The Groves–Ledyard mechanism:
An experimental study of institutional design

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Abstract

The Groves–Ledyard mechanism theoretically can solve the 'free-rider' problem in
public good provision in certain environments. Two questions are of overriding
importance in implementing the mechanism. The first is related to the actual
performance of the mechanism in general. The second is the choice of a 'punishment
parameter', γ, which is the only parameter that is available for those who may want
to actually use the mechanism. Thus the determination of the role of this variable on
mechanism performance is fundamental for any advances along the lines of actual
implementation. In studying the Groves–Ledyard mechanism, we show that the
punishment parameter, γ, plays a crucial role in the performance of the mechanism.
By using $\gamma = 1$ and 100, we show that under the higher punishment parameter, the
Groves–Ledyard equilibrium is chosen much more frequently; a higher level of the
public good is provided and efficiency is higher. By examining two behavioral
models, we show that a higher γ leads to an increase in the probability of an
individual choosing a best response predicted by the model. The parameter γ alone
explains nearly 70% of the data in both the Cournot and the Carlson–Auster
behavioral model. We also found that convergence to Cournot behavior is faster and
more stable under a high γ than under a low γ.

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1. Introduction

A widely recognized problem for economics and political science has been to explore institutional designs that might facilitate cooperation in an environment with public goods. For years a fundamental belief was that the achievement of a Pareto-optimal allocation of resources via decentralized mechanisms in the presence of public goods is incompatible with individual incentives (Samuelson, 1954). Theoretical and experimental work on the voluntary contribution mechanism indicates an underprovision of public goods, as a result of free-riding.

Groves and Ledyard (1977) proposed a decentralized mechanism in a general equilibrium model, in which through a government allocation-taxation scheme the behavioral equilibria (Nash) are Pareto optimal. That is, given the allocation-taxation scheme, "consumers find it in their self-interest to reveal their true preferences for the public goods" and the public goods are produced at an optimal level. Therefore the mechanism is incentive compatible, and it balances the budget both on and off the equilibrium path.

So far, the Groves–Ledyard (GL) mechanism has only been a paper process that exists only on the pages of a journal, but its importance should not be underestimated. It might be possible to take the idea of a process discovered by Groves and Ledyard, refine it, make it operational and put it to use as an actual political/economic process that solves naturally occurring problems. When, and if, that occurs, the institutional design problem would have evolved to its next logical step. That possibility motivates the research reported in this paper.

The research strategy is to observe the behavior of the GL process in the context of the simple situations that can be created in a laboratory and assess its performance relative to what it was created to do and relative to the theory upon which its creation rests.

Two questions are of overriding importance if we want to implement the mechanism. The first is related to the actual performance of the GL mechanism in general. The second is the role of a 'punishment parameter', γ, which is the only parameter that is available for those who may want to actually use the GL mechanism. The GL mechanism is actually a family of mechanisms, depending on the choice of this punishment parameter.[1] For practical implementation of the mechanism, information is needed about the performance of the system in response to an increase or decrease of this punishment parameter. Theory does not address this question except to suggest that if this particular type of punishment is 'too high' the process will

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[1] Muench and Walker (1979, 1983) discussed some effects of parameter choices.

not respond at all. Such testing of the sensitivity of the different GL mechanisms under different parameters has not been performed.

In choosing the actual experimental environments, two drawbacks of the mechanism must be considered: it does not satisfy voluntary participation, i.e. an individual can be worse off as a result of participating in the process; in a general environment multiple equilibria[2] can exist. The way we deal with the first problem is to give every subject an initial endowment. For the second problem, since the focus in this paper is to assess the mechanism relative to the theory and the role of the punishment parameter, a quasilinear environment is used in which a unique Nash equilibrium exists. The equilibrium selection problem in a general environment is left for future research.

The paper is organized as follows. Section 2 reviews the theoretical features of the GL mechanism. Section 3 reviews previous experimental work motivated by the GL mechanism, with comparisons of experimental designs. Section 4 contains a description of the experimental design – the environment, the process and the procedures. Section 5 gives a descriptive summary of data and some preliminary results, and then compares the predictions of different behavioral models to the data. In this section a logit analysis is used to identify and discuss the impact of the different parameters on the behaviors of the subjects. Section 6 concludes the paper.

2. The Groves–Ledyard mechanism

The GL mechanism allocates each individual's share of the cost of public good provision by

$$C_i(x_i \mid \mu_i, \sigma_i) = \frac{X}{I} \cdot q + \frac{\gamma}{2} \left\{ \frac{I-1}{I} (x_i - \mu_i)^2 - \sigma_i^2 \right\},$$

where $\gamma > 0$ is the punishment parameter, I is the number of people in the economy, x_i is individual i's message, indicating his proposed addition to the total amount of public good provided, and $X = \Sigma_i x_i$ is the total amount of public good. Define $S_i = \Sigma_{j \neq i} x_j$ as the sum of the proposed increments by all other members of the group except i, $\mu_i = S_i / I$ as the mean of others' messages, and $\sigma_i^2 = \Sigma_{h \neq i} (x_h - \mu_i)^2 / (I - 2)$ as the squared standard error of the mean of others' messages. q is the per unit cost of the public good.

Some features of the mechanism are important for understanding and implementing the mechanism. As can be observed from the tax function of the GL mechanism, two parameters, γ and I, define a family of GL

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[2] See Bergstrom et al. (1983).

mechanisms. Variations of the punishment parameter, γ, changes the penalty that is imposed on an individual for deviating from the mean of other players' messages. The other parameter is the size of the economy, I, i.e. the number of individuals in the economy.

In the experiments reported here the influence of size on the properties of the equilibria is not considered, especially as the size of the economy grows towards infinity. Technology is not up to the task. So a fixed size of the economy was chosen, and given the fixed size, the punishment parameter was varied. Accordingly, the effects of the punishment parameter on the performance of the mechanism were assessed.

Preferences are induced on units of the abstract public good by an individually specified value function, $V_i(X)$, which indicates the amount of money an individual will receive if the group choice of the public good is X and if the individual pays nothing for it. At each level of public good decided by the group, an individual's net earning in dollars is $NV_i = V_i(X) - C_i(x_i \mid S_i, \sigma_i)$, where $C_i(x_i \mid S_i, \sigma_i)$ is the amount of tax individual i pays if his proposed addition to the total amount of public good provided is x_i, the sum of the proposed increments by all other members of the group except i is S_i, and the squared standard error of the mean of others' messages is σ_i^2.

Therefore, each individual has a monetary profit, and if one assumes that individuals have a strictly monotone increasing utility of money, then the problem becomes

$$\max_{x_i} U_i[V_i(X) - C_i(x_i \mid S_i, \sigma_i)].$$

Then in equilibrium, individual i would submit a message, x_i^e, such that

$$V_i'(X) = C_i'(x_i^e \mid S_i, \sigma_i).$$

This equation simply says that each individual will report a 'desired quantity' of the public good which equates the marginal private benefit perceived with the marginal private cost perceived given the decisions of others.

The marginal cost of the public good to individual i is

$$C_i'(x_i \mid S_i, \sigma_i) = \frac{q}{I} + \gamma \frac{I-1}{I} (x_i - \mu_i).$$

Therefore, changes in γ will affect an individual's equilibrium message, x_i. The effects of punishment parameters on individuals' behaviors will be developed further later.

The *Lindahl equilibrium* $[X^e, \{V_i(X^e)\}]$ satisfies

$$\sum_{i=1}^{I} V_i'(X^e) = q$$

for the experimental environment. So, the sum of individual marginal values for the public good equals the marginal rate of transformation.

Another important feature in the GL mechanism is that it balances the budget both on and off the equilibrium path, i.e. it guarantees a balanced budget for every $X > 0$, i.e. $\sum_{i=1}^{?} C_i(x_i \mid S_i, \sigma_i) = qX$. This is achieved by the last term in the GL rule, the squared standard error of the mean of others' messages, σ_i^2. Including this term causes additional difficulties in implementation by adding another dimension to the individuals' decision problem. But, it is crucial for keeping a balanced budget.

