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Direct Kalman Filtering Approach for GPS/INS Integration

We present a novel Kalman filtering approach for GPS/INS integration. In the approach, GPS and INS nonlinearities are preprocessed prior to a Kalman filter. The GPS preprocessed data are taken as measurement input, while the INS preprocessed data are taken as additional information for the state prediction of the Kalman filter. The advantage of this approach, over the well-studied (extended) Kalman filtering approaches is that a simple and linear Kalman filter can be implemented to achieve significant computation saving with very competitive performance figures.

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I. INTRODUCTION

The Global Positioning System (GPS) is a worldwide radio navigation system and has applications in aviation, aircraft automatic approach and landing, land vehicle navigation and tracking, marine applications, and surveying, etc. [1–2]. A GPS receiver is a low frequency response navigation sensor and can provide instantaneous position accuracy in the order of 15 to 100 m normally at 1 Hz rate. Inertial navigation systems (INS) are one of the most widely used dead reckoning systems. They can provide continuous position, velocity, and also orientation estimates, which are accurate for a short term, but are subject to drift due to sensor drifts. The integration of GPS and INS can limit the shortcomings of the individual systems, namely, the typically low rate of GPS measurements as well as the long term drift characteristics of INS. Integration can also exploit advantages of the two systems, such as the uniform high accuracy trajectory information of GPS and the short term stability of INS.

There are several methods to integrate INS and GPS, such as, loosely coupled or tightly coupled integration, closed-loop or open-loop integration, separated INS and GPS unit or embedded GPS with INS hardware, etc. [19–21]. In all these designs, GPS or the GPS/INS integration filter is typically some form of a Kalman filter. Usually an indirect (extended) Kalman filter is used with inertial errors as its state to achieve acceptable performance. A high-order filter is required to achieve, at best, near optimal performance. The computation load associated is very heavy, since on-line Kalman gains have to be calculated. A fixed Kalman gain is often used to decrease the computational load but with sacrifice of performance. In this paper, we present a direct Kalman filter integration approach in order to eliminate the computational complexity drawbacks as discussed above. Here, the so-called direct Kalman filter is a filter with the vehicle's position and velocity among its states. We aim to design a low order, linear Kalman filter to achieve simple computations but with competitive performance figures when compared with the best published data. The two stage GPS filtering methodology in [6–12] is applied here to preprocess the nonlinearity prior to the Kalman filter. The same methodology is also applied here to the nonlinear INS system, but this is not so straightforward since the nonlinearity for INS is dynamic rather than simply static as for GPS.

The paper is organized as follows. A direct Kalman filter integration approach is given in Section

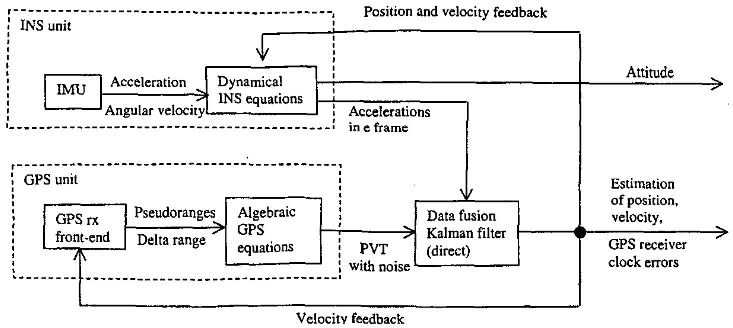

Fig. 1. Diagram of GPS/INS integration via direct Kalman filter approach.

II. Simulation results are shown in Section III. Finally, a conclusion is made in Section IV.

II. A DIRECT KALMAN FILTERING APPROACH FOR GPS/INS INTEGRATION

The diagram of the proposed GPS/INS integration is shown in Fig. 1. The functions of the blocks are as follows.

Dynamical INS Equations: The navigation equations convert the inertial measurement unit (IMU) measured accelerations and angular velocity, along with feedback position and velocity estimates, to INS estimated accelerations in an Earth centered Earth fixed (ECEF) coordinate frame and to attitude estimates. The IMU and navigation equations together are referred to as the INS unit.

