# INTERMEDIATE 



## How Should a Monopoly Price?

-So far a monopoly has been thought of as a firm which has to sell its product at the same price to every customer. This is uniform pricing.
-Can price-discrimination earn a monopoly higher profits?


## Types of Price Discrimination

-1st-degree: Each output unit is sold at a different price. Prices may differ across buyers.

- 2nd-degree: The price paid by a buyer can vary with the quantity demanded by the buyer. But all customers face the same price schedule. E.g., bulk-buying discounts.


## Types of Price Discrimination

-3rd-degree: Price paid by buyers in a given group is the same for all units purchased. But price may differ across buyer groups. E.g., senior citizen and student discounts vs. no discounts for middle-aged persons.

## First-degree Price Discrimination

- Each output unit is sold at a different price. Price may differ across buyers.
- It requires that the monopolist can discover the buyer with the highest valuation of its product, the buyer with the next highest valuation, and so on.



## First-degree Price Discrimination

 \$/output unit

## First-degree Price Discrimination

 \$/output unit

## First-degree Price Discrimination

 \$/output unit

## First-degree Price Discrimination

 \$/output unit The gains to the monopolist on these trades are: $p\left(y^{\prime}\right)-M C\left(y^{\prime}\right), p\left(y^{\prime \prime}\right)-M C\left(y^{\prime \prime}\right)$ and zero.

## First-degree Price Discrimination

## \$/output unit

 So the sum of the gains to the monopolist on all trades is the maximum possible total gains-to-trade.
## First-degree Price Discrimination

 \$/output unit The monopolist gets the maximum possible gains from trade.First-degree price discrimination

## First-degree Price Discrimination

- First-degree price discrimination gives a monopolist all of the possible gains-to-trade, leaves the buyers with zero surplus, and supplies the efficient amount of output.



## Third-degree Price Discrimination

- Price paid by buyers in a given group is the same for all units purchased. But price may differ across buyer groups.



## Third-degree Price Discrimination

- A monopolist manipulates market price by altering the quantity of product supplied to that market.
- So the question "What discriminatory prices will the monopolist set, one for each group?" is really the question "How many units of product will the monopolist supply to each group?"


## Third-degree Price Discrimination

- Two markets, 1 and 2.
$-\mathrm{y}_{1}$ is the quantity supplied to market 1. Market 1's inverse demand function is $p_{1}\left(y_{1}\right)$.
$\bullet y_{2}$ is the quantity supplied to market 2. Market 2's inverse demand function is $p_{2}\left(y_{2}\right)$.


## Third-degree Price Discrimination

$\bullet$ For given supply levels $y_{1}$ and $y_{2}$ the firm's profit is
$\Pi\left(y_{1}, y_{2}\right)=p_{1}\left(y_{1}\right) y_{1}+p_{2}\left(y_{2}\right) y_{2}-c\left(y_{1}+y_{2}\right)$.
$\bullet$ What values of $y_{1}$ and $y_{2}$ maximize profit?

$$
\begin{gathered}
\text { Third-degree Price } \\
\text { Discrimination } \\
\text { D } \left.y_{1}, y_{2}\right)=p_{1}\left(y_{1}\right) y_{1}+p_{2}\left(y_{2}\right) y_{2}-c\left(y_{1}+y_{2}\right) .
\end{gathered}
$$

The profit-maximization conditions are

$$
\begin{aligned}
\frac{\partial \Pi}{\partial y_{1}} & =\frac{\partial}{\partial y_{1}}\left(p_{1}\left(y_{1}\right) y_{1}\right)-\frac{\partial c\left(y_{1}+y_{2}\right)}{\partial\left(y_{1}+y_{2}\right)} \times \frac{\partial\left(y_{1}+y_{2}\right)}{\partial y_{1}} \\
& =0
\end{aligned}
$$