3. Previous implementation

There have been two groups of experiments with mechanisms motivated by the GL mechanism. First, Smith (1979) did two sets of experiments, using a simplified version of the mechanism, which only balanced the budget in equilibrium, i.e. one needs to know the equilibrium in order to balance the budget. The complete GL mechanism balances the budget both on and off the equilibrium path. In the Smith experiments the punishments parameter was set to one.

Secondly, Harstad and Marrese (hereafter abbreviated to HM) (1981a,b, 1982) had two sets of experiments motivated by the GL mechanism. The first set of experiments did not satisfy a balanced budget condition: they used the Smith parameters, but with a different process – the Seriatim process.[3] Their second set of experiments was a computerized version with a balanced budget both on and off the equilibrium path.

Table 1 summarizes the main differences of the Smith, HM and our experimental designs. The two Smith and the Harstad–Marrese (1981a,b) experiments (1) do not satisfy the balanced budget constraint off the

Table 1
Comparison of three sets of experiments

	Balanced budget	Incentive parameter	No. of subjects	Process
Smith	No	$\gamma = 1$	5	Smith
Smith	No	$\gamma = 1$	8	Smith
Harstad–Marrese (1)	No	$\gamma = 1$	3	Seriatim
Harstad–Marrese (2)	Yes	$\gamma = 0.67$	3	Seriatim
	Yes	$\gamma = 3$	4	Seriatim
Chen–Plott (1)	Yes	$\gamma = 1$	5	Periodic
Chen–Plott (2)	Yes	$\gamma = 100$	5	Periodic

[3] To be discussed later in this section.

equilibrium path, so the mechanism they studied was not the actual GL mechanism. Harstad and Marrese (1982) (2) use the complete version of the mechanism, with different punishment parameters and number of subjects. We argue that changing the punishment parameters and the number of subjects simultaneously, as was done in Harstad and Marrese (1982), does not allow one to study the exact impact of the two parameters; besides, the magnitude of the changes was so small that the effects would be very difficult to discern in a laboratory environment. Indeed, the effect of parameters are not discussed in Harstad and Marrese (1981a,b). Neither experiment addressed the role of the punishment parameters in the performance of the mechanism.

Another important difference between our implementation and the previous attempts resides in the processes used. Both the Smith process and the Seriatim process require unanimity, which might add unwanted complexity to the static GL mechanism. They have the common shortcoming of involving much cheap talk and manipulation. Since the subjects are only paid when agreements are reached, they need not be responsible for each decision they make. From our pilot experiments using the Smith process[4] and from Banks et al. (1988), unanimity was found to decrease the efficiency of the system. Therefore, because of the unanimity feature we discard these two processes and use a completely different process in the experiments reported here.

4. Testbed environment

The testbed environment reflects both technical and theoretical considerations. A major consideration of any field application is that the process of the public goods provision covers the cost of the public good. Thus we want to study only processes that satisfy the balanced budget property under both conditions of 'equilibrium' and 'disequilibrium'. In addition we are interested in the influence of the magnitude of the punishment parameter. These considerations, taken together with the technological problems that they can cause, motivated an experimental design in which the size of the economy is fixed and the punishment parameter is varied. The economic environment, the institutional process and the experimental procedures are discussed in the subsections below.

4.1. The economic environment

In all experiments a simple constant unit cost, q, is used to produce the public good. Preferences are induced on units of the abstract public good by

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[4] Data from the pilot experiments are available from the authors upon request.

an individually specified value function, $V_i(X)$, which indicates the amount of money an individual will receive if the group choice of the public good is X and if the individual pays nothing for it.

The parameters chosen for the experiments involve five individuals, $I = 5$. The constant unit cost of the public good is $q = 5$. The valuation functions are quadratic,

$$V_i(X) = A_i X - B_i X^2 + \alpha_i \,,$$

and the GL cost function, in this specific design is

$$C_i(x_i \mid S_i, \sigma_i) = X + \frac{\gamma}{2} \left[\frac{4}{5}(x_i - \mu_i)^2 - \sigma_i^2 \right] \,.$$

Table 2 lists the parameters of each individual subject's valuation functions and their equilibrium values under both punishment parameters. Note that the subjects have quite diverse tastes for the public good. The marginal valuation function, $V_i'(X)$, are shown in Fig. 1. Subject's 1's marginal valuation for the public good is negative at all levels, i.e. it is a public bad for him. The other four players' marginal valuations are also quite different from each other. At the equilibrium, where $X = 5$, both subjects 4 and 5 have marginal valuations higher than the marginal cost of the public good, while subjects 1 and 2 have marginal valuations below the marginal cost. In a voluntary contribution situation, we would expect subjects 4 and 5 to contribute close to the optimal amount of the public good.

One question to be posed is whether, or how likely it is that individual subjects follow their Lindahl equilibria under different punishment parameters. As shown in Table 2, when $\gamma = 1$ the punishment for deviation from the mean of others is not severe, therefore their Lindahl equilibrium messages vary from each other. When $\gamma = 100$, however, the incentive for converging to the mean of others' messages is so strong that all equilibrium messages are 'squeezed' towards one. The distribution of costs also moves

Table 2
Parameter and Lindahl equilibrium values

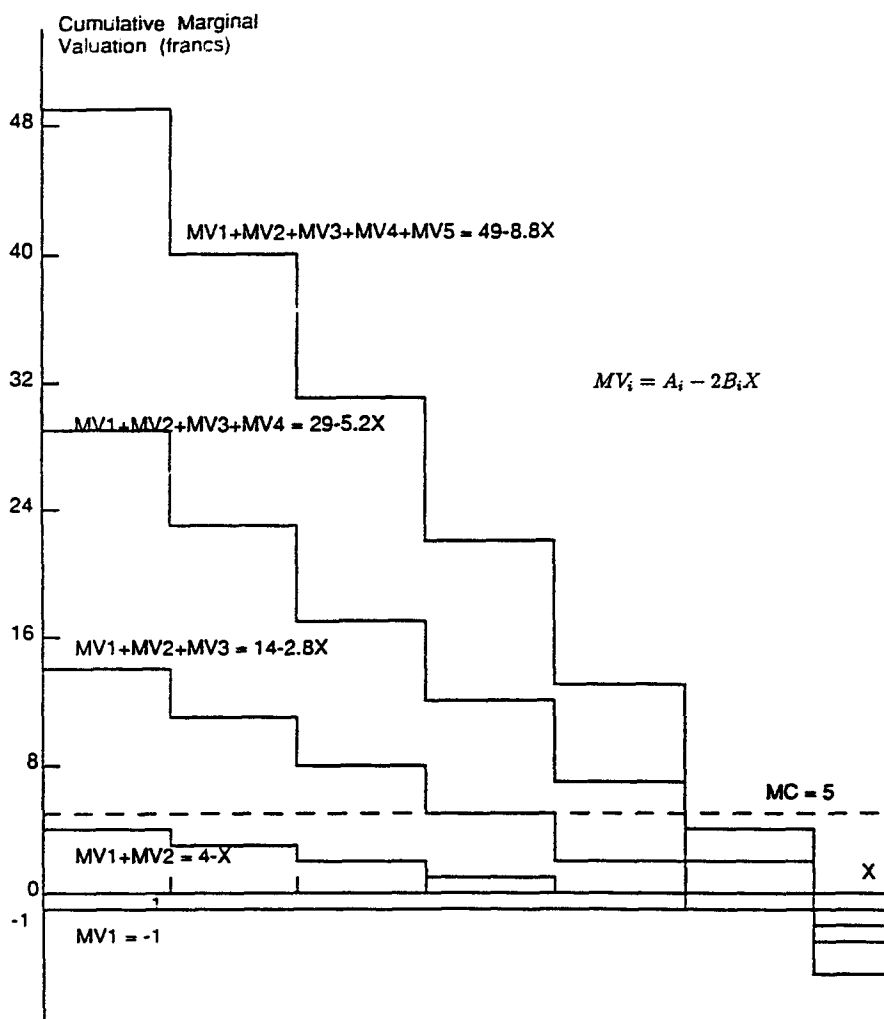| Parameter / Subject ID	A_i	B_i	α_i	x_i^e ($\gamma = 1$)	x_i^e ($\gamma = 100$)
1	−1	0	55	−1	1
2	5	0.5	35	0	1
3	10	0.9	20	1	1
4	20	1.8	0	2	1
5	15	1.2	5	3	1
Σ	49	4.4	115	5	5

Fig. 1. The environment.

toward uniform. In both cases the group optimal quantity of public good is 5.