Algebraic GPS Equations: As in [3–5], the algebraic GPS equations give position, velocity, and clock error estimates by solving the nonlinear GPS pseudo-range and delta range equations. The GPS front end and algebraic GPS equations are referred to as the GPS unit.

Data Fusion Kalman Filter (Direct): This filter obtains estimates of position, velocity, GPS receiver clock errors, as well as bias and drift in the INS estimated accelerations, based on the information data from both the INS and GPS units. Its estimates are fed back to the INS and GPS units as required.

The essential feature of the proposed integration all the various nonlinear operations prior to linear Kalman filtering and to put as much of the necessary dynamics as possible into the Kalman filter. Recall that the Kalman filter is the optimal minimum variance filter for a known linear stochastic model with zero mean Gaussian noise of known covariance. Should the model be linear but the noise be non-Gaussian, then the Kalman filter is the best linear minimum variance filter. Of course, the more sophisticated inertial error models with extended Kalman filtering may improve performance, although perhaps not always significantly, but will cost an order of magnitude or more in computational effort.

The GPS nonlinearities are preprocessed in the block denoted "Algebraic GPS equations" at the GPS sample rate. The nonlinearities of the INS are preprocessed in the block denoted "Dynamical INS equations." These involve an integration and so are dynamical and not algebraic; they calculate an orthogonal coordinate transformation matrix. The outputs from the INS block are estimates of the vehicular attitude, and estimates of accelerations in the ECEF frame, denoted by superscript e here. The INS sampling rate is typically an order of magnitude or so faster than the GPS rate. The acceleration estimates from the INS unit are fed forward to the data fusion Kalman filter, along with the GPS estimates.

In this approach, vehicle position and velocity are chosen as states in the Kalman filter. The propagation equations are simply the equations of motion of the vehicle. Accelerations are included in the Kalman filter to make the propagation equations a better reflection of the real world. The accelerations are estimates from INS and taken as known inputs to the Kalman filter. Additional error states can be included as filter states. The GPS indicated position and velocity from the algebraic GPS equation are the measurements of the Kalman filter, which make the measurement matrix of the Kalman filter very simple.

In traditional approaches, the inertial errors are chosen as states in the Kalman filter. The inertial error model and its linear approximation are well known and well developed. They are taken as the propagation equations of the Kalman filter. The measurements of the Kalman filter are taken to be the differences of GPS measured pseudo-range and INS estimated range. Thus measurement matrix of the Kalman filter incorporates the nonlinearity of GPS pseudo-range equations.

A. Dynamical INS Equations

The computations performed by a strapdown navigator may be regarded as comprising two major parts: propagation of the attitude reference, and solution of the navigation equations. The former uses the gyro outputs to calculate the attitude of the body coordinate frame with respect to a reference coordinate frame. The latter uses this relationship to transform the coordinates of vectors (which may include accelerometer outputs, velocity, gravity effects, Earth rotation) between the frames and hence to calculate acceleration of the body. In our proposed GPS/INS integration method, consequent velocity and position calculations frequently carried out in an INS unit are carried out in the data fusion Kalman filter.

The coordinate frame in which computation is performed, is usually chosen such that it agrees with the coordinate frame for the output. Here we choose the ECEF frame, so as to obtain directly the geocentric Cartesian coordinates which in turn are convenient for integration with GPS information.

The continuous-time nonlinear dynamical equations in the ECEF frame are of the form [19]

$$\begin{bmatrix} \dot{\mathbf{r}}^e \\ \dot{\mathbf{v}}^e \\ \dot{\mathbf{R}}_b^e \end{bmatrix} = \begin{bmatrix} \mathbf{v}^e \\ \mathbf{R}_b^e \mathbf{f}^b - 2\Omega_{ie}^e \mathbf{v}^e + \mathbf{g}^e(\mathbf{r}^e) \\ \mathbf{R}_b^e \Omega_{eb}^b \end{bmatrix} \tag{1}$$

