Selection Dynamics, Asymptotic Stability, and Adaptive Behavior

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Selection Dynamics, Asymptotic Stability, and Adaptive Behavior

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Selection dynamics are often used to distinguish stable and unstable equilibria. This is particularly useful when multiple equilibria prevent a priori comparative static analysis. This paper reports an experiment designed to compare the accuracy of the myopic best-response dynamic and an inertial selection dynamic. The inertial selection dynamic makes more accurate predictions about the observed mutual best-response outcomes.

Stability arguments are often used to select among multiple equilibria. This approach to the equilibrium selection problem is based on the interpretation of an equilibrium point as a potential convention that might arise among players interacting repeatedly. Mutually consistent behavior is not deduced from the description of the situation, but rather is the outcome of some evolutive process.

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An equilibrium point is unstable if it does not correspond to an asymptotically stable fixed point of some explicit selection dynamic. An unstable equilibrium point is unlikely to emerge as the result of an evolutive process and is an unlikely convention. Hence, the analyst should select from the set of stable equilibria. When there is a unique stable equilibrium, this approach may appear to preserve the analyst's ability to abstract from the evolutive process itself with its undesirable dependence on historical accident.[1]

However, the asymptotic stability of an equilibrium point depends on the assumed selection dynamic (compare Lucas [1987] and Woodford [1990], e.g., and see Guesnerie and Woodford [1993] for a survey of alternative stability concepts). Moreover, selection dynamics need not converge to any fixed point corresponding to any equilibrium of the model. Even in simple settings it is possible to construct examples of selection dynamics that predict cyclical or chaotic behavior.

A venerable selection dynamic is the myopic best-response dynamic, which dates back at least to Cournot's (1838) duopoly analysis of firms that best respond to the other firm's last action. Alternative selection dynamics are often used when the myopic best-response dynamic fails to converge to a fixed point. Selection dynamics based on a best response to slowly changing beliefs, inertial beliefs, will often converge when the myopic best-response dynamic does not (see, e.g., Bray 1982; Marcet and Sargent 1989; Thorlund-Peterson 1990). The selection dynamics we consider here are "relaxation algorithms," which include the myopic best-response dynamic, the partial adjustment dynamic, and least-squares learning (see Sargent 1993).

Lucas (1987, p. 241) discusses stability theory based on adaptive behavior and concludes that to be useful "stability theory must be more than simply a fancy way of saying that one does not want to think about certain equilibria. I prefer to view it as an experimentally testable hypothesis, as a special instance of the adaptive laws that we believe govern all human behavior."

This paper examines human behavior in a generic game with multiple equilibria in which the myopic best-response dynamic and inertial selection dynamic make different predictions about stability. The inertial selection dynamic makes more accurate predictions than the myopic best-response dynamic in our experiment.

[1] For those who think that path dependence is a fact of life, see Van Huyck, Battalio, and Beil (1991), Van Huyck, Cook, and Battalio (1993), and Van Huyck et al. (in press).

I. Analytical Framework

Let $e^1, \ldots, e^n$ denote the actions taken by n players, where n is odd and greater than one. Let e denote this action combination, and let $M(e)$ denote the median of e. The game $\Gamma(\omega)$ is defined by the following payoff function and action space for each of these n players, which are indexed by i:

$$\pi(e^i, e^{-i}) = c_1 - c_2 \left| e^i - \omega M(e)[1 - M(e)] \right|, \quad (1)$$

where $\omega \in (1, 4]$, $e^i \in \mathbf{E} = [0, 1]$, e^{-i} denotes $\{e^1, \ldots, e^{i-1}, e^{i+1}, \ldots, e^n\}$, c_1 and c_2 are positive parameters, and $|\cdot|$ is the absolute value function. Assume that the payoff functions and feasible actions are common knowledge.

Before we proceed with our analysis of game $\Gamma(\omega)$, two remarks on why we chose this particular game are in order. First, in game $\Gamma(\omega)$, a player's best response to a given median M is $b(M) = \omega M(1 - M)$, which is a best-response function that has been widely studied in the literature on nonlinear dynamics. The parameter ω "tunes" $b(M)$. Figure 1*a* and *b* graphs $b(M)$ for $\Gamma(2.47222)$ and $\Gamma(3.86957)$, respectively. Second, we use the median to determine payoffs rather than the sum or the mean, in order to capture the anonymity of a many-person economy without using enormous group sizes in our experiments (see Van Huyck, Cook, and Battalio [1993] for a discussion and Rassenti et al. [1993] for a comparison).

The principle of individual rationality prescribes that a player should not use dominated strategies. Let $\Delta(\mathbf{E}^{n-1})$ denote the set of probability distributions on $\mathbf{E}^{n-1} = [0, 1]^{n-1}$. A strategy $e^i \in \mathbf{E}$ is dominated by another strategy $x^i \in \mathbf{E}$ if, for all $s^{-i} \in \Delta(\mathbf{E}^{n-1})$, $\pi(e^i, s^{-i}) < \pi(x^i, s^{-i})$. In $\Gamma(\omega)$, e^i in the interval $(.25\omega, 1]$ are not a best response to any element of $\Delta(\mathbf{E}^{n-1})$ and, hence, are dominated strategies.

Common knowledge that players are individually rational requires the serial deletion of dominated strategies. The set of serially undominated action combinations in game $\Gamma(\omega)$ is $[0, .25\omega]^n$ when $\omega \geq 2$ and $[0, 1 - (1/\omega)]^n$ otherwise. Let $\mathbf{U}(\omega)$ denote the two-dimensional space of serially undominated action combinations. Figure 1*a* and *b* graphs $\mathbf{U}(2.47222)$ and $\mathbf{U}(3.86957)$, respectively; the set $\mathbf{U}(\omega)$ is indicated by grey shading.

The principle of individual rationality does not make very precise predictions in game $\Gamma(\omega)$. Requiring a mutual consistency condition allows one to make more precise predictions. An action combination e^* constitutes a strict equilibrium if it satisfies the following mutual best-response condition:

$$\pi(e^i, e^{-i*}) < \pi(e^{i*}, e^{-i*}) \quad (2)$$

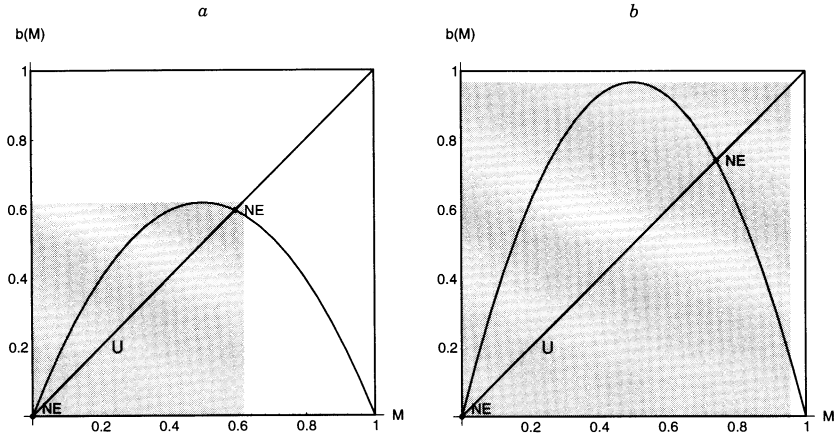

FIG. 1.—*a*, Graph of $b(M)$ and $\mathbf{U}(2.47222)$ for $\Gamma(2.47222)$. The intersection of $b(M)$ and the 45-degree line is a strict equilibrium, *NE*. *b*, Graph of $b(M)$ and $\mathbf{U}(3.86957)$ for $\Gamma(3.86957)$; $\mathbf{U}(\omega)$ is denoted by grey shading.

for all $e^i \in [0, 1]$ and for all i. An observed action combination is a mutual best-response outcome if it satisfies (2).