4.2. The institutional process

Implementation of the mechanism is based on a *Periodic Process*. That is, in each trial each subject i chooses a message x_i, and sends it to the central computer. The computer calculates the total level of public good,

$X = \sum_{i=1}^{I} x_i$, the sum of others' proposals, the variances of others' proposals and each subject's net payoff, and sends the information to the subjects' screens. The subjects are paid each trial for each decision they make. The process repeats for T periods, which are announced in the instructions.

This process differs from those used in other experiments. Here, subjects are paid for each decision. All messages involve commitment and are communicated under condition of incentives. Theoretically, relative to other experiments there is less incentive for cheap talk. Our pilot experiments comparing the Periodic Process with the Smith Process suggest that less cheap talk and manipulative behaviors existed in the Periodic Process, which is also more faithful to the original static mechanism.

4.3. Experimental procedures

Four experiments were conducted using Caltech undergraduates. While most of the subjects had participated in computerized economic experiments before, none had participated in a Groves–Ledyard experiment. Each experiment consisted of two sessions. Each session consisted of 30 periods, with the first five periods being the practice rounds without payment. The practice rounds were used to instruct the subjects about the functions of different keys, how to send in a proposal and how to read and record a result from the screen (see *Computer Instructions* in the appendix). Two experiments started with 30 trials of $\gamma = 1$ design followed by 30 trials of the $\gamma = 100$ design; another two experiments had the reversed order. Each experiment lasted between 1 and 1.5 hours. Table 3 summarizes these four experiments.

At the beginning of each experiment each subject had a set of instructions, a set of payoff tables and record sheet. Because we use the complete version of the GL mechanism, the payoff tables are necessarily three-dimensional.[5] The experimenter read the instructions and taught the subjects how to use the keyboard, how to send messages and how to record results from the computer. After the Computer Instructions, the subjects were

Table 3
Features of experiments

Experiments	Period 1–30 (Session a)	Period 31–60 (Session b)
0219-93	$\gamma = 1$	$\gamma = 100$
0304-93	$\gamma = 100$	$\gamma = 1$
0305-93	$\gamma = 100$	$\gamma = 1$
0401-93	$\gamma = 1$	$\gamma = 100$

[5] See the Appendix for an example of the structure of the payoff tables.

required to finish the Review Questions, which were designed to test subjects' understanding of instructions. Afterwards, the experimenter reviewed the answers to the questions and answered any questions. After this, the subjects read and signed the Financial Agreement, which required them to work in the laboratory in case of negative earning (see the appendix).

When a period of an experiment began, each subject sent his/her proposed addition of the public good through the computer. The central computer calculated the total level of the project (X), the sum of other subjects' proposed additions (S_i), the variance of other subjects' additions (σ_i^2, or o_i as in the instructions), and the net value of the project for each subject (NV_i, or P_i as in the instructions). The information was sent back to the subjects' screens. The subjects then wrote the information in the record sheet. Subjects were strongly encouraged to refer to their payoff tables before and after each decision. Most subjects appeared to study their payoff tables before sending messages and after receiving the feedback. The process was repeated for 25 periods. At the end of an experiment the subjects added their total earnings (in francs) for all 25 periods and converted them to dollar payments. The conversion rate was announced at the beginning of the experiments and was written on the blackboard for their attention.

5. Results

5.1. Descriptive summary of data

The important basic results obtained from the raw data are listed as Result 1 through Result 4. Together these four results provide the first facts about the overall performance of the classic GL mechanism. A more detailed examination of individual behavioral models and the principles that might underlie individual decisions is reserved for Subsections 5.2 and 5.3.

Table 4 contains the aggregate results of the experiments. Each session has two sets of experiments, marked by a and b. Experiment a precedes experiment b. The order of the experiments is a treatment variable. In two sessions (0219-93 and 0401-93) $\gamma = 1$ trials are conducted before the $\gamma = 100$ trials in each session, and each session has 25 trials except for 0305-93b which has 26 trials. The notation f_i is used to denote the frequency that a subject proposes the addition i, and f_i^* is used to denote the equilibrium proposal for the subject(s). Though the aggregate level of public good can range from -10 to 30, only the values actually chosen in the experiments are listed.

Results 1–3 are group level results. Results 1 and 2 tell us that the promise provided by theory, that the GL mechanism can be used to solve

Table 4
Aggregate frequency of choices and efficiency

Incentives	Session	N	f_0	f_1	f_2	f_3	f_4	f_5^*	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	Average level	Efficiency
$\gamma = 1$	0219-93a	25	0.04	0.12	0.16	0.20	0.04	0.12*	0.12	0.08	0.04	0.04	0.00	0.04	0.00	0.00	4.20	0.845
	0304-93b	25	0.04	0.04	0.04	0.16	0.08	0.36*	0.12	0.04	0.08	0.00	0.00	0.00	0.00	0.04	4.88	0.883
	0305-93b	26	0.00	0.00	0.04	0.12	0.27	0.38*	0.11	0.04	0.00	0.00	0.00	0.00	0.04	0.00	4.85	0.942
	0401-93a	25	0.00	0.00	0.00	0.12	0.32	0.24*	0.24	0.04	0.04	0.00	0.00	0.00	0.00	0.00	4.88	0.975
	Average		0.02	0.04	0.06	0.15	0.18	0.28*	0.14	0.05	0.04	0.01	0.00	0.01	0.01	0.01	4.70	0.911
$\gamma = 100$	0219-93b	25	0.00	0.00	0.00	0.04	0.20	0.52*	0.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.96	0.987
	0304-93a	25	0.00	0.00	0.04	0.12	0.24	0.44*	0.04	0.12	0.00	0.00	0.00	0.00	0.00	0.00	4.68	0.967
	0305-93a	25	0.00	0.00	0.04	0.12	0.20	0.28*	0.24	0.12	0.00	0.00	0.00	0.00	0.00	0.00	4.92	0.963
	0401-93b	25	0.00	0.00	0.00	0.00	0.16	0.64*	0.16	0.04	0.00	0.00	0.00	0.00	0.00	0.00	5.08	0.989
	Average		0.00	0.00	0.02	0.07	0.20	0.47*	0.17	0.07	0.00	0.00	0.00	0.00	0.00	0.00	4.91	0.977

the public goods problems in this type of environment, is true in fact. The variable γ is important because when it is increased the efficiency of the process increases and the aggregate level of the public good is closer to the optimal. Result 3 further confirms that γ has a role to play in the performance of the mechanism.

Result 1. The average group efficiency increases when γ increases.

Support. The last column of Table 4 shows the efficiencies of every experiment and the average efficiencies of the two sets of experiments. The average efficiency is 91.1% when $\gamma = 1$, and 97.7% when $\gamma = 100$.   □

Result 2. The average level of public good provided increases when γ increases and the efficient level is chosen more frequently under higher γ.

Support. The level of public good provided for each experiment and the mean level are presented in Table 4. The average level of public good provided is 4.70 when $\gamma = 1$, and 4.91 when $\gamma = 100$. The group efficient level, $X = 5$, is chosen 28% of the time when $\gamma = 1$ and 47% of the time when $\gamma = 100$.   □

Although the group efficiency level (the GL equilibrium) is chosen significantly less frequently when γ is low, the overall efficiency is still above 90%. This is because the actual group payoff, $\Sigma_i (V^i(X) - C_i) = \Sigma_i V_i(X) - qX$, aggregates out γ's incentive effect on the individual's cost share. The function of the punishment parameter is to induce the group efficient level of the public good to be chosen more frequently. So the efficiency is slightly higher when $\gamma = 100$, but in either case it is above 90% on average.