where $\mathbf{r}^e, \mathbf{v}^e$ are the position and velocity vectors in the ECEF frame (e-frame), $\mathbf{f}^b$ the specific force vector in the body frame (b-frame), $\mathbf{g}^e$ the gravity vector in the e-frame and is $\mathbf{r}^e$ dependent, $\mathbf{R}_b^e$ is the transformation matrix from the b-frame to the e-frame, so that $\mathbf{f}^e = \mathbf{R}_b^e \mathbf{f}^b$ is the specific force vector in the e-frame, Ω_{eb}^b is the skew-symmetric matrix of the angular velocity vector ω_{eb}^b of the b-frame with respect to the e-frame coordinated in the b-frame, and Ω_{ie}^e is the skew-symmetric matrix of the Earth's rotation rate ω_{ie}^e which is known precisely. The skew-symmetric matrices are of the form

$$\Omega = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}. \tag{2}$$

The various vectors are dependent on time t and the super dot denotes derivatives with respect to t. The angular velocity vector ω_{eb}^b can be obtained by

$$\omega_{eb}^b = \omega_{ib}^b - \mathbf{R}_e^b \omega_{ie}^e, \qquad \text{with} \quad \mathbf{R}_e^b = \mathbf{R}_b^{e^T} \tag{3}$$

where ω_{ib}^b is the gyro measured angular velocities with respective to the inertial frame coordinated in the body frame. After solving (1), a rotation matrix $\mathbf{R}_b^n$ from the body frame to the navigation frame (n-frame) can then be obtained

$$\mathbf{R}_b^n = \mathbf{R}_e^n \mathbf{R}_b^e \tag{4}$$

where $\mathbf{R}_e^n$ is a rotation matrix from the e-frame to the n-frame, which is position $\mathbf{r}^e$ dependent. The rotation matrix $\mathbf{R}_b^n$ is composed of three successive rotations with angles called yaw, pitch, and roll, which are the attitude of a vehicle. Therefore the attitude of the vehicle can be obtained through the rotation matrix $\mathbf{R}_b^n$ [21].

1) *Discrete Time Navigation Equations*: The INS unit seeks to implement (1) in discrete time at the INS sampling rate with period δT. This rate is chosen so that the discrete-time equations follow closely the continuous-time equations (1).

To this end, let us first construct a sampled version of (1) with state matrix $\mathbf{R}_b^e(l)$ and output $\dot{\mathbf{v}}_e(l)$, driven by $\Omega_{eb}^b(l)$, $\Omega_{ie}^e(l)$, $\mathbf{f}^b(l)$, $\mathbf{v}^e(l)$, $\mathbf{g}^e(\mathbf{r}^e(l))$:

$$\mathbf{R}_b^e(l+1) = \mathbf{R}_b^e(l)\exp(\Omega_{eb}^b(l)\delta\mathrm{T}) \tag{5}$$

$$\dot{\mathbf{v}}^e(l) = \mathbf{R}_b^e(l)\mathbf{f}^b(l) - 2\Omega_{ie}^e(l)\mathbf{v}^e(l) + \mathbf{g}^e(\mathbf{r}^e(l)). \tag{6}$$

Since (5) holds only when $\Omega_{eb}^b(l)$ is constant during the period of δT from l to $l+1$, the accuracy of $\mathbf{R}_b^e$ is dependent on the sample period δT. The smaller the δT, the more accurate the $\mathbf{R}_b^e$ calculation. Now $\exp(\Omega_{eb}^b(l)\delta\mathrm{T})$ can be approximated using a (1,1) Padè approximation [22] denoted with a superbar as

$$\overline{\exp(\Omega_{eb}^b(l)\delta\mathrm{T})} = (2\mathbf{I} + \Omega_{eb}^b(l)\delta\mathrm{T})(2\mathbf{I} - \Omega_{eb}^b(l)\delta\mathrm{T})^{-1}. \tag{7}$$

This is readily verified to preserve orthogonality. Higher accuracy approximations can be made by working with $(\exp(\Omega_{eb}^b(l)(\delta\mathrm{T}/2)))^2$ and applying the (1,1) Padè approximations. (Actually, it is straightforward to show that (n,m) Padè approximations preserve orthogonality with $n = m$, but otherwise do not.)