An action combination is a symmetric equilibrium if it satisfies condition (2) and assigns the same action to all the players. All the strict equilibria of $\Gamma(\omega)$ are symmetric. Hence, it is convenient to denote the equilibria by the ordered pair (e, M). The requirement that $\omega \in (1, 4]$ results in two strict equilibria: a corner equilibrium (0, 0) and an interior equilibrium $(1 - (1/\omega), 1 - (1/\omega))$. Figure 1 indicates these equilibria with *NE*. The equilibria occur at the intersection of $b(M)$ and the 45-degree line.

While imposing the mutual consistency requirement has significantly increased the precision of our prediction, it still leaves an equilibrium selection problem. Moreover, deductive selection principles, such as payoff dominance and symmetry, fail to reduce the set of equilibria, since both of the strict equilibria are efficient and symmetric.[2] Hence, even if subjects are individually rational, giving them common information about $\Gamma(\omega)$ is not likely to produce mutually consistent behavior.

II. Selection Dynamics and Asymptotic Stability

If an equilibrium point is viewed as a potential convention that might arise among the players when they interact repeatedly, then some

[2] For the reader familiar with our previous work, we note in passing that the best response to a uniform prior on M is $\omega/6$.

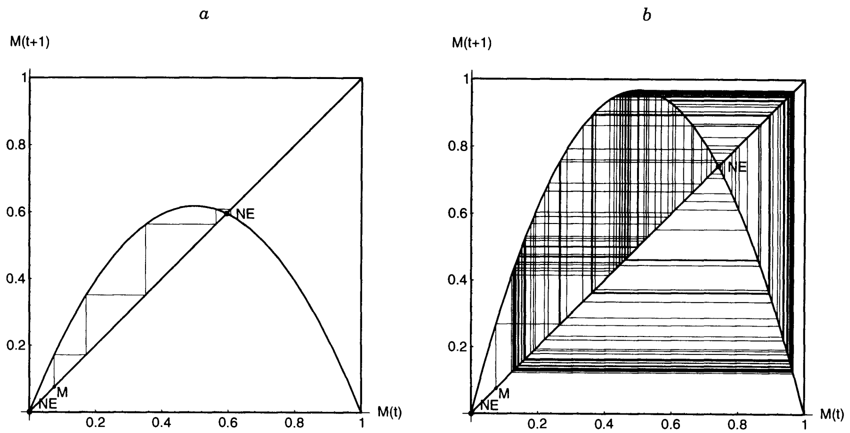

FIG. 2.—*a*, Path of a selection dynamic for $\Gamma(2.47222)$ with the property that the interior equilibrium is stable and the corner equilibrium is unstable; M denotes the initial condition. *b*, Path for $\Gamma(3.86957)$ under the same selection dynamic as fig. 2*a*. Neither equilibrium is stable.

equilibria can be ruled unstable and, hence, unlikely conventions. An equilibrium point is unstable if it does not correspond to an asymptotically stable fixed point of some selection dynamic. The selection dynamics we consider here are "relaxation algorithms," which include the myopic best-response dynamic, the partial adjustment dynamic, and least-squares learning (see Sargent 1993).

Specifically, consider the following dynamical system:

$$\begin{aligned} M_t &= b(M_t^e), \\ M_t^e &= M_{t-1}^e + \alpha_{t-1}(M_{t-1} - M_{t-1}^e) \end{aligned} \tag{3}$$

for $t \geq 2$, where $0 < \alpha_t \leq 1$ for all t and M_t^e can be interpreted as the expectation of the "representative agent." For $t = 1$, let $M_1 = b(M_1^e)$, where M_1^e is an initial condition.

An example is the myopic best-response dynamic. Suppose that players choose their current action as a best response to last period's median action. Then $\alpha_t = 1$ for all t, and dynamical system (3) can be reduced to the following difference equation:

$$M_{t+1} = \omega M_t(1 - M_t). \tag{4}$$

This difference equation has been studied recently in Baumol and Benhabib (1989), Boldrin and Woodford (1990), and Eckalbar (1993).

Figure 2*a* illustrates the use of the myopic best-response dynamic (4) as a selection dynamic for $\Gamma(2.47222)$. Notice that for an initial condition close to the corner equilibrium, the dynamic diverges toward the interior equilibrium. However, initial conditions close to the interior equilibrium converge to the interior equilibrium. Hence, the myopic best-response dynamic implies that the corner equilibrium is unstable and the interior equilibrium is stable.

This distinction between unstable and stable equilibria in $\Gamma(\omega)$ is useful to an economist conducting a priori comparative static analysis. If e^i is effort, then does increasing ω increase effort? The unstable corner equilibrium (0, 0) is not a function of ω, but the stable interior equilibrium $(1 - (1/\omega), 1 - (1/\omega))$ is an increasing function of ω. Hence, appealing to something like Samuelson's correspondence principle would allow the analyst to conclude that increasing ω does increase effort (see Brock and Malliaris [1989] on Samuelson's correspondence principle).

However, increasing ω globally in game $\Gamma(\omega)$ leads to complex dynamics under the myopic best-response dynamic. Let $\mathbf{A}(\omega)$ denote the attractor set under the myopic best-response dynamic. For ω in (1, 3), $\mathbf{A}(\omega)$ contains the single element $\{1 - (1/\omega)\}$; but for ω in [3, 4], the dynamics get complicated. For ω in [3, 3.449499), globally attracting two-period cycles appear. This bifurcation continues until $\mathbf{A}(4) = \mathbf{E}$.

Figure 2*b* illustrates the path for game $\Gamma(3.86957)$ under the myopic best-response dynamic starting at .075, which is the same initial condition used in figure 2*a*. (Simulations reported in the text were produced by a Mathematica package available on request.) While the path still diverges from the corner equilibrium, it does not converge to the interior equilibrium. Instead it wanders in the space $[.122, .967]^2$. Neither the corner nor the interior equilibrium is stable.

There is an uncountable number of initial values yielding bounded time paths that *never* repeat any past behavior in $\Gamma(3.86957)$ no matter how long a set of time periods one permits the calculation to encompass. This phenomenon, an aperiodic series that does not repeat itself, is sometimes called chaos, and the attractor set [.122, .967] is sometimes called a strange attractor. Here it will be denoted $\mathbf{A}(3.86957)$. Note that the attractor set is a subset of the set of serially undominated actions, that is, $\mathbf{A}(\omega)^2 \subset \mathbf{U}(\omega)$.