The aggregate data can have a tendency to hide the potential importance of γ. First, the cost of adjustment as created by γ is a type of zero-sum game. The cost paid by one individual is a benefit received by another. Thus the cost of adjustment cannot appear in the aggregate data. In addition, because the efficiency levels of the mechanism are so high, even under low levels of γ there would seem to be little room for the variable to have an effect. The next result signals that significant effects of γ exist in the data and thus the result serves as a basis for a more detailed analysis of individual behavior.

Result 3. The increase of γ reduces dispersion of outcomes across experiments.

Table 5
Subject 1's frequency of choices

Incentives	Session	N	f_{-2}	f_{-1}	f_0	f_1	f_2	f_3	f_4	f_5	f_6
	0129-93a	25	0.04	0.56*	0.04	0.16	0.20	0.00	0.00	0.00	0.00
	0304-93b	25	0.96	0.00*	0.00	0.00	0.00	0.00	0.00	0.00	0.04
$\gamma = 1$	0305-93b	26	0.58	0.23*	0.11	0.04	0.00	0.00	0.00	0.00	0.04
	0401-93a	25	0.12	0.76*	0.12	0.00	0.00	0.00	0.00	0.00	0.00
	Average		0.42	0.39*	0.07	0.05	0.05	0.00	0.00	0.00	0.02
	0219-93b	25	0.00	0.00	0.04	0.92*	0.04	0.00	0.00	0.00	0.00
	0304-93a	25	0.00	0.04	0.20	0.56*	0.20	0.00	0.00	0.00	0.00
$\gamma = 100$	0305-93a	25	0.00	0.04	0.24	0.56*	0.16	0.00	0.00	0.00	0.00
	0401-93b	25	0.00	0.00	0.12	0.80*	0.08	0.00	0.00	0.00	0.00
	Average		0.00	0.02	0.15	0.71*	0.14	0.00	0.00	0.00	0.00

Support. Table 4 shows that when $\gamma = 1$, 12 out of 26 nonoptimal levels of public good are chosen with positive frequencies; while only the 5 alternatives closest to the group efficient level are chosen in the $\gamma = 100$ case. □

Result 3 indicates that the role of γ can be detected at the aggregate level of analysis. However, even though the aggregate may be of interest, the details of individual decisions are more instructive. The nature of Result 4 can best be introduced by a detailed study of the patterns of individual behavior.

Tables 5–9 present the frequencies of choosing each alternative by each subject.[6] The subjects are numbered so that an individual indexed k in one experiment has exactly the same induced preferences as the individual indexed k in the other experiments. A brief review of the individual

Table 6
Subject 2's frequency of choices

Incentives	Session	N	f_{-2}	f_{-1}	f_0	f_1	f_2	f_3	f_4	f_5	f_6
	0129-93a	25	0.08	0.32	0.48*	0.12	0.00	0.00	0.00	0.00	0.00
	0304-93b	25	0.28	0.24	0.36*	0.12	0.00	0.00	0.00	0.00	0.00
$\gamma = 1$	0305-93b	26	0.00	0.08	0.31*	0.61	0.00	0.00	0.00	0.00	0.00
	0401-93a	25	0.00	0.04	0.84*	0.12	0.00	0.00	0.00	0.00	0.00
	Average		0.09	0.17	0.50*	0.24	0.00	0.00	0.00	0.00	0.00
	0219-93b	25	0.00	0.00	0.12	0.80*	0.08	0.00	0.00	0.00	0.00
	0304-93a	25	0.00	0.04	0.28	0.56*	0.08	0.04	0.00	0.00	0.00
$\gamma = 100$	0305-93a	25	0.00	0.00	0.20	0.68*	0.12	0.00	0.00	0.00	0.00
	0401-93b	25	0.00	0.00	0.04	0.96*	0.00	0.00	0.00	0.00	0.00
	Average		0.00	0.01	0.16	0.75*	0.07	0.01	0.00	0.00	0.00

[6] The raw data and computer programs are available from the authors by request.

Table 7
Subject 3's frequency of choices

Incentives	Session	N	f_{-2}	f_{-1}	f_0	f_1	f_2	f_3	f_4	f_5	f_6
	0129-93a	25	0.00	0.04	0.40	0.24*	0.20	0.12	0.00	0.00	0.00
	0304-93b	25	0.00	0.00	0.12	0.72*	0.16	0.00	0.00	0.00	0.00
$\gamma = 1$	0305-93b	26	0.00	0.00	0.12	0.27*	0.38	0.23	0.00	0.00	0.00
	0401-93a	25	0.00	0.00	0.60	0.40*	0.00	0.00	0.00	0.00	0.00
	Average		0.00	0.01	0.31	0.41*	0.18	0.09	0.00	0.00	0.00
	0219-93b	25	0.00	0.00	0.04	0.96*	0.00	0.00	0.00	0.00	0.00
	0304-93a	25	0.00	0.00	0.08	0.88*	0.04	0.00	0.00	0.00	0.00
$\gamma = 100$	0305-93a	25	0.00	0.00	0.04	0.80*	0.16	0.00	0.00	0.00	0.00
	0401-93b	25	0.00	0.00	0.00	1.00*	0.00	0.00	0.00	0.00	0.00
	Average		0.00	0.00	0.04	0.91*	0.05	0.00	0.00	0.00	0.00

Table 8
Subject 4's frequency of choices

Incentives	Session	N	f_{-2}	f_{-1}	f_0	f_1	f_2	f_3	f_4	f_5	f_6
	0129-93a	25	0.00	0.00	0.04	0.24	0.48*	0.24	0.00	0.00	0.00
	0304-93b	25	0.00	0.00	0.12	0.80	0.08*	0.00	0.00	0.00	0.00
$\gamma = 1$	0305-93b	26	0.00	0.04	0.08	0.88	0.00*	0.00	0.00	0.00	0.00
	0401-93a	25	0.00	0.00	0.00	0.04	0.68*	0.28	0.00	0.00	0.00
	Average		0.00	0.01	0.06	0.49	0.31*	0.13	0.00	0.00	0.00
	0219-93b	25	0.00	0.00	0.00	1.00*	0.00	0.00	0.00	0.00	0.00
	0304-93a	25	0.00	0.00	0.00	0.96*	0.04	0.00	0.00	0.00	0.00
$\gamma = 100$	0305-93a	25	0.00	0.00	0.24	0.68*	0.04	0.00	0.04	0.00	0.00
	0401-93b	25	0.00	0.00	0.00	0.96*	0.04	0.00	0.00	0.00	0.00
	Average		0.00	0.00	0.06	0.90*	0.03	0.00	0.01	0.00	0.00

Table 9
Subject 5's frequency of choices

Incentives	Session	N	f_{-2}	f_{-1}	f_0	f_1	f_2	f_3	f_4	f_5	f_6
	0129-93a	25	0.00	0.04	0.28	0.20	0.16	0.12*	0.12	0.04	0.04
	0304-93b	25	0.00	0.00	0.00	0.00	0.00	0.00*	0.16	0.44	0.40
$\gamma = 1$	0305-93b	26	0.00	0.00	0.00	0.04	0.08	0.88*	0.00	0.00	0.00
	0401-93a	25	0.00	0.00	0.00	0.12	0.64	0.20*	0.04	0.00	0.00
	Average		0.00	0.01	0.07	0.09	0.22	0.30*	0.08	0.12	0.11
	0219-93b	25	0.00	0.00	0.20	0.56*	0.24	0.00	0.00	0.00	0.00
	0304-93a	25	0.00	0.00	0.20	0.64*	0.16	0.00	0.00	0.00	0.00
$\gamma = 100$	0305-93a	25	0.00	0.00	0.00	0.88*	0.12	0.00	0.00	0.00	0.00
	0401-93b	25	0.00	0.00	0.04	0.88*	0.08	0.00	0.00	0.00	0.00
	Average		0.00	0.00	0.11	0.74*	0.15	0.00	0.00	0.00	0.00

statistics will help the reader interpret the tables and understand the peculiar aspects of the detailed behaviors.

