8TH EDITION

INTERMEDIATE

MICROECONONICS HAL R. VARIAN

Profit-Maximization

A firm uses inputs j = 1...,m to make products i = 1,...n.

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- Output levels are y_1, \dots, y_n .
- Input levels are x_1, \dots, x_m .
- ◆ Product prices are p₁,...,p_n.
- Input prices are w_1, \ldots, w_m .

The Competitive Firm

The competitive firm takes all output prices p₁,...,p_n and all input prices w₁,...,w_m as given constants.

The economic profit generated by the production plan (x₁,...,x_m,y₁,...,y_n) is

$$\Pi = \mathbf{p}_1 \mathbf{y}_1 + \cdots + \mathbf{p}_n \mathbf{y}_{\mathbf{h}} = -\mathbf{w}_1 \mathbf{x}_1 - \cdots + \mathbf{w}_n \mathbf{x}_n$$

Economic Profit Output and input levels are typically flows.

- ♦ E.g. x₁ might be the number of labor units used per hour.
- And y₃ might be the number of cars produced per hour.

Consequently, profit is typically a flow also; e.g. the number of dollars of profit earned per hour.

Economic Profit How do we value a firm?

- Suppose the firm's stream of periodic economic profits is Π₀, Π₁, Π₂, ... and r is the rate of interest.
- Then the present-value of the firm's economic profit stream is

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$$PV = \Pi_{0} + \frac{\Pi_{1}}{1+r} + \frac{\Pi_{2}}{1+r}^{2} + \cdots$$

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A competitive firm seeks to maximize its present-value.

♦ How?

- ◆ Suppose the firm is in a short-run circumstance in which $x_2 \equiv \tilde{x}_2$.
- ♦ Its short-run production function is $y = f(x_1, \tilde{x}_2).$

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- ◆ Suppose the firm is in a short-run circumstance in which $x_2 \equiv \tilde{x}_2$.
- ♦ Its short-run production function is $y = f(x_1, \tilde{x}_2).$
- The firm's fixed cost is F C = W 2 X 2 and its profit function is

$$\Pi = \mathbf{p}\mathbf{y} - \mathbf{w}_1\mathbf{x}_1 - \mathbf{w}_2\mathbf{\tilde{x}}_2.$$

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Short-Run Iso-Profit Lines

♦ A \$∏ iso-profit line contains all the production plans that provide a profit level \$∏.

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♦ A \$ Π iso-profit line's equation is $\Pi = py - W_1 X_1 - W_2 \tilde{X}_2.$

Short-Run Iso-Profit Lines

- ♦ A \$∏ iso-profit line contains all the production plans that yield a profit level of \$∏.
- ◆ The equation of a \$ Π iso-profit line is $\Pi \equiv py - w_1 x_1 - w_2 \tilde{x}_2$.

 $\frac{W_{1}}{X_{1}} +$

W 2 X 2

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◆ I.e.



Short-Run Iso-Profit Lines



Short-Run Profit-Maximization

- The firm's problem is to locate the production plan that attains the highest possible iso-profit line, given the firm's constraint on choices of production plans.
- ♦ Q: What is this constraint?

Short-Run Profit-Maximization

- The firm's problem is to locate the production plan that attains the highest possible iso-profit line, given the firm's constraint on choices of production plans.
- ♦ Q: What is this constraint?
- ♦ A: The production function.















Short-Run Profit-Maximization $MP_1 = \frac{W_1}{p} \Leftrightarrow p \times MP_1 = W_1$

 $p \times M P_1$ is the marginal revenue product of input 1, the rate at which revenue increases with the amount used of input 1. If $p \times M P_1 > w_1$ then profit increases with x_1 . If $p \times M P_1 < w_1$ then profit decreases with x_1 . Short-Run Profit-Maximization; A Cobb-Douglas Example Suppose the short-run production function is $y = x_1^{1/3} \tilde{x}_2^{1/3}$.

The marginal product of the variable input 1 is $MP_1 = \frac{\partial y}{\partial x_1} = \frac{1}{3} x_1^{-2/3} \tilde{x}_2^{1/3}$.

The profit-maximizing condition is

 $M R P_1 = p \times M P_1 = \frac{p}{2} (x_1)^{-2/3} \tilde{x}_2^{1/3} = w_1.$

Short-Run Profit-Maximization; A Cobb-Douglas Example Solving $\frac{p}{3}(x_1^*)^{-2/3} \tilde{x}_2^{1/3} = w_1$ for x_1 gives $(x_1^*)^{-2/3} = \frac{3 w_1}{p \tilde{x}_2^{1/3}}.$

Short-Run Profit-Maximization; A Cobb-Douglas Example Solving $\frac{p}{3}(x_1^*)^{-2/3} \tilde{x}_2^{-1/3} = w_1$ for x_1 gives $(\mathbf{x}_{1}^{*})^{-2/3} = \frac{3 \mathbf{w}_{1}}{\mathbf{p} \,\widetilde{\mathbf{x}}_{2}^{1/3}}.$ That is, $(x_1^*)^{2/3} = \frac{p \tilde{x}_2^{1/3}}{3 w_1}$ © 2010 W. W. Norton & Company, Inc. 26