The prediction that the interior equilibrium is unstable when ω equals 3.86957 depends crucially on the assumption $\alpha_t = 1$. In order to characterize the local stability of the interior equilibrium for the partial adjustment dynamic generally, let $\alpha_t = \alpha \in (0, 1]$. The linearized system for the partial adjustment dynamic around the interior equilibrium is $u_t = \mathbf{A}u_{t-1}$, where u_t is the vector $(M_t - [1 - (1/\omega)], M_t^e - [1 - (1/\omega)])$ and $\mathbf{A}$ is the Jacobian matrix for the system. Since $\mathbf{A}$ can be diagonalized, the solution to the linearized system is

$$u_k = \begin{bmatrix} 1 - \dfrac{1}{\omega} & 2 - \omega \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1^{k-1} & 0 \\ 0 & \lambda_2^{k-1} \end{bmatrix} \begin{bmatrix} \alpha & -\alpha(2-\omega) \\ -\alpha & \alpha - 1 \end{bmatrix} \frac{u_1}{-\lambda_2}, \quad (5)$$

where $\lambda_1 = 0$ and $\lambda_2 = 1 + \alpha - \alpha\omega$ are the eigenvalues of $\mathbf{A}$. For the linearized system to converge to zero, both λ_1^{k-1} and λ_2^{k-1} must converge to zero as $k \to \infty$. Hence, asymptotic stability requires $|1 + \alpha - \alpha\omega| < 1$.

To check our simulation results for the myopic best-response dynamic, note that when $\alpha = 1$ the stability condition becomes $1 < \omega < 3$. Hence, when $\omega = 2.47222$ the interior equilibrium is locally stable, and when $\omega = 3.86957$ the interior equilibrium is predicted to be locally unstable.

When $\alpha < 1$, we call the partial adjustment dynamic inertial since it responds sluggishly to changes in the current value of M_t. How much inertia must there be in the dynamics before the interior equilibrium of $\Gamma(3.86957)$ is predicted to be locally stable? Manipulating the stability condition gives the requirement that $\alpha < 2/(\omega - 1)$. So the interior equilibrium is stable if $\alpha < .7$. While introducing inertia can stabilize the interior equilibrium, no amount of inertia can stabilize the corner equilibrium.

The final version of (3) considered here sets $\alpha_t = 1/t$. Specifically, consider the following dynamical system:

$$\begin{aligned} M_t &= b(M_t^e), \\ M_t^e &= \frac{t-1}{t} M_{t-1}^e + \frac{1}{t} M_{t-1} \end{aligned} \quad (6)$$

for $t \geq 2$. For $t = 1$, let $M_1 = b(M_1^e)$, where M_1^e is an initial condition. We shall denote this dynamical system the L map, which is mnemonic for Lucas (1987).

Figure 3 graphs the L map for our examples. Unlike the myopic best-response dynamic, which starts each new step on the 45-degree line, the L map starts the next step between $b(M)$ and the 45-degree line. The initial point of each step is denoted with a small dot. The first step starting at M_1^e (denoted M in the figure) is the same for both dynamics; compare figure 2*a* and *b* with figure 3*a* and *b*. But the second step under the L map begins halfway between $b(M_1^e)$ and the 45-degree line, and the third step begins a third of the way between $b(M_2^e)$ and the 45-degree line and so on. The L map predicts that the corner equilibrium is unstable and the interior equilibrium is stable for both $\Gamma(2.47222)$ and $\Gamma(3.86957)$.

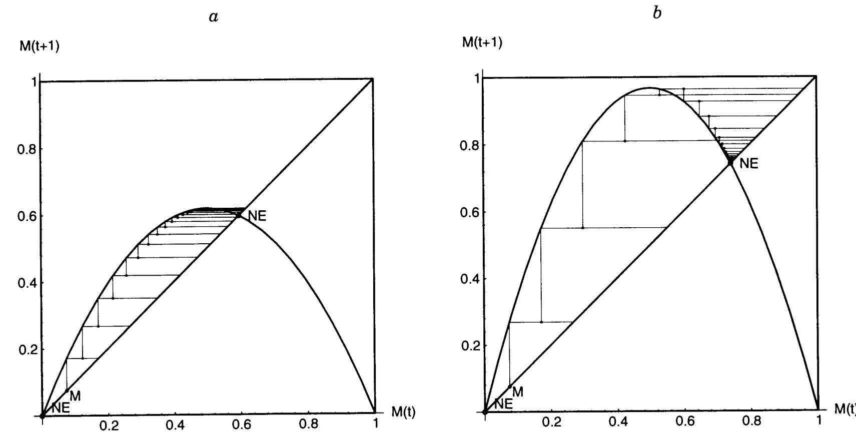

FIG. 3.—*a*, L map for $\Gamma(2.47222)$. *b*, L map for $\Gamma(3.86957)$

The L map has the desirable property that its attractor set contains the unique element $\{1 - (1/\omega)\}$ for $\omega \in (1, 4]$. The interior equilibrium is always globally stable under the L map in $\Gamma(\omega)$. Hence, one would conclude that increasing ω in $\Gamma(\omega)$ will increase the observed median level of effort, M.

III. Adaptive Behavior

We consider both the myopic best-response dynamic and the L map to be selection dynamics rather than reasonable models of adaptive behavior. In our view, a realistic model of adaptive behavior would have to allow for heterogeneity and random exploration. A dynamical system consisting of stochastic difference equations for each player can have properties different from those of a representative deterministic difference equation system. Searching through the vast number of possibilities and then actually characterizing the solution paths or attractor sets are daunting tasks.

Fortunately, Milgrom and Roberts (1991) have developed an elegant and general theory of adaptive learning that does not require one to commit to any specific system of stochastic difference equations. A sequence of actions is consistent with adaptive learning if player i eventually chooses only actions that are nearly best responses to some probability distribution over the other agents' actions, where near zero probability is assigned to actions that have not been played for a sufficiently long time. If behavior is consistent with this concept of adaptive learning, then, as Milgrom and Roberts show, the observed sequence of actions will converge to the set of serially undominated action combinations (see also Bernheim 1984; Moulin 1986; Gul 1990). One of the important results of their analysis is that convergence to equilibrium can be almost solely a property of the game being studied: "nearly everything" converges to equilibrium in Cournot's duopoly model and in their general equilibrium with gross substitutes model.

If human behavior in $\Gamma(\omega)$ is consistent with adaptive learning, then the observed sequence of actions will converge to $\mathbf{U}(\omega)$. Notice that all the relaxation algorithms are consistent with adaptive learning (recall figs. 1–3). While these selection dynamics make much stronger—even implausible—assumptions about behavior, they do make more precise predictions in $\Gamma(\omega)$. Since the area contained in $\mathbf{U}(\omega)$ varies inversely with ω, Milgrom and Roberts's general theory can also make very precise predictions in $\Gamma(\omega)$. In fact, as $\omega \rightarrow 1$, the set $\mathbf{U}(\omega) \rightarrow [0, 0]^2$, which is the unique equilibrium of the limiting game $\Gamma(1)$. Conversely, $\mathbf{U}(4) = [0, 1]^2$, which is the space of feasible outcomes.

IV. Experimental Design

Our experiment consists of two treatments: sessions 1–2 are $G(2.47222)$ and sessions 3–8 are $G(3.86957)$. The game $G(\omega)$ is derived from $\Gamma(\omega)$ using the parameters $c_1 = \$0.50$ and $c_2 = \$1.00$ in equation (1). Payoffs were rounded to the nearest ten-thousandths of a dollar. The sessions repeated $G(\omega)$ 40 periods, which was announced at the beginning of the session. (The initial session, session 3, was run for 70 periods.)

Figure 4 is a halftone image of the main screen used in the experiment. On the computer the box and two "gutters" are in blue. Hence, we call this graphical user interface the "blue box." Once the subject has clicked on the blue box, he can slide the mouse on the mouse pad and read the payoffs associated with all feasible combinations of (e^i, M). The gutters above and to the right of the blue box allow the subject to adjust either e^i or M against a fixed value of M or e^i, respectively. This is useful when searching for a best response to M or checking the security of e^i. We believe that the blue box interface is an effective way to communicate a best-response function to undergraduate students.