The INS unit implements an approximation to (6) using accelerometer measurements of accelerations $\mathbf{f}^b(l)$ which typically are noisy with bias and drift; likewise for the gyro rotation rates ω_{ib}^b, and thus for ω_{eb}^b. Also in the INS unit, the position and velocity vectors $\mathbf{r}^e, \mathbf{v}^e$ are estimated from the data fusion Kalman filter. This estimation introduces further errors (noise). Using carets to denote estimates or measurements, then the navigation equations are implemented in the INS unit as

$$\begin{aligned} \hat{\mathbf{R}}_b^e(l+1) &= \hat{\mathbf{R}}_b^e(l)\overline{\exp(\hat{\Omega}_{eb}^b(l)\delta\mathrm{T})} \\ \hat{\dot{\mathbf{v}}}^e(l) &= \hat{\mathbf{R}}_b^e(l)\hat{\mathbf{f}}^b(l) - 2\Omega_{ie}^e\hat{\mathbf{v}}^e(l) + \mathbf{g}^e(\hat{\mathbf{r}}^e(l)). \end{aligned} \tag{8}$$

Recalling that there are drifts and biases in the measurements from gyros and accelerometers, we expect that these in turn lead to biases and drifts in the estimates of the e-frame acceleration $\dot{\mathbf{v}}_e$. The details are as follows.

B. Bias and Drift in INS

Let angular velocity errors on ω_{eb}^b caused by gyro drifts be $\Delta\omega_{eb}^b$, leading to drifts $\Delta\Omega_{eb}^b$ in Ω_{eb}^b and $\Delta\mathbf{R}_b^e(l)$ in $\mathbf{R}_b^e(l)$. Then (5) can be rewritten as

$$\begin{aligned} \mathbf{R}_b^e(l+1) + \Delta\mathbf{R}_b^e(l+1) &= \mathbf{R}_b^e(l)\exp((\Omega_{eb}^b(l) + \Delta\Omega_{eb}^b(l))\delta\mathrm{T}) \\ &= \mathbf{R}_b^e(l+1) + \mathbf{R}_b^e(l)\Delta\Omega_{eb}^b(l)\delta\mathrm{T} + \mathrm{HOT} \end{aligned} \tag{9}$$

where HOT denotes higher order terms. Let accelerometer bias be $\Delta \mathbf{f}^b$. Recalling that $\mathbf{f}^e = \mathbf{R}_b^e \mathbf{f}^b$, then

$$(\mathbf{R}_b^e(l+1) + \mathbf{R}_b^e(l)\Delta\Omega_{eb}^b(l)\delta T)(\mathbf{f}^b(l+1) + \Delta\mathbf{f}^b(l+1))$$
$$= \mathbf{f}^e(l+1) + \mathbf{R}_b^e(l+1)\Delta\mathbf{f}^b(l+1)$$
$$+ \mathbf{R}_b^e(l)\Delta\Omega_{eb}^b(l)(\mathbf{f}^b(l+1) + \Delta\mathbf{f}^b(l+1))\delta T$$
$$:= \mathbf{f}^e(l+1) + \Delta\mathbf{f}_1^e(l+1) + \Delta\mathbf{f}_2^e(l+1)\delta T. \qquad (10)$$

From (10), it can be concluded that gyro drift and accelerometer bias cause bias $\Delta\mathbf{f}_1^e$ and drift $\Delta\mathbf{f}_2^e$ on specific forces in the e-frame, where

$$\Delta\mathbf{f}_1^e(l+1) := \mathbf{R}_b^e(l+1)\Delta\mathbf{f}^b(l+1) \qquad (11)$$

$$\Delta\mathbf{f}_2^e(l+1) := \mathbf{R}_b^e(l)\Delta\Omega_{eb}^b(l)(\mathbf{f}^b(l+1) + \Delta\mathbf{f}^b(l+1)). \qquad (12)$$

Since equations are in discrete form, $\Delta\mathbf{f}_1^e$, $\Delta\mathbf{f}_2^e$ are considered as the bias and drift, respectively, of the e-frame acceleration $\dot{\mathbf{v}}_e$, which are also caused by gyro drift and accelerometer bias. Actually the errors on the e-frame acceleration are also dependent on e-frame velocity errors and position errors which are introduced by the Kalman filter.