Short-Run Profit-Maximization; A Cobb-Douglas Example Solving $\frac{p}{2}(x_1^*)^{-2/3} \tilde{x}_2^{1/3} = w_1$ for x_1 gives $(\mathbf{x}_{1}^{*})^{-2/3} = \frac{3 \mathbf{W}_{1}}{\mathbf{p} \,\widetilde{\mathbf{x}}_{2}^{1/3}}.$ That is, $(\mathbf{x}_{1}^{*})^{2/3} = \frac{\mathbf{p} \, \widetilde{\mathbf{x}}_{2}^{1/3}}{3 \, \mathbf{w}_{1}}$ $x_{1}^{*} = \left(\begin{array}{c} p \tilde{x}_{2}^{1/3} \\ 3 w_{1} \end{array} \right)^{3/2}$ SO © 2010 W. W. Norton & Company, Inc. 27 Short-Run Profit-Maximization; A Cobb-Douglas Example $x_1^* = \left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2}$ is the firm's short-run demand

for input 1 when the level of input 2 is fixed at \tilde{x}_2 units.

Short-Run Profit-Maximization; A Cobb-Douglas Example $x_1^* = \left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2}$ is the firm's short-run demand

for input 1 when the level of input 2 is fixed at \tilde{x}_2 units.

The firm's short-run output level is thus

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$$y^{*} = (x_{1}^{*})^{1/3} \tilde{x}_{2}^{1/3} = \left(\frac{p}{3 w}\right)^{1/2} \tilde{x}_{2}^{1/2}.$$
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Comparative Statics of Short-Run Profit-Maximization

What happens to the short-run profitmaximizing production plan as the output price p changes? Comparative Statics of Short-Run Profit-Maximization The equation of a short-run iso-profit line is $y = \frac{W_1}{p} x_1 + \frac{\prod + W_2 \tilde{x}_2}{p}$

so an increase in p causes -- a reduction in the slope, and -- a reduction in the vertical intercept.







Comparative Statics of Short-Run Profit-Maximization An increase in p, the price of the firm's output, causes

- an increase in the firm's output level (the firm's supply curve slopes upward), and
- –an increase in the level of the firm's variable input (the firm's demand curve for its variable input shifts outward).






Comparative Statics of Short-Run Profit-Maximization

What happens to the short-run profitmaximizing production plan as the variable input price w₁ changes?

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Comparative Statics of Short-Run Profit-Maximization The equation of a short-run iso-profit line is $y = \frac{W_1}{p}x_1 + \frac{\Pi + W_2 \tilde{x}_2}{p}$

so an increase in w₁ causes -- an increase in the slope, and -- no change to the vertical intercept.







Comparative Statics of Short-Run Profit-Maximization An increase in w₁, the price of the firm's variable input, causes

- a decrease in the firm's output level (the firm's supply curve shifts inward), and
- a decrease in the level of the firm's variable input (the firm's demand curve for its variable input slopes downward).

Comparative Statics of Short-**Run Profit-Maximization** The Cobb-Douglas example: When $y = x_1^{1/3} \tilde{x}_2^{1/3}$ then the firm's short-run demand for its variable input 1 is $\mathbf{x}_{1}^{*} = \left(\frac{p}{3 w_{1}}\right)^{3/2} \widetilde{\mathbf{x}}_{2}^{1/2} \text{ and its short-run} \\ \mathbf{y}^{*} = \left(\frac{p}{3 w_{1}}\right)^{1/2} \widetilde{\mathbf{x}}_{2}^{1/2}.$





- Now allow the firm to vary both input levels.
- Since no input level is fixed, there are no fixed costs.

Both x₁ and x₂ are variable.
Think of the firm as choosing the production plan that maximizes profits for a given value of x₂, and then varying x₂ to find the largest possible profit level.

The equation of a long-run iso-profit line is $y = \frac{W_1}{p} x_1 + \frac{\Pi + W_2 x_2}{p}$

so an increase in x₂ causes -- no change to the slope, and -- an increase in the vertical intercept.















 ◆ Profit will increase as x₂ increases so long as the marginal profit of input 2
 p × M P₂ - w ₂ > 0.

The profit-maximizing level of input 2 therefore satisfies

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 $\mathbf{p} \times \mathbf{M} \mathbf{P}_2 - \mathbf{w}_2 = \mathbf{0}.$

Profit will increase as x₂ increases so long as the marginal profit of input 2
 p × M P₂ - w₂ > 0.

The profit-maximizing level of input 2 therefore satisfies

 $\mathbf{p} \times \mathbf{M} \mathbf{P}_2 - \mathbf{w}_2 = \mathbf{0}.$

And p × M P₁ - w₁ = 0 is satisfied in any short-run, so ...

The input levels of the long-run profit-maximizing plan satisfy

 $p \times MP_1 - w_1 = 0$ and $p \times MP_2 - w_2 = 0$.