When $\gamma = 1$, subject 1's equilibrium choice is $x_1^* = -1$. In the four sessions, half of the subjects who have the incentive structure of subject 1 choose -2 more frequently, and the other half chooses their equilibrium, -1, more frequently. On average, -1 is chosen slightly less frequently than -2, though it is still one of the bimodal distributions. When $\gamma = 100$, however, the equilibrium choice for subject 1, $x_1^* = 1$, is chosen 71% of the time on average. And, it is the most frequent choice for every subject; it is chosen more than 56% of the time. When $\gamma = 1$, the payoff for $x_1 = -1$, denoted by P_{-1}, is strictly greater than the payoffs of the other choices only at the equilibrium, $S_1 = 6$. For any slight disturbances, P_{-1} no longer dominates other choices. For $S_1 \in [3, 5]$, P_{-2} and P_{-1} round up to exactly the same integer values. And for $S_1 \leqslant 2$, we have $P_{-2} > P_{-1}$. Therefore, when the choices vary around the equilibrium value, the probability of choosing -2 instead of -1 is rather high. When $\gamma = 100$, however, the equilibrium choice strongly dominates the other choices not only at the equilibrium, but also in the neighborhood of the equilibrium.

When $\gamma = 1$, three out of the four subject 2's choose their equilibrium, $x_2^* = 0$, more frequently than the other alternatives. On average, the Nash equilibrium, which is chosen 50% of the time, is the mode of distribution. Compared with subject 's payoff structure, the equilibrium choice weakly dominates the other payoffs at three values: $S_2 = 4$, 5, 6. When $S_2 < 4$, $P_1 > P_2$; when $S_2 > 6$, $P_{-1} > P_0$. Therefore the equilibrium value of 1 is chosen most frequently, and both -1 and 0 are chosen with substantial frequencies. When $\gamma = 100$, the Nash equilibrium, $x_2^* = 1$, is chosen 75% of the time on average. Again, the equilibrium choice strongly dominates other choices at and around the equilibrium, thus providing strong incentives for the subjects to play Nash.

For subject 3's, the average frequency of choosing the equilibrium $x_3^* = 1$, when $\gamma = 1$, though highest among the frequencies of the same punishment parameter, is less than half of the frequency of choosing the equilibrium when $\gamma = 100$. When $\gamma = 1$, for $S_3 < 4$, $P_2 \geqslant P_1$, and for $S_3 > 4$, $P_0 \geqslant P_1$, which partly explains why 0 is chosen 31% of the time, and 2 is chosen 18% of the time.

For subject 4's, when $\gamma = 1$, $P_1 > P_2$ for $S_4 > 3$; $P_3 > P_2$ for $S_4 < 3$. Conjecture: in sessions 0304-93b and 0305-93b, most of the time $S_4 > 3$. Again, when we increase γ to 100, the average frequency of attained Nash is almost three times as high as when $\gamma = 1$.

Subject's 5's choice distributions follow a similar pattern as the other subjects.

The next result indicates that the role of γ becomes very pronounced at the individual level of analysis. Result 4 is built from an application of the

equilibrium properties implicit in the behavioral theory of the GL mechanism and asks to what extent the static equilibrium behavior can be detected in the choice behavior of individuals. The result provides both absolute and relative measurements of the accuracy of the equilibrium model when the model is applied at the individual level of analysis under both conditions of $\gamma = 1$ and $\gamma = 100$.

Result 4. The equilibrium model applied to individual choice behavior increases in accuracy when γ increases.

Support. In Tables 5–9 the column f_i^* indicates the frequency of each individual's choice of their GL equilibria. The mean frequency of equilibrium choice is 38% when $\gamma = 1$ and 80% when $\gamma = 100$; it is the mode choice (i.e. most frequent choice) of 10 individuals out of 20 when $\gamma = 1$, and 20 out of 20 when $\gamma = 100$. □

5.2. Behavioral models

The above analysis makes clear that individual behavior is important. This subsection is an attempt to develop some intuition about the principles of individual behavior that might be operating in the context of the mechanism. Two standard models (Ledyard, 1978) can be used as benchmarks. These are the Cournot model, which has individuals using information only one period back, and the other is the Carlson–Auster model (Carlson, 1967; Auster, 1971), which has individuals using information from all past periods and giving them equal weight.

The underlying rationale of these models is described below. Recall, that an individual's value function for the public good is

$$V_i(X) = A_i X - B_i X^2 + \alpha_i ,$$

and the GL cost function is

$$C_i(x_i \mid S_i, \sigma_i) = \frac{X}{I} \cdot q + \frac{\gamma}{2} \left[\frac{I-1}{I} (x_i - \mu_i)^2 - \sigma_i^2 \right] .$$

In equilibrium, from $V_i' = C_i'$, we get

$$x_i = a_i S_i + b_i ,$$

where

$$a_i = \frac{(\gamma/I) - 2B_i}{\gamma(I-1)/I + 2B_i} , \qquad b_i = \frac{A_i - q/I}{\gamma(I-1)/I + 2B_i} .$$

For our design and environment, the set of parameters is presented in Table 10.

Table 10
Cournot response coefficients

Incentive	$\gamma = 1$					$\gamma = 100$				
Subject ID	1	2	3	4	5	1	2	3	4	5
a_i	0.25	−0.44	−0.62	−0.77	−0.69	0.25	0.23	0.22	0.20	0.21
b_i	−2.5	2.22	3.46	4.32	4.38	−0.03	0.05	0.11	0.23	0.17

5.2.1. The Cournot model

Individual players follow Cournot behavior, i.e. $x_i^t = a_i S_i^{t-1} + b_i$. To test the accuracy of the Cournot hypothesis, the raw data are classified to measure the frequency of Cournot reactions. A clear pattern, summarized by the next result, is that Cournot behavior explains over half of the choices when $\gamma = 100$, but does not explain a majority of choices when $\gamma = 1$. Most of the subjects can be classified as Cournot players when $\gamma = 100$. When $\gamma = 1$, however, only a few subjects seem to play Cournot, such as subjects 1 and 2 in session 0401-93a. Refer to Table 11.

Table 11
Frequency and statistics of Cournot behaviors

Incentives	Session	1	2	3	4	5	Statistics
	0219-93a	0.29	0.38	0.21	0.25	0.08	Mean = 0.40100
	0304-93b	0.08	0.32	0.44	0.36	0.48	Stdv = 0.20512
$\gamma = 1$	0305-93b	0.25	0.54	0.46	0.42	0.42	Skewness = 0.81255
	0401-93a	0.75	0.96	0.50	0.33	0.50	Kurtosis = 3.86778
	0219-93b	0.88	0.83	1.00	1.00	0.58	Mean = 0.80000
	0304-93a	0.54	0.58	0.92	0.92	0.58	Stdv = 0.16887
$\gamma = 100$	0305-93a	0.50	0.71	0.79	0.71	0.83	Skewness = −0.41573
	0401-93b	0.79	0.96	1.00	0.96	0.92	Kurtosis = 1.63586

Table 12
Frequency and statistics of Carlson–Auster behaviors

Incentives	Session	1	2	3	4	5	Statistics
	0219-93a	0.25	0.50	0.17	0.45	0.17	Mean = 0.52000
	0304-93b	0.04	0.29	0.67	0.67	0.50	Stdv = 0.26288
$\gamma = 1$	0305-93b	0.24	0.44	0.48	0.76	0.92	Skewness = −0.14939
	0401-93a	0.75	0.88	0.88	0.67	0.67	Kurtosis = 1.72501
	0219-93b	0.92	0.83	1.00	1.00	0.58	Mean = 0.81100
	0304-93a	0.54	0.58	0.88	0.96	0.63	Stdv = 0.16141
$\gamma = 100$	0305-93a	0.58	0.71	0.79	0.71	0.88	Skewness = −0.37408
	0401-93b	0.79	0.96	1.00	0.96	0.92	Kurtosis = 1.53620

Result 5. Cournot behavior is predominant when $\gamma = 100$; it is used less than half of the times when $\gamma = 1$.

Support. Table 12 shows that 40% of the choices on average are Cournot best response when $\gamma = 1$, while 80% of the choices can be categorized as Cournot behavior when $\gamma = 100$.   □

The possibility of convergence to Cournot messages under the two punishment parameters is also of interest. We define $\text{Cvg} = g(\cdot)/(T - n)$, where $g(S_i^{t-1}) = a_i S_i^{t-1} + b_i$, T is the total rounds and n is the initial number of rounds. The purpose of this definition of convergence is to see if the subjects make Cournot responses more frequently as they play along, and if they converge in probability to Cournot behavior.