C. Data Fusion Kalman Filter Design

Let the state space model for the design of the data fusion Kalman filter be

$$\xi = [\mathbf{x}^T \; b \; \dot{\mathbf{x}}^T \; \dot{b} \; (\Delta\mathbf{f}_1^e)^T \; (\Delta\mathbf{f}_2^e)^T]^T \qquad (13)$$

where $\mathbf{x}$ is the GPS receiver's position coordinates in the ECEF frame, and b is the GPS receiver's clock range bias. Consider a discrete time signal model for data fusion, operating at a fast sample rate with sampling period δT, as

$$\xi_{l+1} = \mathbf{A}_l\xi_l + \mathbf{B}_l\mathbf{u}_l + \mathbf{w}_l \qquad (14)$$

with $l = 0,1,2,\ldots$. We assume first that the fast sample rate is equal to the INS sample rate and is N times the GPS rate, where N is some integer, typically between 10 and 200. Thus the GPS sample period ΔT is $N\delta T$. Here the known inputs $\mathbf{u}_l$ are the e-frame acceleration estimates from the INS unit via $\mathbf{u}_l = \dot{\mathbf{v}}^e$, and

$$\mathbf{B}_l = \begin{bmatrix} 0_{4\times 3} \\ \mathbf{I}_{3\times 3}\delta T \\ 0_{7\times 3} \end{bmatrix}. \qquad (15)$$

The unknown inputs $\mathbf{w}_l$ represent the estimation errors in the accelerations (including estimation errors in biases and drifts in accelerations), and other modeling errors. For Kalman filter design proposes only, these are assumed here to be zero mean and with known or estimated covariance matrix Q, although more sophisticated models of bias and time-varying statistics can be used at the cost of increased complexity of Kalman filter design. The Kalman filter, even designed on simple and conservative model assumptions is known to be robust to more real-world noises.

The transition matrix $\mathbf{A}_l$ has the form

$$\mathbf{A}_l = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{0} & \mathbf{A}_{22} \end{bmatrix} \qquad (16)$$

with

$$\mathbf{A}_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 & \delta T & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \delta T & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \delta T & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \delta T \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{A}_{12}^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta T & 0 & 0 & \Delta T^2 & 0 & 0 \\ 0 & \Delta T & 0 & 0 & \Delta T^2 & 0 \\ 0 & 0 & \Delta T & 0 & 0 & \Delta T^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (17)$$

$$\mathbf{A}_{22}^* = \begin{bmatrix} c_1\mathbf{I}_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & c_2\mathbf{I}_{3\times 3} \end{bmatrix} \qquad (18)$$

$$\mathbf{A}_{12} = \begin{cases} \mathbf{A}_{12}^*, & \text{if } l = mN \\ 0_{8\times 6}, & \text{otherwise} \end{cases},$$
$$\mathbf{A}_{22} = \begin{cases} \mathbf{A}_{22}^*, & \text{if } l = mN \\ 0_{6\times 6}, & \text{otherwise} \end{cases} \qquad (19)$$

for integers $m = 0,1,2,\ldots$, where $c_1, c_2 \leq 1$ are forgetting rates for the biases and drift rate, respectively. Actually the GPS clock bias and drift can be processed at the GPS rate.

The measurement equation for the data fusion Kalman filter model can be represented by

$$\mathbf{Z}_l = \mathbf{C}_l^T\xi_l + \mathbf{v}_l \qquad (20)$$

where for integers m (as above)

$$\mathbf{C}_l^T = \begin{cases} [\mathbf{I}_{8\times 8} \quad 0_{8\times 6}] & \text{for } l = mN \\ 0 & \text{otherwise} \end{cases}. \qquad (21)$$

Here $\mathbf{Z}_l$ are the computed position, velocity, and clock errors from the algebraic GPS equations, and the covariance matrix $\mathbf{R}_l$ of the measurement noise v_l is covariance matrix of the solution from the algebraic GPS equation [3, 6] which is time varying. The measurement equation is identical to that for the GPS Kalman filter in [23] at times $0, N, 2N, \ldots$, and is zero otherwise.

For the data fusion signal model (14)–(20), the standard Kalman filter algorithm can be applied. Three cases are now considered.