That is, marginal revenue equals marginal cost for all inputs.



The Cobb-Douglas example: When y = $x_1^{1/3} \tilde{x}_2^{1/3}$ then the firm's short-run demand for its variable input 1 is



$$Long-Run Profit-Maximization
\Pi = py' - w_1x_1' - w_2\tilde{x}_2
= p\left(\frac{p}{3w_1}\right)^{1/2} \tilde{x}_2^{1/2} - w_1\left(\frac{p}{3w_1}\right)^{3/2} \tilde{x}_2^{1/2} - w_2\tilde{x}_2$$

Long-Run Profit-Maximization $\Pi = py^* - w_1x_1^* - w_2\tilde{x}_2$







Long-Run Profit-Maximization

$$\Pi = \left(\frac{4 p^{3}}{27 w_{1}}\right)^{1/2} \tilde{x}_{2}^{1/2} - w_{2} \tilde{x}_{2}.$$

What is the long-run profit-maximizing level of input 2? Solve

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27 w w

$$0 = \frac{\partial \Pi}{\partial \tilde{\mathbf{x}}_2} = \frac{1}{2} \left(\frac{4 p^3}{27 w_1} \right)^{1/2} \tilde{\mathbf{x}}_2^{-1/2} - w_2$$

 $\widetilde{\mathbf{X}}_2 = \mathbf{X}_2$

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to get

What is the long-run profit-maximizing input 1 level? Substitute



What is the long-run profit-maximizing input 1 level? Substitute



What is the long-run profit-maximizing output level? Substitute



What is the long-run profit-maximizing output level? Substitute



Long-Run Profit-Maximization So given the prices p, w_1 and w_2 , and the production function $y = x_1^{1/3} x_2^{1/3}$

the long-run profit-maximizing production plan is



Returns-to-Scale and Profit-Maximization

 If a competitive firm's technology exhibits decreasing returns-to-scale then the firm has a single long-run profit-maximizing production plan.


Returns-to-Scale and Profit-Maximization

 If a competitive firm's technology exhibits exhibits increasing returnsto-scale then the firm does not have a profit-maximizing plan.



Returns-to-Scale and Profit-Maximization

So an increasing returns-to-scale technology is inconsistent with firms being perfectly competitive.

Returns-to-Scale and Profit-Maximization

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What if the competitive firm's technology exhibits constant returns-to-scale?



Returns-to Scale and Profit-Maximization

So if any production plan earns a positive profit, the firm can double up all inputs to produce twice the original output and earn twice the original profit.



Returns-to Scale and Profit-Maximization

- Therefore, when a firm's technology exhibits constant returns-to-scale, earning a positive economic profit is inconsistent with firms being perfectly competitive.
- Hence constant returns-to-scale requires that competitive firms earn economic profits of zero.



- Consider a competitive firm with a technology that exhibits decreasing returns-to-scale.
- For a variety of output and input prices we observe the firm's choices of production plans.
- What can we learn from our observations?

If a production plan (x',y') is chosen at prices (w',p') we deduce that the plan (x',y') is revealed to be profitmaximizing for the prices (w',p').

















Revealed Profitability (x''',y''') is chosen at prices (w''',p''') so $y \uparrow (x''',y''')$ maximizes profit at these prices. Slope (x'', y'') would provide higher v '' profit but it is not chosen because it is not feasible so v ′′′ the technology set lies under the iso-profit line.







Observing more choices of production plans by the firm in response to different prices for its input and its output gives more information on the location of its technology set.









What else can be learned from the firm's choices of profit-maximizing production plans?

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Revealed Profitability $p'y' - w'x' \ge p'y'' - w'x''$ and $p''y'' - w''x'' \ge p''y'' - w''x'$ so $p'y' - w'x' \ge p'y'' - w'x''$ and $-p''y' + w''x' \ge -p''y'' + w''x''.$

Adding gives

$$(p' - p'')y' - (w' - w'')x' \ge$$

Revealed Profitability

$$(p' - p'')y' - (w' - w'')x' \ge$$

 $(p' - p'')y'' - (w' - w'')x''$
so
 $p' - p'')(y' - y'') \ge (w' - w'')(x' - x'')$
That is,
 $\Delta p \Delta y \ge \Delta w \Delta x$
is a necessary implication of profit-
maximization.

Revealed Profitability $\Delta \mathbf{p} \Delta \mathbf{y} \geq \Delta \mathbf{w} \Delta \mathbf{x}$ is a necessary implication of profitmaximization. Suppose the input price does not change. Then $\Delta w = 0$ and profit-maximization implies $\Delta p \Delta y \ge 0$; *i.e.*, a competitive firm's output supply curve cannot slope downward.

Revealed Profitability $\Delta \mathbf{p} \Delta \mathbf{y} \geq \Delta \mathbf{w} \Delta \mathbf{x}$ is a necessary implication of profitmaximization. Suppose the output price does not change. Then $\Delta p = 0$ and profit-maximization implies () $\geq \Lambda \otimes \Lambda \times ; i.e., a competitive$ firm's input demand curve cannot slope upward.