Fig. 2 shows player 1's rate of convergence to Cournot behavior under $\gamma = 1$ and $\gamma = 100$ in experiments 0219-93a and 0219-93b, respectively. The pattern exhibited in the figure is typical in most of the experiments, i.e.

Result 6. When $\gamma = 1$, convergence to Cournot behavior is rare, slow and unstable; when $\gamma = 100$, the convergence is fast and stable for most of the subjects.
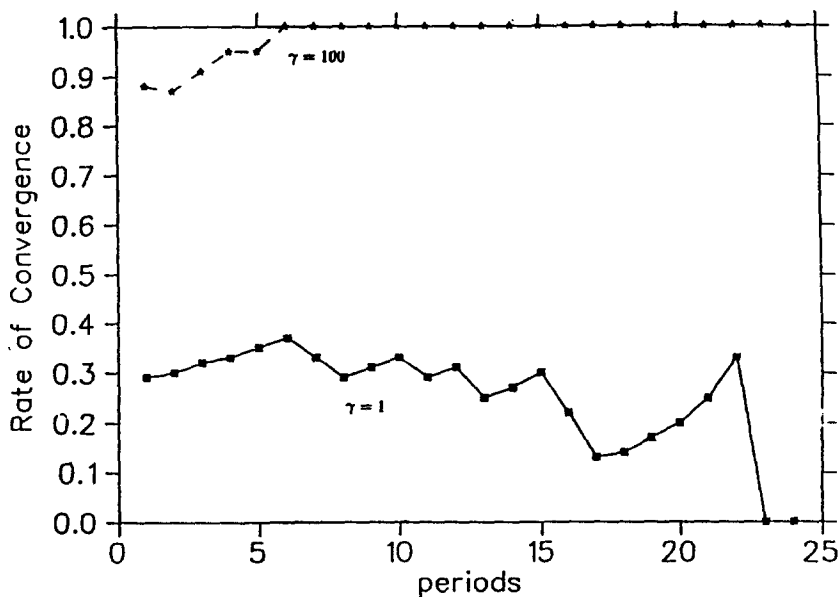
Fig. 2. Rate of convergence under two punishment parameters.

Support. All of the experiments exhibit similar patterns as those shown in Fig. 2, i.e. when $\gamma = 100$ convergence to Cournot behavior is fast and stable, when $\gamma = 1$ the convergence is slow and unstable, and over half of the time it does not converge at all. $\square$

The Cournot model postulates that subjects base their best responses only on the information they receive in the previous period. An alternative model is that subjects base their best responses on all the information they receive in the previous periods. How much weight each subject puts on past information might differ among periods and subjects. Here, we examine a simple version of such a model, when all previous periods are given equal weight by all subjects. The Carlson–Auster expectations model postulates that each subject bases best responses upon the average of all previous period's information.

5.2.2. Carlson–Auster expectations model

Individual subjects follow the Cournot response based on the average of all previous period's information, i.e.

$$x_i^t = a_i \left(\frac{1}{t-1} \sum_{r=1}^{t-1} S_i^r \right) + b_i .$$

Result 7. The subjects use Carlson–Auster best responses over half of the time under both punishment parameters. The prediction of the Carlson–Auster model is more accurate than the Cournot model.

Support. The frequency with which the choice is the prediction of the Carlson–Auster model is contained in Table 12. As shown, when $\gamma = 1$, 52% of the choices on average are Carlson–Auster best responses; when $\gamma = 100$, 81% of the choices on average are Carlson–Auster best responses. In only one of the eight experiments is the rate of CA behavior less than 50%. Compared with the Cournot model, the Carlson–Auster expectations model explains a higher percentage of the data under both punishment parameters. $\square$

The result is that the Carlson–Auster model, in which subjects are seen as averaging out all the past information and then optimizing, is more accurate than the Cournot model which predicts that the subjects only look at the previous period before optimizing.

5.3. Logit analysis: Incentives and choice behavior

From the above classification of raw data, the role of γ in individual subjects' decision to use Cournot best responses or Carlson–Auster best

responses is obvious. The purpose now is to explore the possibility that other factors might contribute to individuals' tendency to use either Cournot or Carlson–Auster best responses. Apart from γ, could the probability of individual choices be related to the parameters of their preferences for the public good? What other factors affect the probability of individual choices?

The principle of 'design consistency' (Plott, 1993) requires that the reasons for choices be studied. If a process is expected to have robust performance properties, it should be working for the right reasons. That is, the process should be working according to the basic theory and principles that were used to design the process in the first place. Therefore, we proceed by an examination of possible relationships among the induced preferences, the punishment parameter and individual subjects' probabilities of choosing Cournot responses. Analysis of the Carlson–Auster model can be done in a similar way.

A widely held belief in the experimental literature is that the predictive capacity of game-theoretic or economic-theoretic models improves as the level of incentive increases. This presumption plays such an active role in the analysis of this section that we give it a name.

The General Incentive Hypothesis. The error of game-theoretic and economic-theoretic models decreases as the level of incentive increases.

Applying the General Incentive Hypothesis to this analysis, let us consider a subject's probability of choosing his Cournot response, $P_i(C)$, as a decreasing function of his net gain from deviating from Cournot. We use NV_i^c to denote a subject's net value from choosing the Cournot response, and $NV_i^{\varepsilon_i}$ to denote his net value from choosing a message ε_i away from his Cournot response, $x_i^c = a_i S_i^{I-1} + b_i$. His deviation, $\varepsilon_i \in [\underline{x} - x_i^c, \bar{x} - x_i^c]$, where $\underline{x}$ and $\bar{x}$ denote the upper and lower bounds of the subject's message space. Therefore, omitting the subscript i for simplicity, a subject's net value from deviating from the Cournot response is

$$
NV^\varepsilon = A(x^c + \varepsilon + S) - B(x^c + \varepsilon + S)^2 + \alpha
$$
$$
- \left[\frac{q}{I}(x^c + \varepsilon + S) + \frac{\gamma}{2}\left(\frac{I-1}{I}\left(x^c + \varepsilon - \frac{S}{I-1}\right)^2 - \sigma^2\right) \right],
$$

and his net gain from deviating is

$$
NG^\varepsilon = A\varepsilon - B\varepsilon(2x^c + \varepsilon + 2S) - \left[\frac{q}{I}\varepsilon + \frac{\gamma}{2}\frac{I-1}{I}\varepsilon\left(2x^c + \varepsilon - \frac{2S}{I-1}\right) \right]
$$
$$
= -\left(B + \frac{I-1}{2I}\gamma\right)\varepsilon^2 .
$$

It follows that an increase in either B_i or γ causes a decrease of the net gain from deviation. Application of the General Incentive Hypothesis leads to:

Proposition. An increase in γ or B_i will cause the subjects to choose Cournot responses with a higher probability.

Therefore, in the logit analysis, we consider two independent variables: γ, the punishment parameters, and B_i, the coefficient of individual's value functions for the public good. The dependent variable is a discrete choice variable, y, which equals one if a subject makes a Cournot response, and zero otherwise. Therefore, the model is

$$P[y = 1] = \Phi(\beta'x) .$$

Coefficients, t-statistics (in parentheses), log-likelihood and the percentages correctly predicted for each model are given in Table 13.

For the Cournot hypothesis, we consider two logit models, C_1 and C_2. The simple basic model C_1 has only one independent variable, $x = \gamma$, i.e. a player's decision depends only upon γ. In C_2, B_i is added as an independent variable to the basic model.

In testing the impact of different parameters on the probability of the Carlson–Auster hypothesis, we devise similar logit models, and get models CA_i, which are tabulated in the last two columns of Table 13.

As we see in the model C_1, in the basic model for the Carlson–Auster hypothesis, CA_1, γ alone explains nearly 70% of the data. A consistent pattern in all four models is the positive and significant impact of γ and B_i on the choice of the best responses behavior, which confirms our observations from the classification of the raw data and theoretical deduction.