Case 1: *Identical GPS and INS Sampling Rates* ($N = 1$). Even if the GPS and INS sampling rates are identical ($N = 1$), the data fusion Kalman filter is somewhat more sophisticated than the GPS Kalman filter of [6]. This is because of the external "inputs" from the INS unit, the bias and drift state estimates, and associated (Kalman) gain terms. In this case the data fusion filter will, in general, be time varying. There is a time-varying Riccati error covariance and Kalman gain, to take account of initial transients and slow variations in the noise covariance. However, such a filter can be quite accurately approximated in our context (performance wise) by a time-invariant filter using limiting Riccati equation solutions working with typical noise covariance. Of course, the Kalman gain can also be gain-scheduled depending on some of the variables.

The crucial design variables for the linear filter are then the forgetting rates c_1, c_2 and the noise variances $\mathbf{Q}_l, \mathbf{R}_l$. Selection of these design variables will benefit from knowledge of the sensor noise, bias, and drift characteristics

$$\text{Initialization} \quad \begin{aligned} \hat{\xi}_{l-1/l-1} &= \xi_0 \\ \mathbf{P}_{l-1/l-1} &= \mathbf{P}_0 \end{aligned} \tag{22}$$

$$\text{Prediction} \quad \begin{aligned} \hat{\xi}_{l/l-1} &= A_{l-1}\hat{\xi}_{l-1/l-1} + \mathbf{B}_{l-1}\mathbf{u}_{l-1} \\ \mathbf{P}_{l/l-1} &= \mathbf{A}_{l-1}\mathbf{P}_{l-1/l-1}\mathbf{A}_{l-1}^{\mathrm{T}} + \mathbf{Q}_{l-1} \end{aligned} \tag{23}$$

$$\text{Update} \quad \begin{aligned} \mathbf{K}_l &= \mathbf{P}_{l/l-1}\mathbf{C}_l[\mathbf{C}_l^{\mathrm{T}}\mathbf{P}_{l/l-1}\mathbf{C}_l + \mathbf{R}_l]^{-1} \\ \hat{\xi}_{l/l} &= \hat{\xi}_{l/l-1} + \mathbf{K}_l[\mathbf{Z}_l - \mathbf{C}_l^{\mathrm{T}}\hat{\xi}_{l/l-1}] \\ \mathbf{P}_{l/l} &= [\mathbf{I} - \mathbf{K}_l\mathbf{C}_l^{\mathrm{T}}]\mathbf{P}_{l/l-1}. \end{aligned} \tag{24}$$

Case 2: *Integer ratios of GPS and INS Sampling Rates* ($N = 0, 1, 2, \ldots$). In this case, the $\mathbf{C}_l$ matrix is periodic with its period N being the ratios of the sampling rates, assumed here to be an integer (typically between 10 and 100). The associated Riccati error covariance matrix has a consequent periodic component, as then has the Kalman gains. However, such a filter can be quite accurately approximated (performance wise) by a filter with precisely periodic Kalman gains. One choice is with the gains to the bias and drift states being constant and the gains to the position and velocity and GPS clock drift states being obtained by the Kalman filter algorithm when $l = 0, N, 2N, \ldots$ and zero otherwise. In the follow, equations used for a Kalman filter with fixed gains to the bias and drift states are shown

$$\text{Initialization} \quad \begin{aligned} \hat{\xi}_{l-1/l-1} &= \xi_0 \\ \mathbf{p}_{l-1/l-1} &= \mathbf{p}_0 \end{aligned} \tag{25}$$

$$\text{Prediction} \quad \begin{aligned} \hat{\xi}_{l/l-1} &= \mathbf{A}_{l-1}\hat{\xi}_{l-1/l-1} + \mathbf{B}_{l-1}\mathbf{u}_{l-1} \\ \mathbf{p}_{l/l-1} &= \mathbf{A}_{11}\mathbf{p}_{l-1/l-1}\mathbf{A}_{11}^{\mathrm{T}} + \mathbf{q}_{l-1} \end{aligned} \tag{26}$$

