complex geometry. (ii) It allows one to correlate in a simple way properties and performances of a system to its structure and to the dynamics of its formation. (iii) In many instances, the choice is either to use the limited-range data or to discard it altogether and not have even an approximate picture of the studied object. Opting for the former can be emphatically understood. (iv) Fractal geometry provides a proper language and symbolism for studies of ill-defined geometries.

It is important to reiterate, however, that the ability to fit data to Eq. 1 does not imply fractality and that the label "fractal" is not needed. So should one refer to such results in terms of a fractal object? If by "fractal" one refers to the original Mandelbrot teaching of many orders of magnitude, then the data we collected do not seem to support it in an unequivocal way. If by "fractal" one means an object that obeys Eq. 1 over a limited range, then the use of this label may be acceptable, not only because of its usefulness, but because of the following additional reasons: (i) Interestingly, the sense of self-similarity in irregular objects is comprehended visually even for a limited range. (ii) In some cases, experimentally derived objects resemble simulated objects obtained from fractal models. (iii) The empirical values of $D$ for spatial objects fall in the fractal regime of $0 < D < 3$. (iv) And, it may be too late to make any changes

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in a terminology that, at this stage, seems to be deeply rooted in practice. A drift from an original meaning of a concept is common in science, representing adaptability of the original ideal definition to realistic restrictions that emerge when put to practice.

We arrive at our final question: Is the geometry of nature fractal? Several key processes involving equilibrium-critical phenomena (in magnets, liquids, percolations, and phase transitions, for example) and some nonequilibrium growth models (such as aggregation) are backed by intrinsically scale-free theories and lead therefore to power-law scaling behavior on all scales. However, the majority of the data that was interpreted in terms of fractality in the surveyed *Physical Review* journals does not seem to be linked (at least in an obvious way) to existing models and, in fact, does not have theoretical backing. Most of the data represent results from nonequilibrium processes. The common situation is this: An experimentalist performs a resolution analysis and finds a limited-range power law with a value of $D$ smaller than the embedding dimension. Without necessarily resorting to special underlying mechanistic arguments, the experimentalist then often chooses to label the object for which she or he finds this power law a "fractal." This is the fractal geometry of nature.

### References


1. M. Marder, *Science* **277**, 647 (1997).
2. R. V. Kalia *et al.*, *Phys. Rev. Lett.* **78**, 2144 (1997).
3. For more detail, see O. Malcai *et al.*, *Phys. Rev. E* **56**, 2817 (1997).
4. Our earlier version of the histogram (*3*) had two cases with a range of 3.7 to 3.8 decades. It turns out that we were too "liberal" in our interpretation: One case was a deterministically built exact Koch fractal [B. Sapoval *et al.*, *ibid.* **48**, 3333 (1993)], and the other was described by the authors as representing "almost no deviations . . . for the first three decades" [T. Holten *et al.*, *ibid.* **50**, 754 (1994)].
5. For earlier critical analyses, see L. P. Kadanoff, *Phys. Today* **39**, 6 (February 1986), and O. R. Shenker, *Stud. Hist. Philos. Sci.* **25**, 967 (1994).


**VIROLOGY**

# Even Viruses Can Learn to Cope with Stress

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More than 5 years ago, a commentary in *Science* announced that viruses engage in "Star Wars" strategies against the immune system. Some of the viral invaders make receptors (viroceptors) that imitate normal cellular receptors and so can sequester and inactivate molecules that the immune system tries to use to fight the virus (*1*). Since that time, numerous other viral subterfuges for evading or subverting host defense mechanisms have been exposed (*2–4*), and viruses now are known to use an extraordinary spectrum of proteins to target immune molecules of the host cells. One particularly effective host defense is for the infected cell to self-destruct by programmed cell death, and in fact, cell death is triggered by infection with a wide variety of viruses (*5*). In response some viruses use specific proteins to suppress the cell suicide that would normally curtail the infection (*5*, *6*). Other classes of intracellular responses have elicited their own array of viral countermeasures as well (see the table). To this growing list, we can now add reactive oxidative species (oxidative stress) as a worthy target for viral inhibition. On page 102 of this issue, Shisler *et al.* (*7*) report that molluscum contagiosum virus (MCV) encodes a novel anti-oxidant protein (MC066L) that functions as a scavenger of reactive oxygen metabolites and protects cells from ultraviolet- or peroxide-induced damage. Equally intriguing, MC066L is also the first bona fide selenoprotein expressed by a virus.

The story began last year when B. Moss and his colleagues at the National Institutes of Health sequenced the genome of MCV, a human poxvirus that causes benign tumorlike skin lesions that can become problematic in immunosuppressed patients, including those with AIDS (*8*). Given the proclivity of the larger DNA viruses to engage in widespread gene piracy, it was expected that MCV would encode a variety of host-derived proteins. But what was most unexpected was how extraordinarily different the nonessential gene repertoire of MCV was from those of previously sequenced poxviruses, particularly vaccinia and variola (*9*). Not only was MCV bereft of most of the better studied immunoregulators, such as the secreted viroceptors that precipitated the original Star Wars analogy, but 77 of the 182 predicted MCV open reading frames had no obvious viral counterparts at all. Moreover, some of these novel candidates were predicted to antagonize immune responses on the basis of their sequence similarities to other known host genes, and this list included such luminaries as a major histocompatability complex–1 heavy chain homolog, a β-chemokine, and two related death effector domain–containing proteins (*8*, *9*). Particularly notable among these host-derived candidates was a predicted

**Intracellular Defense Strategies by Viruses**

| Host cell anti-virus mechanisms | Virus counter-strategies | Virus examples |
|---|---|---|
| Apoptosis (cell suicide) | Homologs of bcl-2 | **BHRF1** Epstein-Barr virus; **E1B/19K** adenovirus |
| | Caspase inhibitors | **crmA** cowpox virus; **p35** baculovirus |
| | Death effector | **MC159** molluscum contagiosum virus; **E8** equine herpes-2 |
| | Serpins | **SPI-1** rabbitpox virus |
| | p53 binding proteins | **T-Ag** simian virus 40; **E1B/55K** adenoviru |
| | Rb binding proteins | **E7** papilloma virus; **IE2** cytomegalovirus |
| | Ankyrin-repeat host range proteins | **CHOhr** cowpox virus; **M-T5** myxoma virus |
| | ER-retained protein | **M-T4** myxoma virus |
| Intracellular signaling | PKR inhibition | **E3L, K3L** vaccinia virus |
| | Tyrosine kinase modulation | **Tip** herpesvirus saimiri<br>**LMP-2A** Epstein-Barr virus |
| | Receptor mimicry | **M-T2** myxoma virus |
| | Signal transducer protein | **LMP-1** Epstein-Barr virus; **IAP** baculovirus |
| Viral antigen presentation | MHC-1 suppression | **E3/19K** adenovirus; **US11** cytomegalovirus |
| | TAP inhibition | **ICP47** herpes simplex virus; **US6** cytomegalovirus |
| **Oxidative stress response** | **Anti-oxidant selenoprotein** | **MCO66L (molluscum contagiosum virus)** |


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