## Third-degree Price Discrimination

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$$
=0
$$

$\frac{\partial \Pi}{\partial y_{2}}=\frac{\partial}{\partial y_{2}}\left(p_{2}\left(y_{2}\right) y_{2}\right)-\frac{\partial c\left(y_{1}+y_{2}\right)}{\partial\left(y_{1}+y_{2}\right)} \times \frac{\partial\left(y_{1}+y_{2}\right)}{\partial y_{2}}$
$=0$

> Third-degree Price $\frac{\partial\left(y_{1}+y_{2}\right)}{\partial y_{1}}=1$ and $\frac{\left.\text { anination }_{1}+y_{2}\right)}{\partial y_{2}}=1$ so
the profit-maximization conditions are

$$
\begin{aligned}
\frac{\partial}{\partial y_{1}}\left(p_{1}\left(y_{1}\right) y_{1}\right) & =\frac{\partial c\left(y_{1}+y_{2}\right)}{\partial\left(y_{1}+y_{2}\right)} \\
\text { and } \frac{\partial}{\partial y_{2}}\left(p_{2}\left(y_{2}\right) y_{2}\right) & =\frac{\partial c\left(y_{1}+y_{2}\right)}{\partial\left(y_{1}+y_{2}\right)} .
\end{aligned}
$$

$$
\begin{gathered}
\text { Third-degree Price } \\
\text { Disgrimination } \\
\frac{\partial}{\partial y_{1}}\left(p_{1}\left(y_{1}\right) y_{1}\right)=\frac{\partial c\left(y_{1}+y_{2}\right)}{\partial y_{2}}\left(p_{2}\left(y_{2}\right) y_{2}\right)=\frac{d\left(y_{1}+y_{2}\right)}{\partial(1)}
\end{gathered}
$$

> Third-degree Price Disgrimination $\left.y_{1}\right)=\frac{\partial c\left(y_{1}+y_{2}\right)}{\partial y_{2}}\left(p_{2}\left(y_{2}\right) y_{2}\right)=\frac{\partial\left(y_{1}+y_{2}\right)}{\partial\left(y_{1}\right)}$
$\operatorname{MR}_{1}\left(\mathrm{y}_{1}\right)=\mathrm{MR}_{2}\left(\mathrm{y}_{2}\right)$ says that the allocation $y_{1}, y_{2}$ maximizes the revenue from selling $y_{1}+y_{2}$ output units.
E.g., if $\mathrm{MR}_{1}\left(\mathrm{y}_{1}\right)>\mathrm{MR}_{2}\left(\mathrm{y}_{2}\right)$ then an output unit should be moved from market 2 to market 1 to increase total revenue.

$$
\begin{gathered}
\text { Third-degree Price } \\
\text { Disgrimination } \\
\frac{\partial}{\partial y_{1}}\left(p_{1}\left(y_{1}\right) y_{1}\right)=\frac{\partial c\left(y_{1}+y_{2}\right)}{\partial y_{2}}\left(p_{2}\left(y_{2}\right) y_{2}\right)=\frac{\partial\left(y_{1}+y_{2}\right)}{\partial y_{2}}
\end{gathered}
$$

The marginal revenue common to both markets equals the marginal production cost if profit is to be maximized.




## Third-degree Price Discrimination

- In which market will the monopolist cause the higher price?



## Third-degree Price Discrimination

- In which market will the monopolist cause the higher price?
- Recall that ${ }_{1}\left(y_{1}\right)=p_{1}\left(y_{1}\right)\left[1+\frac{1}{\varepsilon_{1}}\right]$
and

$$
M R_{2}\left(y_{2}\right)=p_{2}\left(y_{2}\right)\left[1+\frac{1}{\varepsilon_{2}}\right]
$$

## Third-degree Price Discrimination

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and

$$
M R_{2}\left(y_{2}\right)=p_{2}\left(y_{2}\right)\left[1+\frac{1}{\varepsilon_{2}}\right] .
$$