While the rotation of the mouse ball is an analog process, the monitor used VGA graphics to display this information. The box formed by the intersection of the vertical and horizontal line in figure 4 is 4×4 pixels; the rotation of the mouse ball moves at a ratio of one–two hundredth of an inch per pixel, and the blue box itself is 360×360 pixels. Hence, the blue box interface restricts the action

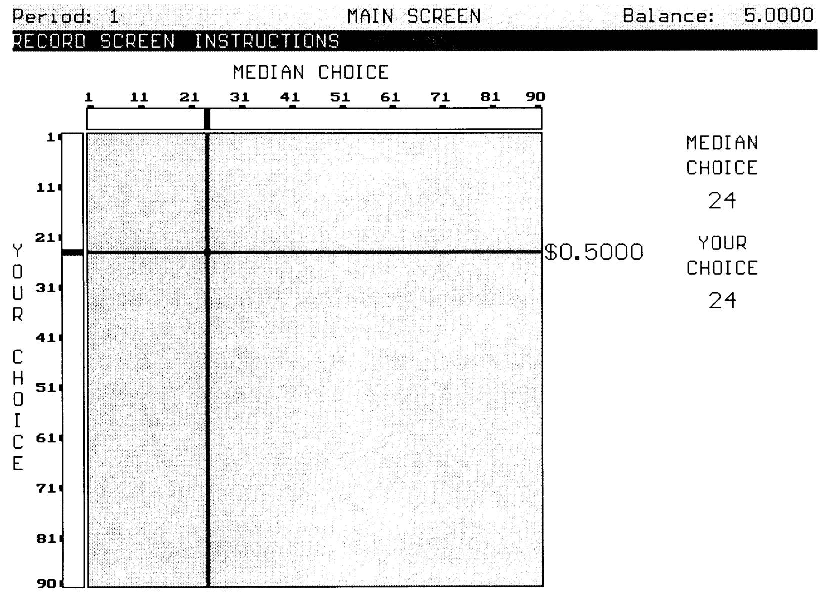

FIG. 4.—Graphical user interface used to communicate the best response function. Using a mouse, subjects can change "YOUR CHOICE" and "MEDIAN CHOICE" independently or both simultaneously.

space to 90 feasible choices. This restricts the players to a finite action space.

Let $\Phi = \{1, \ldots, 90\}$ denote the subjects' finite set of actions. The function f: $\Phi \rightarrow \mathbf{E}$ mapping subjects' actions into the unit interval is $f(e^i) = (90 - e^i)/89$. Notice that this flips the best-response function and the 45-degree line and moves the corner equilibrium to the lower right corner of the blue box. We framed the game this way to increase the chances of observing an interesting initial condition. Let Φ^n denote the set of all possible action combinations. There were five players in each session, so there were 90^5 elements in Φ^5.

The strict equilibria for $G(2.47222)$ are (37, 37) and (90, 90). The strict equilibria for $G(3.86957)$ are (24, 24) and (90, 90). The values for ω were chosen to ensure that the interior equilibria existed in pure strategies and, hence, remained strict given the 90×90 grid, which is why we have inflicted 2.47222 and 3.86957 on the reader. Using f to map e back to the unit interval implies that effort in the interior equilibrium increases from approximately .5955 to .7416 as ω goes from 2.47222 to 3.86957. Both equilibria have a payoff of $0.50 per subject per period.

When the action space is Φ, the set of serially undominated actions is $\{35, 36, \ldots, 89, 90\}$ and $\{4, 5, \ldots, 89, 90\}$, respectively. Hence, individual rationality or behavior consistent with adaptive learning implies that subjects will not choose $e^i \in \{1, 2, \ldots, 34\}$ in $G(2.47222)$ and will not choose $e^i \in \{1, 2, 3\}$ in $G(3.86957)$ either initially or after behavior has converged.

The discreteness introduced by using a graphical user interface has some important implications for our analysis. The analysis no longer predicts chaos since it is impossible to construct a bounded path that never repeats any past value. There are only 90 elements in Φ, so M_t must repeat itself at least once in 91 periods. Analysis of $G(3.86957)$ reveals that the myopic best-response dynamic converges to a stable seven cycle when $M_1 \in \Phi \backslash \{1, 24, 67, 90\}$. The attractor set is the sequence $\{6, 72, 34, 10, 59, 12, 53\}$. The attractor set for $M_1 \in \{24, 67\}$ is the interior equilibrium and for $M_1 \in \{1, 90\}$ is the corner equilibrium.

The instructions were read aloud while the subjects followed along on their monitors (see App. A for the text of the instructions). The instructions covered the general information about the experiment as well as the use of the graphical user interface. There were three screens used in the experiments: the instructions screen, containing a copy of the instructions; the record screen, where the history of play was recorded; and the main screen, where subjects studied the payoff matrix and made their decisions.

After the instructions, a questionnaire was given to the subjects. The questionnaire had two sections. The first required the subjects to use their main screen to determine the payoffs for various combinations of their individual choice and the median choice for their group. These values were a subset of the values in the table forming a coarser grid over the payoff surface. Specifically, rows and columns 1, 15, 30, 45, 60, 75, and 90 of the payoff table were included on the questionnaire. The second section had the subjects calculate the median for two sets of five numbers.

The experiment was conducted in the Texas A&M University economic science laboratory. The laboratory uses networked 386SX personal computers linked over an IBM token ring with Novell software. Seating at the terminals was determined by lot. Forty subjects participated in the experiment, five in each session. All were recruited from undergraduate economics courses at Texas A&M. The sessions took about 2 hours to conduct. If the subjects coordinated on either the corner or the interior equilibrium for all 40 periods, they would each earn \$20.

V. Experimental Results

Figures 5 and 6 report the empirical distribution function for period 1 of the $G(2.47222)$ and $G(3.86957)$ sessions, respectively. The data

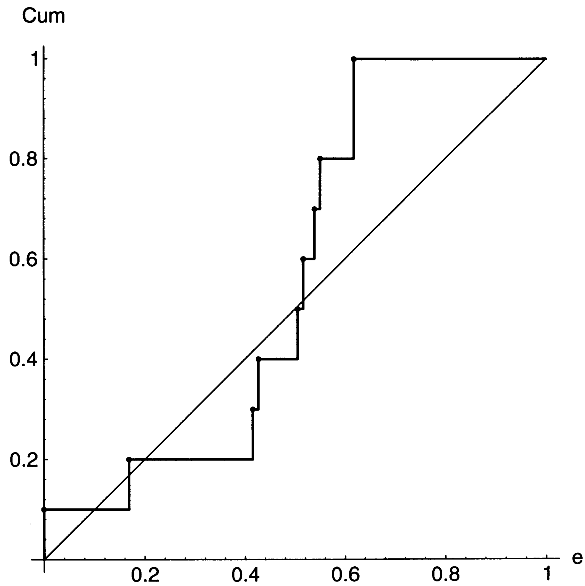

FIG. 5.—Period 1 empirical distribution function for $G(2.47222)$ sessions. The T-statistic of 0.378 rejects uniform play at the 10 percent level of statistical significance.