Result 8. The single most important factor that affects the subjects' probabilities of choosing best responses is γ. An increase in γ leads to an

Table 13
Logit models for Cournot and Carlson–Auster hypotheses

Independent variables	C_1	C_2	CA_1	CA_2
Ones	−0.424	−0.564	6.452e-002	−0.504
	(−4.524)	(−4.004)	(0.703)	(−3.556)
γ	1.809e-002	1.814e-002	1.388e-002	1.433e-002
	(12.187)	(12.193)	(9.304)	(9.411)
B_i		0.159		0.649
		(1.347)		(5.315)
Log-likelihood	−566.6	−565.69	−568.9	−554.2
% predicted	69.948	69.948	66.425	69.534

increase in the probability of an individual choosing his best response, in both the Cournot and the Carlson–Auster behavioral models.

Support. In the basic model C_1, γ alone is able to correctly predict 69.948% of the observations. In CA_1, γ alone explains 66% of the data. In all four models, the coefficients of γ are significant at the 99% level, and are positive, which says that an increase in γ leads a subject to choose Cournot responses with higher probability.   □

Result 9. The preference parameter, B_i, has a significant and positive impact on the probability of an individual choosing his best response. An increase in B_i leads to an increase in the probability of an individual choosing his best response, in both the Cournot and the Carlson–Auster behavioral models.

Support. In C_2, B_i is significant at the 90% level; in CA_2, B_i is significant at the 99% level. In both models, the coefficients of B_i are positive. In CA_2, the percentage of data predicted rises from 66.425% to 69.534% after B_i is added as an independent variable.   □

The tendency of an individual to use a Cournot-type response is related to the details of the individual's preferences. The level of B_i, which has a negative impact on an individual's marginal value of the public good, also influences his tendency to give a Cournot or Cournot-related response. An increase in B_i leads to an increase in the probability of an individual choosing Cournot. This is consistent with the General Incentive Hypothesis and the proposition about the probability of an individual choosing best responses. Such relationships have been observed before in voluntary contribution experiments.

Observation. The influence of B_i in these data is consistent with the influence of public goods valuation on voluntary contributions observed in other experiments.

Support. The experimental literature suggests that the greater is the marginal value of a public good, the more is the tendency of an individual to voluntarily contribute to public goods (e.g. Isaac et al., 1985; Isaac and Walker, 1988; Palfrey and Prisbrey, 1993). That is, if the marginal rate of substitution between the public good and the private good increases, then the individual values the public good more relative to the private good, and is willing to contribute more private good for the production of the public good.[7] Thus the individual is less likely to follow the Cournot strategy for no

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[7] See Ledyard (1993) for a more rigorous treatment.

provision of public goods. That is, as the benefit of the Cournot responses go down, the frequency of its use goes down. Therefore, in two completely different mechanisms, the parameters of an individual's induced preferences have a consistent impact on an individual's probability of choosing the best responses. Thus all of these observations are consistent with a general pattern of observation that connects the level and structure of rewards to the accuracy of an economic- or game-theoretic model.[8] □

The logit analysis is consistent with the observation and proposition about the impact of the punishment parameter, γ, and the preference parameter, B_i, on an individual's probability of choosing the best responses. Regardless of which behavioral model is imposed, an increase in γ supports the performance of the model.

6. Conclusions

On paper the Groves–Ledyard mechanism solves the free-rider problem in certain economic environments. The free-rider problem has been the cornerstone of the problem of public goods provision and the Groves–Ledyard process promises a solution. The research reported here demonstrates that if the GL process is made operational through an implementation called a 'periodic process', then in a simple quasilinear environment the promise of the theory can be realized.

The effectiveness of the GL solution to the public goods problem is closely related to a special parameter which we have called the 'punishment' parameter. As the level of 'punishment' is elevated from a level $\gamma = 1$ to a level $\gamma = 100$, the efficiency of the operation of the process increases from 91% to 98% and the average level of provision increases from 4.7 units to 4.9 units, which is to be compared with an optimum of 5 units. Furthermore, an increase in the level of γ substantially decreases the dispersion of the outcomes across experiments, thereby suggesting that it influences the reliability of the process.

In any 'testbed' experiments of the type reported here, it is useful to perform what has been called 'design consistency checks' (see Plott, 1993) to determine if the reasons that a process is working are given by the basic theory and principles that were used to design the process in the first place. A process might be observed working but it might be working for the wrong reason.

The consistency check on the mechanism reveals that over half of the individuals are exhibiting the type of behavior that is assumed by the

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[8] See Fiorina and Plott (1978) for an example in which two public goods are involved.

principles of the GL model. That is, over half of the individual choices can be viewed as Cournot responses or, more accurately, as optimal responses based on a belief that other individuals will be choosing on average as they have chosen in the past (the Carlson–Auster model). The response of individual behavior to an increase in γ is to increase the frequency of Cournot-type responses and coverage more rapidly to such responses.

The focus on the punishment parameter creates another interesting question relevant to the actual implementation of the GL processes. Our results demonstrate that an increase in punishment increases the instance of Cournot-type responses on which the mechanism depends. However, observing that the mechanism performs better when $\gamma = 100$ than when $\gamma = 1$ does not lead to the conclusion that the higher is the punishment parameter, the better the mechanism performs. To illustrate the point, we consider what happens when $\gamma \rightarrow \infty$. For simplicity, we use Cournot behavior as an example. At time t, player i's Cournot reaction is

$$
\begin{aligned}
x_i^t &= a_i S_i^{t-1} + b_i \\
&= \frac{A - q/I + (\gamma/I - 2B_i)S_i^{t-1}}{\gamma(I-1)/I + 2B_i} \\
&\rightarrow \frac{S_i^{t-1}}{I-1}, \text{ as } \gamma \rightarrow \infty.
\end{aligned}
$$

So when γ is very large, the subject's best response, if he follows Cournot behavior, is to choose the mean of the other subject's last period message, to avoid being punished by the large γ. Then we can induce a subject's best response at period t, given the initial choices of all subjects. Let subject i's initial move at time zero be x_i^0, then $X^0 = \Sigma_i x_i^0$. It is easy to prove that

$$
\begin{aligned}
x_i^t &= \left[\frac{1}{I} + \frac{(-1)^{t+1}}{I(I-1)^t} \right] X^0 + (-1)^t \frac{x_i^0}{(I-1)^t} \\
&\rightarrow \frac{X^0}{I}, \text{ as } t \rightarrow \infty.
\end{aligned}
$$

We can see that, given a large enough γ and long enough repetition, all subjects' choices converge towards the mean of the initial choices, which can be anything. So, from our experiments and theoretical deduction, it is clear that as γ increases from one on, the performance of the mechanism improves, but as it goes to infinity, the performance declines. The optimal choice of γ remains an open question.

Another open question for future research is the performance of the mechanism when there are multiple equilibria. We studied a quasilinear environment with a unique equilibrium. It would be interesting to see which equilibrium will be selected in a more general environment with multiple equilibria.

The institutional design problem identified in the opening paragraphs of this paper are beginning to be solved. It is possible to align at least one normative criterion (efficiency) with the proper incentives. The paper processes, when brought into the context of operational process, work substantially as expected. The magnitude and nature of the incentives are important but they are important in ways that make intuitive sense. Whether or not the process themselves (like the GL process) will ultimately provide the tools needed by those who wish to design a process for implementation in the field (Bates, 1985, 1988; Ostrom, 1990) remains to be seen.

Acknowledgments

Appendix: Experiment instructions

You are about to participate in a decision process in which one of numerous competing alternatives will be chosen. This is part of a study intended to provide insight into certain features of decision processes. If you follow the instructions carefully and make good decisions you may earn a considerable amount of money. You will be paid in cash at the end of the experiment.

This decision process will proceed as a series of trials during which a project level will be determined and financed. The 'level' can be negative, zero or positive, 'units', the exact level of which must be determined. Attached to the instructions you will find a series of tables, which describe the value to you of decisions made during the process, called the Payoff Tables. *You are not to reveal this information to anyone.* It is your own private information.