- But, ${ }^{M R_{1}\left(y_{1}^{*}\right)}=M R_{2}\left(y_{2}^{*}\right)=M C\left(y_{1}^{*}+y_{2}^{*}\right)$

So

$$
\begin{gathered}
\text { Third-degree Price } \\
p_{1}\left(y_{1}^{*}\right)\left[\begin{array}{l}
\text { Pisarimination } \\
\varepsilon_{1}
\end{array}\right]=p_{2}\left(y_{2}\right)\left[1+\frac{1}{\varepsilon_{2}}\right] .
\end{gathered}
$$

## Third-degree Price

$$
p_{1}\left(y_{1}^{*}\right)\left[1+\frac{\text { Disarimination }}{\varepsilon_{1}}\right]=p_{2}\left(y_{2}\right)\left[1+\frac{1}{\varepsilon_{2}}\right]
$$

Therefore, $p_{1}\left(y_{1}^{*}\right)>p_{2}\left(y_{2}^{*}\right)$ if and only if

$$
1+\frac{1}{\varepsilon_{1}}<1+\frac{1}{\varepsilon_{2}}
$$

## Third-degree Price

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$$

## Third-degree Price

So

$$
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$$

The monopolist sets the higher price in the market where demand is least own-price elastic.

## Two-Part Tariffs

- A two-part tariff is a lump-sum fee, $p_{1}$, plus a price $p_{2}$ for each unit of product purchased.
- Thus the cost of buying $x$ units of product is

$$
p_{1}+p_{2} x
$$



## Two-Part Tariffs

- Should a monopolist prefer a twopart tariff to uniform pricing, or to any of the price-discrimination schemes discussed so far?
- If so, how should the monopolist design its two-part tariff?


## Two-Part Tariffs

## $p_{1}+p_{2} x$ <br> $\bullet Q$ : What is the largest that $p_{1}$ can be?

## Two-Part Tariffs

$$
p_{1}+p_{2} x
$$

$\bullet Q$ : What is the largest that $p_{1}$ can be?
A: $p_{1}$ is the "market entrance fee" so the largest it can be is the surplus the buyer gains from entering the market.
-Set $\mathrm{p}_{1}=$ CS and now ask what should be $p_{2}$ ?

## Two-Part Tariffs



## Two-Part Tariffs

## \$/output unit

Should the monopolist set $p_{2}$ above MC?
$\mathrm{p}_{1}=\mathbf{C S}$.
$p_{2}=p\left(y^{\prime}\right) \quad C S$

## Two-Part Tariffs



## Two-Part Tariffs



## Two-Part Tariffs

\$/output unit


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## Two-Part Tariffs

\$/output unit
Should the monopolist set $p_{2}=M C ?$
$p_{1}=C S$.
PS is profit from sales.

$\left.p p^{\prime \prime}\right)$



## Two-Part Tariffs

\$/output unit
Should the monopolist set $p_{2}=M C$ ?
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## Two-Part Tariffs

\$/output unit
Should the monopolist set $p_{2}=M C$ ?
$p_{1}=C S$.
PS is profit from sales.
$p_{2}=p\left(y^{\prime \prime}\right)^{\text {CS }}$


## Two-Part Tariffs

\$/output unit
Should the monopolist set $p_{2}=M C$ ?
$p_{1}=C S$.
PS is profit from sales.

## CS <br> 

Additional profit from setting $p_{2}=M C$.

## Two-Part Tariffs

- The monopolist maximizes its profit when using a two-part tariff by setting its per unit price $p_{2}$ at marginal cost and setting its lumpsum fee $p_{1}$ equal to Consumers' Surplus.



## Two-Part Tariffs

- A profit-maximizing two-part tariff gives an efficient market outcome in which the monopolist obtains as profit the total of all gains-to-trade.



## Differentiating Products

- In many markets the commodities traded are very close, but not perfect, substitutes.
- E.g., the markets for T-shirts, watches, cars, and cookies.
- Each individual supplier thus has some slight "monopoly power."
-What does an equilibrium look like for such a market?