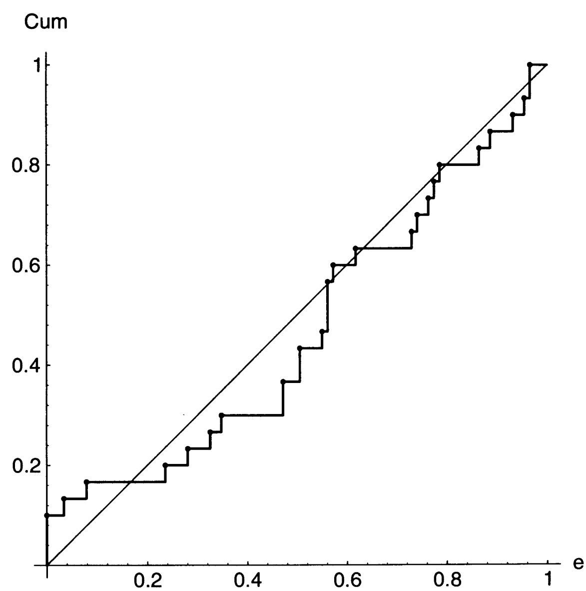

FIG. 6.—Period 1 empirical distribution function for $G(3.86957)$ sessions. The T-statistic of 0.167 fails to reject uniform play at usual significance levels.

have been transformed back into the unit interval using $f(e^i)$. Recall that the set of strictly dominated actions in $G(2.47222)$ is (.618, 1] and in $G(3.86957)$ is (.967, 1]. No subject played a strictly dominated action in the initial period, which can be seen in the figures by noting that the cumulative frequency achieves the value of one at .618 and .966, respectively.

Figure 5 reveals that 80 percent of the initial play in $G(2.47222)$ occurs between .40 and .62. When one is reading figure 5, it is helpful to know that the figure plots the period 1 results of two sessions of five subjects each, so each action accounts for .1 of the cum. For example, the step to .1 at zero denotes one subject whose initial action was consistent with the corner equilibrium. The Kolmogorov T-statistic for a null hypothesis of uniform play is .378, which exceeds a critical value of .369 at the 10 percent significance level (see Conover 1980). Hence, we reject uniform play in the $G(2.47222)$ sessions.

The period 1 empirical distribution function for the $G(3.86957)$ sessions is closer to the uniform distribution (see fig. 6). Recall that the figure plots the period 1 results of six sessions of five subjects each. The Kolmogorov T-statistic for a null hypothesis of uniform play is .167, and the critical value is .218 at the 10 percent significance level. Hence, we fail to reject uniform play in $G(3.86957)$. Comparing figures 5 and 6 leads us to conclude that the principle of individual rationality did influence behavior in the initial period of the experiment.

While four subjects chose an action consistent with the corner equilibrium, only one subject chose an action consistent with the interior equilibrium. Hence, we doubt that subjects imposed a mutual consistency condition on their initial behavior.[3] None of the sessions resulted in a mutual best-response outcome in period 1.

Figures 7 and 8 graph the observed medians for sessions 1 and 2 in the phase space. Sessions 1 and 2 used $G(2.47222)$. The initial median, denoted $M(1)$ in the figure, is used as an initial condition for the L map, and the path of the L map is also graphed. The large dots denote a pair (M_t, M_{t+1}) observed in the session, and the thick line connecting the large dots indicates the actual sequence of play. Note that if M_1 and M_2 were the same, then the data point would lie on the 45-degree line at $M(1)$; if $M_2 = b(M_1)$, the initial data point would lie on $b(M)$ at the vertical ray from $M(1)$. As the figures illustrate, both the data and the L map quickly converge to the interior fixed point of the L map.

[3] It takes only a few seconds for someone familiar with the blue box technology to find all the symmetric equilibria of the game. So it was certainly possible for the subjects to check for mutual consistency.

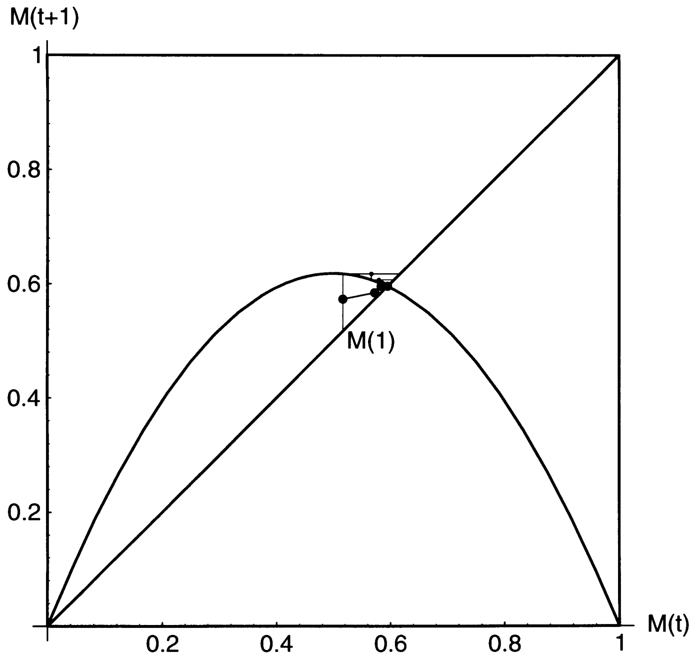

FIG. 7.—Session 1 observed and simulated medians. The large dots denote a pair (M_t, M_{t+1}) actually observed in the session, and the thick line connecting the large dots indicates the actual sequence of play. The small dots and thin lines graph the L map with initial condition $M(1)$.

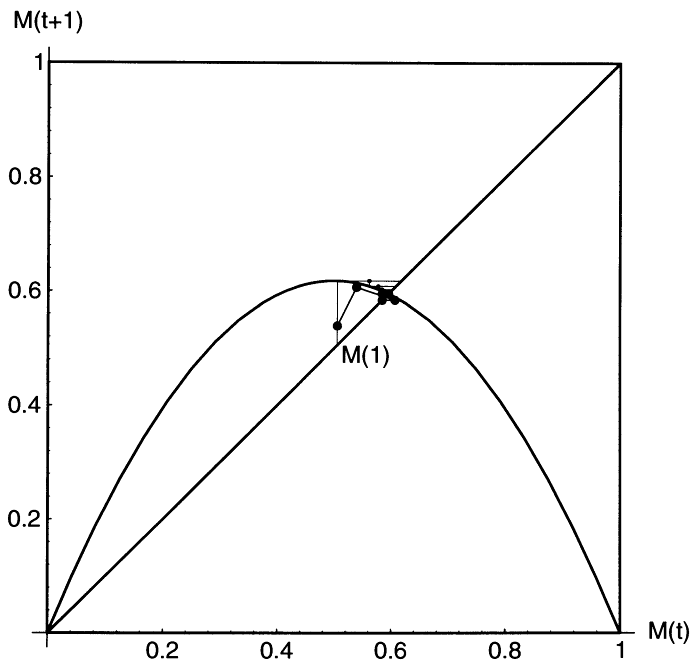

FIG. 8.—Session 2 observed and simulated medians

Behavior in session 1 results in a median consistent with the interior equilibrium from period 4 through the end of the session and mutual best-response outcomes from period 7 through the end of the session. Behavior in session 2 results in a median consistent with the interior equilibrium from period 6 through the end of the session. However, subject 4 in session 2 wanders about—even playing a strictly dominated action six times—and, hence, the session results only in a few mutual best-response outcomes. The other four subjects are all playing an action consistent with the interior equilibrium by period 12 (see App. B). As predicted by all the selection dynamics considered above, the interior equilibrium of $G(2.47222)$ appears to be a stable equilibrium.