$$\text{Update} \quad \begin{aligned} \mathbf{k}_l &= \mathbf{p}_{l/l-1}\mathbf{c}_l[\mathbf{c}_l^{\mathrm{T}}\mathbf{p}_{l/l-1}\mathbf{c}_l + \mathbf{R}_l]^{-1} \\ \mathbf{p}_{l/l} &= [\mathbf{I} - \mathbf{k}_l\mathbf{c}_l^{\mathrm{T}}]\mathbf{p}_{l/l-1} \\ \hat{\xi}_{l/l} &= \hat{\xi}_{l/l-1} + \mathbf{K}_l[\mathbf{Z}_l - \mathbf{C}_l^{\mathrm{T}}\hat{\xi}_{l/l-1}] \\ \mathbf{K}_l &= \begin{cases} \begin{bmatrix} \mathbf{k}_l \\ (\mathbf{kfix})_{6\times 8} \end{bmatrix}, & \text{if} \quad l = mN \\ 0, & \text{otherwise} \end{cases}. \end{aligned} \tag{27}$$

Here $\mathbf{p}, \mathbf{k}, \mathbf{c}$ are covariance matrix, Kalman gain, observation matrix, respectively, of the first 8 states, such as, position, velocity and GPS clock error states

$$\mathbf{c}_l = \begin{cases} \mathbf{I}_{8\times 8} & \text{for} \quad l = mN \\ 0 & \text{otherwise} \end{cases}. \tag{28}$$

$\mathbf{q}$ is the covariance matrix of the process noise vector $\mathbf{w}_l$ associated with states of position, velocity and GPS clock errors and **kfix** is the Kalman gain matrix of the 6 bias and drift states, it is constant, being the form of

$$\mathbf{kfix} = \begin{bmatrix} 0_{3\times 3} & g_1\mathbf{I}_{3\times 3} & 0_{3\times 2} \\ 0_{3\times 3} & g_2\mathbf{I}_{3\times 3} & 0_{3\times 2} \end{bmatrix} \tag{29}$$

where g_1, g_2 are constant values, typically less than 1.0, which are again the design parameters of the Kalman filter.

Case 3: *Nonsynchronized GPS and INS Sampling*: This case can in principle be handled within the context of Kalman filtering theory, see for example [18], but for our purposes since N is typical large, there can be approximate round off to the nearest integer value and the results used as for Case 2 above. We do not explain this further here.

REMARKS Using the Kalman filter for an assumed linear stochastic model (14)–(21), achieves optimality in a minimum error covariance sense when the noise is zero mean, white and Gaussian, and is the best linear filter when the noise is non-Gaussian. Loss of optimality, perhaps to a negligible degree occurs in practice since noise errors may be correlated, non-Gaussian, or the model may also have a degree of error due to finite filter sampling rates and other approximations. Even so, our conjecture is that the extra benefit of using inertial error models with extended Kalman filtering may not be worth the extra computational effort, which would be an order of magnitude or so increase.

The GPS receiver clock error states can be excluded from the Kalman filter to achieve a low-dimensional filter with only a marginally degraded performance. Since GPS algebraic equation

TABLE I
Mean and Variance of Estimated Position Errors for Three Design Options of GPS/INS

IMU Statistics: 3 deg/hr gyro drift, 0.025 m/s^2 accelerometer bias							
		GPS measurement noise variance Pseudorange: 30.0 m Pseudorange rate: 50 cm/s			GPS measurement noise variance Pseudorange: 30.0 m Pseudorange rate: 5 cm/s		
Position Errors		Latitude	Longitude	Height	Latitude	Longitude	Height
First option GPS resetting	Mean (m)	−5.310	−7.722	0.852	−1.003	−1.388	0.172
	Variance (m^2)	28.175	117.932	31.615	0.715	2.489	0.847
Second option as in [20, 23]	Mean (m)	−0.80834	1.317	0.069	1.313	1.361	−0.255
	Variance (m^2)	25.785	102.163	29.906	0.4	1.043	0.6445
Proposed method	Mean (m)	−4.504	−3.373	0.761	−0.936	−1.572	0.137
	Variance (m^2)	27.373	102.576	29.721	0.695	2.530	0.844

gives the point estimates of position, velocity and clock errors (PVT), however they are correlated with each other in their covariance matrix. When the clock errors are excluded from the Kalman filter, actually the correlation of clock errors with position and velocity is ignored, resulting in the degraded performance. However the clock errors cannot be excluded from a Kalman filter if there is no preprocessing of GPS pseudo-ranges as the GPS algebraic equation.