During each period a level of the project will be determined. For the first unit provided during a period you will receive the amount listed in row 1 of the Redemption Value Sheet. If a second unit is also provided during the period, you will receive the additional amount listed in row 2 of the Redemption Value Sheet. If a third unit is provided, you will receive, in addition to the two previous amounts, the amount listed in row 3, etc. As you can see, your individual total payment is computed as a sum of the redemption values of specific units. (These redemption totals are tabulated for your convenience on the right-hand side of the Redemption Value Sheet.)

The payoff each period, which is yours to keep, is the difference between the total of the redemption values of units of the project provided and your individual expenditures on the project. All values are stated in francs and can be converted into cash at a rate of _francs per dollar at the end of the experiment. Suppose, for example, your Redemption Value Sheet was as below and two units were provided.

Project level (units)	Redemption value of specific units (francs)	Total redemption value of all units (francs)
1	2500	2500
2	1500	4000
3	1000	5000

Your redemption value for the two units would be 4000 and your payoffs would be computed by subtracting your individual expenditures from this amount. If three units were provided, the redemption value would be determined by the redemption values of the first and second units plus the redemption value of the third unit, that is

$$2500 + 1500 + 1000 - 5000 .$$

Each unit of the project costs _francs. Hence, the total cost for a project is _times the project size. Your expenditure toward the total project cost for a trial is determined from your decision and the decisions of all others. Note that the redemption values can be negative. Your expenditure can also be negative. That is, rather than paying for the project you are paid.

Your individual decisions will influence both the final level of the project chosen by the group and your individual expenditures on the project. Recall that your payoffs from the experiment will be the difference between the redemption values (positive or negative) that are determined by the level of the project chosen and your individual expenditures (positive or negative). These will be explained in turn.

Project level determination (X). Each period each individual will choose a proposed addition (x) to the status quo of zero provision. This proposed addition can be any amount ranging from _to_. These amounts will be added together to get the total of proposed additions (X). This total is the project level that will be chosen.

Level of individual expenditures (c). The level of your individual expenditures depends upon your individual proposed addition (x), the proposed additions of other participants (S) and the variability among the proposed additions of the other participants (o). The actual formula is somewhat

cumbersome,[9] so a table that summarizes all of the relevant information will be used instead.

Payoff table. The payoff table will summarize both the redemption value of the level of the project chosen and the level of individual expenditures that you will incur depending upon the choices of additions that you and other participants make. This table is a rather large table contained in your instructions. The following example will demonstrate how you read it. The numbers in the example are completely arbitrary and in general have no relationship to the actual table that you will be using. The purpose is only to help you to understand how to read the real table.

*S:13**ID:9**

o \ x	−5	−4	−3	−2	−1	0	1	2	3	4	5
0.13	−323	−119	7	50	34	23	9	−115	−323	−581	−957
0.55	−33	−13	−10	11	23	9	−25	−78	−99	−119	−139
3.46	−55	24	48	67	33	−3	−18	−76	−127	−205	−254
9.57	4	77	95	214	341	348	343	218	10	−281	−670

*S:16**ID:9**

o \ x	−5	−4	−3	−2	−1	0	1	2	3	4	5
0.00	23	19	22	30	64	83	49	−11	−23	−81	−95
0.55	−3	−1	0	14	43	55	−15	−48	−66	−97	−166
3.46	−5	14	35	87	29	−65	−74	−98	−274	−306	−764
9.67	77	136	248	360	657	246	119	34	5	−81	−409

The example table consists of three relevant numbers. The first number is the sum of the additions chosen by the other participants. This number is located in the upper left corner of a table. Since it is a sum it is denoted by S. To start, find the example table for which S, the sum of the additions of others, is equal to 13. The top row of the table lists the possible choices that you might make for your own proposed addition, x. The amounts that you have as options in this example range from −5 to +5. Of course these might differ from the options that might exist on the real table.

The left column of the table contains measures of the variability of the proposed additions of the other participants, o. This variability measure reflects how scattered the additions of others are. For example, if all of the other participants give the exact same number, then there is no scatter at all and the variability is zero. Suppose that everyone gives a different number

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[9] Individual expenditure = (your addition + addition of others) + A(your addition − average addition of others)^2 − B(variability of others). In experiment no. 1, $c = (x + S) + 0.4*(x − S/4)^2 − 0.5*o$, in experiment no. 2, $c = (x + S) + 40*(x − S/4)^2 − 50*o$.

but all numbers differ very little, then the scatter is low, as is the measure of variability of the additions of other participants.

Suppose that S is 13 and that the variance (of the additions of others) is 3.46. If you chose a proposed additional equal to -2, then your payoff is 67. That is, your payoff is determined by the sum of the additions of others, the variance and your own addition. Each entry of the table is your payoff that corresponds to your choice and the choices of the other participants. The payoff could have been calculated from the formulas. Since S is 13 and you choose -2 the project level chosen is 11. The redemption value for 11 units would then be determined and the individual expenditures would also be computed by formula and subtracted. The table does all of these calculations for you.

Another example might be useful. Suppose S is 16, variance is 9.67 and your proposed addition is 5. The example table indicates a payoff of -409 which you would get from such a pattern of decisions.

It is crucial that you check your payoff tables before and after each decision. As you can see, your choice, x, decides which column you will end up; the others' choices decide which table and which row of that table you will end up.

There will be 30 trials for each experiment. The first 5 trials of each experiment will be practice trials. You will not be paid for these practice trials. Starting from the 6th trial, you will be paid for each decision you make.

Your file includes a record sheet at the last page of each set of experiments for you to record the results of each trial. At the end of each trial, you should record your proposed addition, x, in the first row; the sum of proposals of others, S, in the second row; the variances of others, o, in the third row; and your net payoff, P, in the fourth row.

Feel free to earn as much cash as you can. Are there any questions?

Computer instructions. At the beginning of each trial you are free to enter any proposed addition, x, between -2 and 6, and then press the *F-10* key to send it to the central computer. If you want to send a negative number, enter the number first and then the negative sign. If you would like to change your selection, use the *Back Space* key to delete the selection, and then enter your new selection. Now go ahead and enter a number. Notice that if you enter a number out of the -2 and 6 range, the computer will tell you that your choice is out of range and you need to change your selection. Now everybody please use the *Back Space* key to erase your choice, and then type in a negative number by typing the number first and then the negative sign. Now please press the *F-10* key and then confirm it by typing y. Once you confirm your choice by typing y, you cannot change your choice anymore. After everyone sends their choices, the computer will calculate the sum of proposals of others, S, the variances of other members, o, and your corresponding payoff for this trial, P, and send these numbers of your

screen. This process will be repeated at each trial. Now go ahead and record the result of the first trial to the first column of your record sheet.

1. Key function summaries

F-10: send your choice to the central computer.
Back Space: erase your choices.
y: confirm your choices before sending off to the central computer.

2. Review questions

1. If each of you propose the following units: $x_1 = 5$, $x_2 = 4$, $x_3 = 3$, $x_4 = -2$, $x_5 = 1$,
   (1) The total level of the project, $X =$
   (2) The sum of others' proposal, $S =$
   (3) The variances of these proposed additions for each player is: $o_1 = 7.00$, $o_2 = 8.92$, $o_3 = 10.00$, $o_4 = 2.92$, $o_5 = 9.67$. From the payoff table, your payoff for this trial, $P =$

2. Suppose all others have the same proposed addition, you alone raise your addition by 1 unit, then
   (1) The total level of the project, $X =$
   (2) The sum of others' proposal, $S =$
   (3) The variances of these proposed additions for each player is: $o_1 = 7.00$, $o_2 = 8.92$, $o_3 = 10.00$, $o_4 = 2.92$, $o_5 = 9.67$. From the payoff table, your payoff for this trial, $P =$

3. True or false:
   (1) Your share of the total cost depends only on your decisions.
   (2) Each person does not necessarily have the same total value formula.

3. Financial agreement

Should my earnings from the experiment be negative, I agree to work in the Experimental Economics and Political Science Laboratory at a rate of 7 dollars per hour until the loss is repaid.
   Name and Signature
   Date

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