Sessions 3–8 used $G(3.86957)$. The function $G(3.86957)$ discriminates between the myopic best-response dynamic, which predicts that both equilibria are unstable, and the L map, which predicts that the corner equilibrium is unstable and the interior equilibrium is stable. Figures 9–14 graph the observed median for sessions 3–8 in the phase space. As the figures reveal, the observed behavior does not contradict the prediction that the interior equilibrium is stable. Moreover, the data strongly reject the prediction that the interior equilibrium is unstable. Both the L map and the data quickly converge on the interior fixed point of the L map.

Behavior in session 3 results in a median consistent with the interior equilibrium from period 3 through the end of the session and a mutual best-response outcome in periods 9 and 14–18 and from period 22 through the end of the session. Behavior in session 6 results in a median consistent with the interior equilibrium from period 22 through the end of the session, and four of five subjects give a best response from period 26 through the end of the session. The behavior in the other four sessions is bracketed by the examples of session 3, which converged quickly to a mutual best-response outcome, and session 6, which converged occasionally to a mutual best-response outcome. Subjects in sessions 4, 5, 7, and 8 implement a mutual best-response outcome from period 28 to the end of the session (see App. B).

The data clearly reject the hypothesis that the interior equilibrium is unstable. The median in all six sessions with $G(3.86957)$ converged to the interior fixed point of the L map. The corner equilibrium is unstable and the interior equilibrium is stable for both treatments in our experiment.

Does increasing ω increase the median level of effort? Initially, sessions 3 and 8 actually have a lower median level of effort than sessions 1 and 2 (compare figs. 9 and 14 with figs. 7 and 8). However, the predicted relationship emerges in period 2 and is then never