## Differentiating Products

$\bullet$ Free entry $\Rightarrow$ zero profits for each seller.


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- Profit-maximization $\Rightarrow$ MR = MC for each seller.



## Differentiating Products

$\bullet$ Free entry $\Rightarrow$ zero profits for each seller.

- Profit-maximization $\Rightarrow$ MR = MC for each seller.
-Less than perfect substitution between commodities $\Rightarrow$ slight downward slope for the demand curve for each commodity.







## Differentiating Products

- Such markets are monopolistically competitive.
- Are these markets efficient?
- No, because for each commodity the equilibrium price $p\left(y^{*}\right)>\operatorname{MC}\left(y^{*}\right)$.





## Differentiating Products

- Each seller supplies less than the efficient quantity of its product.
- Also, each seller supplies less than the quantity that minimizes its average cost and so, in this sense, each supplier has "excess capacity."



## Differentiating Products by Location

- Think a region in which consumers are uniformly located along a line.
- Each consumer prefers to travel a shorter distance to a seller.
- There are $\mathrm{n} \geq 1$ sellers.
- Where would we expect these sellers to choose their locations?


## Differentiating Products by Location



- If $\mathbf{n}=1$ (monopoly) then the seller maximizes its profit at $\mathrm{x}=$ ? ?


## Differentiating Products by Loogtion



- If $\mathbf{n}=1$ (monopoly) then the seller maximizes its profit at $x=1 / 2$ and minimizes the consumers' travel cost.


## Differentiating Products by Loogation

0


- If $\mathbf{n}=\mathbf{2}$ (duopoly) then the equilibrium locations of the sellers, $A$ and $B$, are $x_{A}=$ ?? and $x_{B}=$ ??


## Differentiating Products by

## Loag̨tion

## $\longrightarrow x$

- If $\mathbf{n}=\mathbf{2}$ (duopoly) then the equilibrium locations of the sellers, $A$ and $B$, are $x_{A}=$ ?? and $x_{B}=$ ??
$\rightarrow$ How about $x_{A}=0$ and $x_{B}=1$; i.e. the sellers separate themselves as much as is possible?


## Differentiating Products by



## Loag̨tion

- If $x_{A}=0$ and $x_{B}=1$ then $A$ sells to all consumers in $[0,1 / 2$ ) and $B$ sells to all consumers in $(1 / 2,1]$.
- Given B's location at $x_{B}=1$, can A increase its profit?

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$\bullet$ If $x_{A}=0$ and $x_{B}=1$ then $A$ sells to all consumers in $[0,1 / 2$ ) and $B$ sells to all consumers in $(1 / 2,1]$.
- Given B's location at $x_{B}=1$, can A increase its profit? What if A moves to $x$ '? Then A sells to all customers in $\left[0,1,2+1 / 2 x^{\prime}\right)$ and increases its profit.


## Differentiating Products by

Loogation
B

- Given $x_{A}=x^{\prime}$, can $B$ improve its profit by moving from $x_{B}=1$ ?



## Differentiating Products by



- Given $x_{A}=x^{\prime}$, can $B$ improve its profit by moving from $x_{B}=1$ ? What if $B$ moves to $x_{B}=x$ '?


## Differentiating Products by



Loogtion

- Given $x_{A}=x^{\prime}$, can $B$ improve its profit by moving from $x_{B}=1$ ? What if $B$ moves to $x_{B}=x$ '? Then $B$ sells to all customers in (( $\left.\left.x^{\prime}+x^{\prime \prime}\right) / 2,1\right]$ and increases its profit.
-So what is the NE?


## Differentiating Products by Loogation



A\&B
$\bullet$ Given $x_{A}=x^{\prime}$, can $B$ improve its profit by moving from $x_{B}=1$ ? What if $B$ moves to $x_{B}=x^{\prime \prime}$ ? Then $B$ sells to all customers in (( $\left.\left.x^{\prime}+x^{\prime \prime}\right) / 2,1\right]$ and increases its profit.
$\bullet$ So what is the $N E ? x_{A}=x_{B}=1 / 2$.