The two-stage estimator concept was developed in [6] for the GPS case. Here the same concept is extended to the case of GPS/INS Integration. The GPS algebraic equation is taken as the first stage, the direct Kalman filter as the second stage estimator. It is found in the next section that it has performance loss. This performance loss is due to the low order model used in the direct Kalman filter, rather than the two stage estimator. A different approach using the same concept of the two stage estimator for GPS/INS integration is discussed in [23]. It uses an indirect Kalman filter as the second stage estimator; near optimal performance can be achieved. It is proved [23] mathematically that the two stage estimator has superior performance than the EKF when the same order of Kalman filter model is used. The algebraic equation does not lose information, since from solution, the raw pseudo-range measurements can be recovered completely. In the two stage case, actually all the information contained in the raw pseudo-range measurements is fed to the Kalman filter, but in a different form of PVT, which is simpler to work with than pseudo-ranges.

III. IMPLEMENTATION AND SIMULATION RESULTS

A trajectory used for simulation is assumed as follows. An airplane's initial position is at latitude of 35 deg south, longitude of 150 west and height of 1000 m, the initial velocities are 700 km/hr, 200 m/hr, −100 m/hr along the three axes of the navigation frame, i.e., north, east, and vertical down, respectively. It is accelerated along the north, east, and down with the acceleration of 6 km/hr2, 6 km/hr2, −6 km/hr2, respectively. The airplane's initial orientation is assumed to be parallel to the navigation frame, i.e., 0 deg of yaw, pitch, and roll and then rotated with 0.005 deg/s for yaw, pitch, and roll. The simulation period is 1 hr.

In the implementation of the new proposed 14 state Kalman filter, the algorithm (25)–(27) is used, therefore fixed Kalman gains for the bias and drift $\Delta\mathbf{f}_1^e$, $\Delta\mathbf{f}_2^e$ are employed. The forgetting rates c_1,c_2 are set to be 0.99, and the fixed Kalman gains, g_1,g_2 are set to be 0.008. The prediction rate is set to be 32 Hz; the measurement update rate is set to be the GPS sample rate, 1 Hz. Therefore $\delta\mathrm{T} = 0.03125$ s, $\Delta\mathrm{T} = 1$ s, and $N = 32$.

For comparison, two other integration options are presented. The first integration option, called GPS resetting, also the simplest from the implementation viewpoint, is resetting the INS-derived position and velocity. Here the GPS receiver employs a Kalman filter as the one discussed in [6, 23]. However, the prediction equations are implemented at a fast sample rate, such as 32 Hz here, while the update equations are implemented at GPS sample rate, 1 Hz. The second option is the one using the GPS pseudo-range measurements in the integration filter as discussed in [20, 23]. Their simulation results are shown in Table I.

From Table I, it can be seen that the proposed integration method gives better performance than the first option, GPS resetting approach, since the former takes INS calculated accelerations as known inputs and acceleration biases and drifts as Kalman filter states. Certainly the proposed method makes use of the information from INS. However, it gives worse performance than the second option. This is because the errors on the calculated accelerations are modeled by the bias and drift states $\Delta\mathbf{f}_1^e$, $\Delta\mathbf{f}_2^e$ in an approximated and simple way as discussed in Section IIB, while a linearized differential equation of the acceleration errors are modeled in the second option. The advantage of the proposed method is that off-line Kalman gain computation can be implemented so that on-line computation is low.

IV. CONCLUSIONS

In this paper we have presented a direct Kalman filtering approach for GPS/INS integration. In the approach, GPS and INS nonlinearities are preprocessed prior to a Kalman filter. The GPS preprocessed data are taken as measurement input; the INS preprocessed data are taken as an additional information to the state prediction of the Kalman filter. The advantage of this approach is that a simple and linear Kalman filter can be implemented to achieve significant computation saving with competitive performance figures.

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