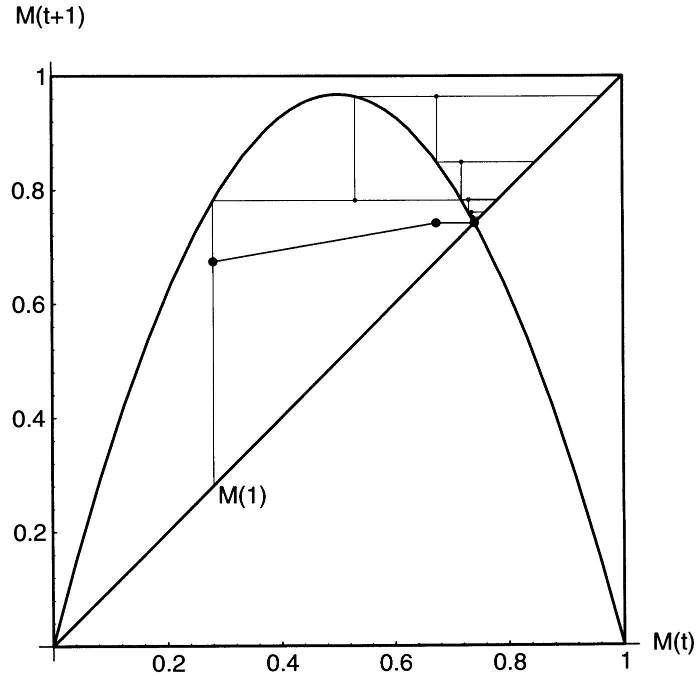

FIG. 9.—Session 3 observed and simulated medians

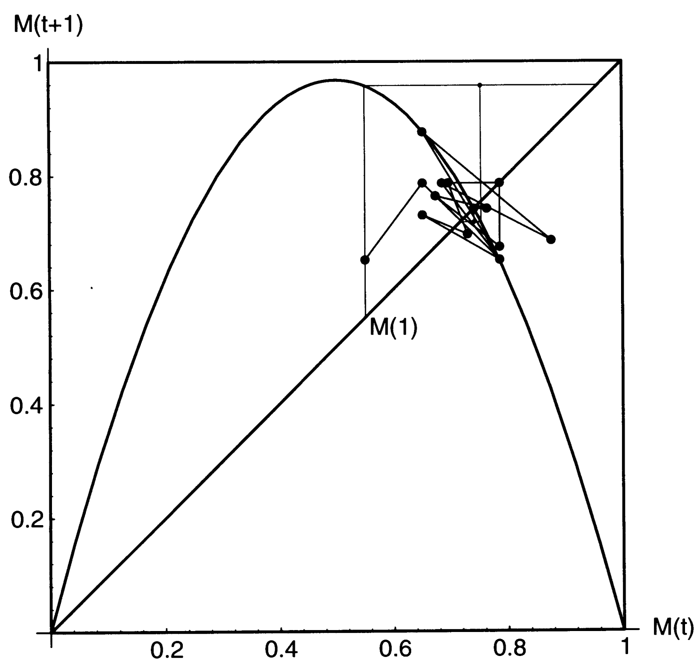

FIG. 10.—Session 4 observed and simulated medians

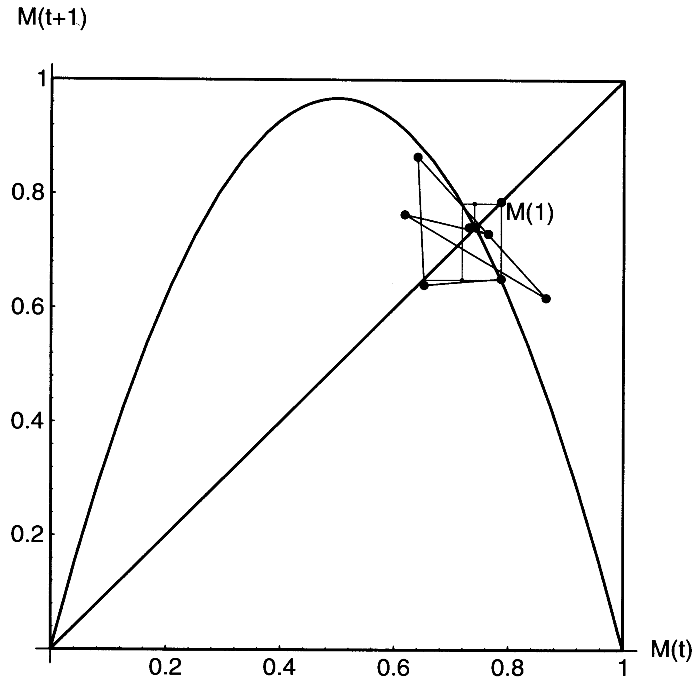

FIG. 11.—Session 5 observed and simulated medians

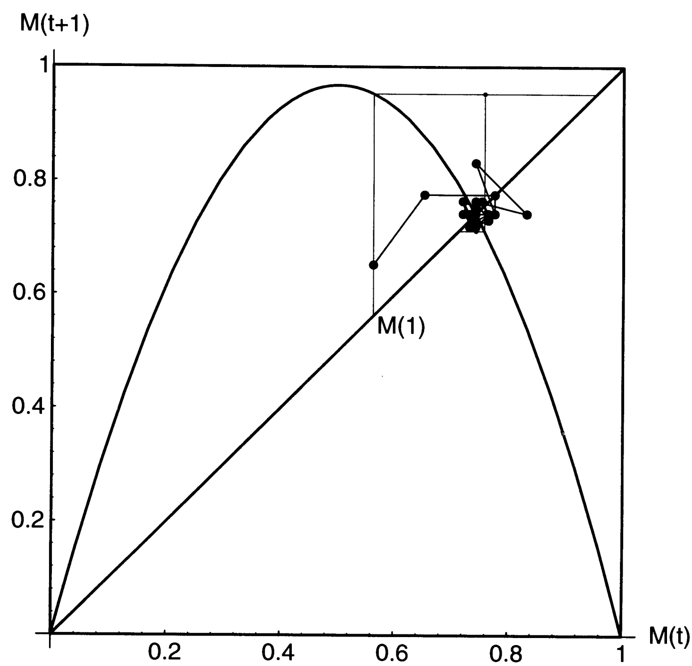

FIG. 12.—Session 6 observed and simulated medians

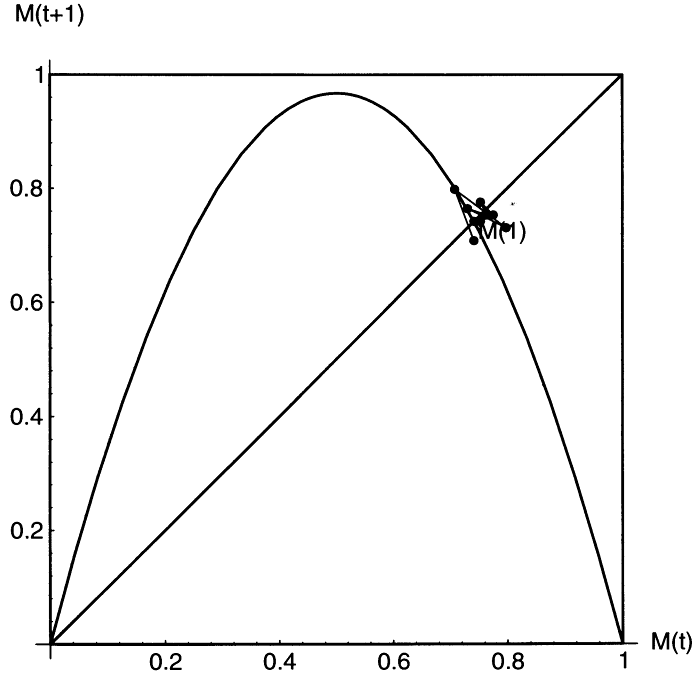

FIG. 13.—Session 7 observed and simulated medians

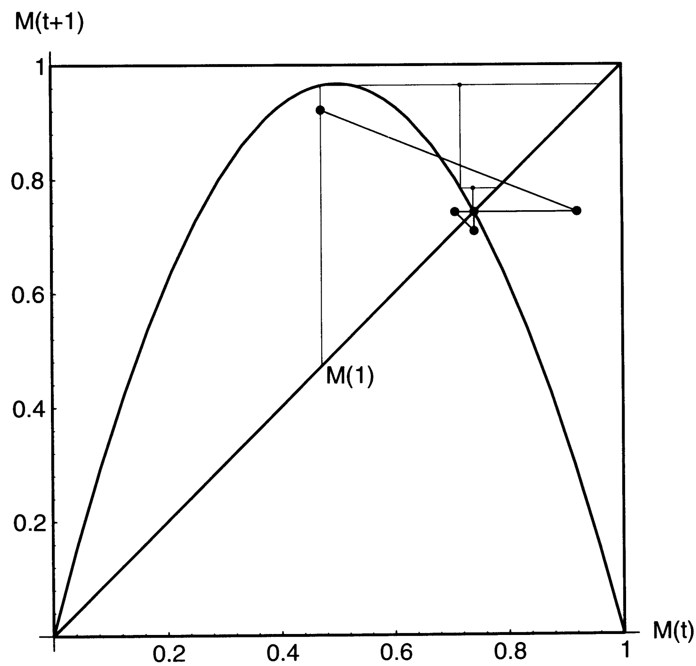

FIG. 14.—Session 8 observed and simulated medians

violated. The data are consistent with the a priori comparative static prediction that increasing ω increases observed effort after some transition to equilibrium.

VI. Conclusion

The inertial selection dynamic accurately predicts the behavior observed in our experiment. The myopic best-response dynamic does not. Given our results, an accurate selection theory must characterize the interior equilibrium of $\Gamma(\omega)$ as a stable fixed point of the selection dynamic. Observed behavior is inertial, and an accurate selection dynamic must reflect this inertia. Whether this inertia is increasing with the reciprocal of time, as in the L map, is a question for future research.

It is also an open question whether the L map is an accurate model of adaptive behavior. A point theory rather than an area theory or a distribution theory can accurately explain the observed behavior in our experiment. Hence, one is tempted to use the L map as a model of adaptive behavior (see also Boylan and El-Gamal [1993, p. 212], who use a Bayesian analysis to conclude that an inertial selection dynamic, specifically fictitious play, is "infinitely more likely" than the myopic best-response dynamic). Marimon and Sunder's (1993) mixed results in favor of least-squares learning make us cautious about giving in to this temptation.

Finally, this experiment does not contradict the traditional view of stability analysis. Behavior always converged to the unique stable fixed point of the inertial selection dynamic. It does so remarkably quickly. In game $\Gamma(\omega)$, it seems reasonable to abstract from the evolutive process and to conduct comparative static exercises. One can be confident that the transition to equilibrium will be brief.

Appendix A

Text File for Graphical User Interface

WELCOME!

This is an experiment in the economics of strategic decision making. Various research foundations have provided funds for this research. If you follow the instructions and make good decisions, you may earn a considerable amount of money, which will be paid to you in cash.

THE LOGITECH MOUSE

You will be making choices using a Logitech mouse, which should be in the middle of your table. If you cannot find the mouse, please raise your hand.

Hold the mouse in a relaxed manner with your thumb and little finger on either side of the mouse. Rest your wrist naturally on the table surface. To move the mouse, let your hand pivot from the wrist. Use a light touch.

In order to participate in this experiment, you will need to be able to POINT, move the cursor on to an object by sliding the mouse, and CLICK, push any one of the mouse buttons. We will call pointing at an object and then clicking your mouse CLICKING ON an object displayed on the screen.

In order to display the next page, slide your mouse so that the pointer is on PAGE DOWN, located on the blue bar below, and click any button. To review a page, CLICK ON PAGE UP.

NO TALKING

As part of the scientific method in this experiment it is important that you remain silent and do not look at other people's work. If you have any questions or need assistance of any kind, please raise your hand and an experimenter will come to you. If you talk, laugh, exclaim out loud, etc., you will be asked to leave the experiment and you will not be paid. We expect and appreciate your cooperation.

GENERAL

In this session there will be five participants in each experiment. There will be 40 market periods. In each period, every participant will pick a value of X. The values of X you may choose are the whole numbers from 1 to 90. The value of YOUR CHOICE of X and the value of the MEDIAN CHOICE of X chosen by all the participants within the experiment will determine the payoff you receive for that period.

THE MEDIAN

The MEDIAN CHOICE is determined as follows. The choices made by the five participants are ordered from smallest to largest in numerical order. The median value is the third from the bottom or the third from the top of the ordered choices. For example, to find the median of the five numbers—

93, 92, 94, 99, 92

—arrange the numbers in ascending order—

92, 92, 93, 94, 99

—find the third choice, either counting from the first number forward or the last number back, of the ordered choices and that is the median value. In this example, the median value is 93.