## Differentiating Products by Loogation



## $\longrightarrow x$

$\rightarrow$ The only NE is $x_{A}=x_{B}=1 / 2$.
$\bullet$ Is the NE efficient?


## Differentiating Products by Loogation


$\rightarrow$ The only NE is $x_{A}=x_{B}=1 / 2$.

- Is the NE efficient? No.
$\bullet$ What is the efficient location of A and $B$ ?


## Differentiating Products by

## $1 / 4 \quad$ Looątion $3 / 4$


$\rightarrow$ The only NE is $x_{A}=x_{B}=1 / 2$.

- Is the NE efficient? No.
$\bullet$ What is the efficient location of $A$ and $B$ ? $x_{A}=1 / 4$ and $x_{B}=3 / 4$ since this minimizes the consumers' travel costs.


## Differentiating Products by Loogation

## 0 <br> $\longmapsto \mathbf{X}$

-What if $\mathrm{n}=3$; sellers $A, B$ and $C$ ?

## Differentiating Products by Loogation

## 0 <br> $\longmapsto \mathbf{X}$

-What if $\mathrm{n}=3$; sellers $\mathrm{A}, \mathrm{B}$ and C ?

- Then there is no NE at all! Why?



## Differentiating Products by Looaztion

## $\longmapsto \mathbf{X}$

$\bullet$ What if $n=3$; sellers $A, B$ and $C$ ?

- Then there is no NE at all! Why?
- The possibilities are:
- (i) All 3 sellers locate at the same point.
- (ii) 2 sellers locate at the same point.
- (iii) Every seller locates at a different point.


## Differentiating Products by

 Aooatien

## $\longmapsto X$

- (iii) Every seller locates at a different point.
-Cannot be a NE since, as for $\mathbf{n}=2$, the two outside sellers get higher profits by moving closer to the middle seller.


## Differentiating Products by



- (i) All 3 sellers locate at the same point.
-Cannot be an NE since it pays one of the sellers to move just a little bit left or right of the other two to get all of the market on that side, instead of having to share those customers.


## Differentiating Products by


$\longmapsto X \quad C$ gets almost $1 / 2$ of the market

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-Cannot be an NE since it pays one of the sellers to move just a little bit left or right of the other two to get all of the market on that side, instead of having to share those customers.


## Differentiating Products by


$\longmapsto X \quad$ A gets about $1 / 4$ of the market
2 sellers locate at the same point.
-Cannot be an NE since it pays one of the two sellers to move just a little away from the other.

## Differentiating Products by



## $\longmapsto \mathbf{X}$

A gets almost $\mathbf{1 / 2}$ of the market
2 sellers locate at the same point.
-Cannot be an NE since it pays one of the two sellers to move just a little away from the other.

## Differentiating Products by



## $\longmapsto \mathbf{X}$

A gets almost $\mathbf{1 / 2}$ of the market
2 sellers locate at the same point.
-Cannot be an NE since it pays one of the two sellers to move just a little away from the other.

## Differentiating Products by

 Location$\bullet$ If $\mathbf{n}=\mathbf{3}$ the possibilities are:
-(i) Alll 3 -sellers locate at the same point.

- (ii) 2 sellers locate at the same point.
- (iii) Cvery-sellerlocates-ata different point.
$\bullet$ There is no NE for $n=3$.


## Differentiating Products by

## Location

- If $\mathbf{n}=\mathbf{3}$ the possibilities are:
-(i) All 3 -sellers locate at the same point.
-(ii) 2 sellers looate at the same point.
-(iii) Cwery-sellerloeates-at-a different point.
$\bullet$ There is no NE for $\mathrm{n}=3$.
- However, this is a NE for every $n \geq 4$.