MAIN SCREEN

You are provided with a screen which tells you the potential payoffs you may receive. This screen, labeled MAIN SCREEN at the top, contains three active boxes that may help you to make your decision. In addition, CLICKING ON the light blue bar at the top of the screen on the word RECORD SCREEN will take you to the RECORD SCREEN or CLICKING ON INSTRUCTIONS, will bring you to this INSTRUCTION screen. By CLICKING ON the word RETURN in the lower right hand corner you can return from these two screens to the MAIN SCREEN. Also displayed are the current PERIOD and the current BALANCE of your earnings are displayed in the upper corners of the screen.

The three active boxes are the large blue square, the vertical bar on its left, and the horizontal bar above it. By CLICKING ON one of these three boxes you can view the payoff associated with any hypothetical combination of YOUR CHOICE of X and the MEDIAN CHOICE of your experiment. The horizontal bar represents all the possible MEDIAN values, the vertical bar represents all the possible values of YOUR CHOICE, and the large blue square represents all their possible combinations. The payoff, in dollars, associated with any possible combination of YOUR CHOICE and the MEDIAN CHOICE of your experiment is displayed to the right of the large blue square.

CLICKING ON any of these three boxes will add several things to the screen display. A vertical green line and a horizontal green line intersecting at a small yellow square to form a set of cross hairs in the large blue square. Two small green boxes, one in the vertical bar next to the YOUR CHOICE label on the left of the MAIN SCREEN and one in the horizontal bar under the label MEDIAN CHOICE at the top of the MAIN SCREEN. And exact values under the labels YOUR CHOICE and MEDIAN CHOICE at the right of the large blue square. The vertical green line and green box in the horizontal bar represent the hypothetical MEDIAN CHOICE. The horizontal green line and green box in the vertical bar represent YOUR CHOICE. The small yellow square is located at the intersection of the hypothetical MEDIAN CHOICE and the value of YOUR CHOICE. Each intersection represents a potentially different payoff.

In a moment you will view the MAIN SCREEN by CLICKING ON the words MAIN SCREEN in the light blue bar at the top of the screen. While you are in the MAIN SCREEN try CLICKING ON the three active boxes. You return to the INSTRUCTIONS by CLICKING ON the word RETURN on the light blue bar at the top of the screen, during the actual experiment this RETURN box will not be on the MAIN SCREEN.

CLICK ON the words MAIN SCREEN, now.

By CLICKING ON the large blue square you select both a hypothetical MEDIAN CHOICE and a value for YOUR CHOICE to be displayed. Whenever you return to the MAIN SCREEN from one of the other two screens the last values you had selected will still be displayed. By CLICKING ON either of the two bars you are selecting only one of the two values. The other value is determined as follows.

If you have already made a selection for both values in the current period the last selection you made will be displayed. At the beginning of Period 1 the unselected value will be set equal to one. For all subsequent periods, if the value of YOUR CHOICE is unselected, YOUR CHOICE will be set equal to one (1). For all subsequent periods it is set equal to the value you selected for YOUR CHOICE in the previous period. If the value of the hypothetical MEDIAN CHOICE was unselected then it is set equal to one in the first period and for subsequent periods it is set equal to the previous period's actual MEDIAN CHOICE.

In summary, the value of the hypothetical MEDIAN CHOICE and the value of YOUR CHOICE are both represented in three ways. First, by a green line in the large blue square. Second, by a green box in the horizontal or vertical bar. And third, by a numeric display of the exact value under the appropriate label to the right of the large blue square.

The difference in the three active boxes is in what they control. CLICKING ON the horizontal bar allows you to change the hypothetical value of the MEDIAN CHOICE by moving its green box to the right or left with the mouse while leaving the value of YOUR CHOICE unchanged. CLICKING ON the vertical bar allows you to change the value of YOUR CHOICE by moving its green box up or down with the mouse while leaving the hypothetical value of the MEDIAN CHOICE of your experiment's choices unchanged. CLICKING ON the large blue box allows you to change both a hypothetical value for the MEDIAN CHOICE and the value of YOUR CHOICE by moving the small yellow square right, left, up, down or diagonally with the mouse. As you change the hypothetical MEDIAN CHOICE, the value of YOUR CHOICE, or both, the representations of these values on the screen will change to reflect these values and the payoff associated with the new hypothetical combination will be displayed on the right.

IF you CLICK while in any of these active boxes the arrow cursor will return to the screen and an ACCEPT box will appear on the right hand side of the screen. This then gives you three options. First, it will allow you to select a different active box by CLICKING ON that box. Second, you may view either of the other two screens, the RECORD SCREEN or the INSTRUCTIONS by CLICKING ON the appropriate title in the light blue bar near the top of the screen. The third option is to make your choice of X for the current period.

This begins by making sure that the value you have chosen for X is currently displayed as YOUR CHOICE on the screen and CLICKING ON the ACCEPT box, you will then have an opportunity to go back to the active boxes and change the value of YOUR CHOICE should you make a mistake or wish to change the value of YOUR CHOICE for the current period. This is discussed fully in the next section, MAKING YOUR CHOICE.

MAKING CHOICES

When the ACCEPT box appears on your screen, CLICKING ON the ACCEPT box will bring up on the screen two CONFIRM? boxes. A red box labeled NO and a green box labeled OK. If you CLICK ON the OK box the value currently displayed as YOUR CHOICE will be your choice of X for that period. CLICKING ON the NO box cancels your choice if you wish to return to the active boxes and change the value for YOUR CHOICE. Once you have confirmed your choice by CLICKING ON the OK box the value of YOUR CHOICE cannot be changed for the current period.

REMEMBER, you are only choosing YOUR CHOICE. The hypothetical MEDIAN CHOICE that is displayed on your screen is there to inform you of the payoffs associated with various combinations of MEDIAN CHOICE values and values of YOUR CHOICE. The actual median, the one that along with YOUR CHOICE determines your actual payoff, is determined by all five of the values of X chosen by the five participants in your experiment for that period.

THE OUTCOME

When all the participants in all the experiments have made their choices for a period, the MEDIAN for each experiment will be calculated and the individual payoffs for each participant will be determined. This period outcome will be displayed on your MAIN SCREEN as follows. A red vertical line will indicate the actual value of the MEDIAN CHOICE for your experiment for that period and a green line the value you chose for YOUR CHOICE. The associated payoff for this combination will be displayed on the right of the blue square, and will be added to your balance as your EARNINGS for that period.

The computer will display the outcome for ten seconds and then switch to the record screen.

RECORD SCREEN

At the beginning of the experiment a loan of $5.0000 will be made to each participant. This loan will be repaid at the end of the experiment by deducting $5.0000 from each participant's BALANCE at the end of the last of the 40 periods, called the TOTAL BALANCE.

Should any participant's BALANCE become negative during the course of the experiment, the experiment will end. The $5.0000 loan will be subtracted from the BALANCE at the end of that period to obtain the ENDING BALANCE which, rounded to the nearest cent, will then be paid to you in cash.

The record screen displays the period outcomes and updates your earnings balance. The following information is displayed on the record screen: PERIOD, YOUR CHOICE, MEDIAN CHOICE, PERIOD EARNINGS, and BALANCE.

We will now view the record screen. It will only contain the loan since you haven't made any choices, but once the screen is full you can PAGE UP, LINE UP, PAGE DOWN, or LINE DOWN to review previous outcomes. Remember to CLICK ON RETURN to return to these instructions. CLICK ON RECORD SCREEN to view the record screen.

During the experiment a period ends once everyone has left the record screen by CLICKING ON RETURN. Remember that you can always return to the record screen from your MAIN SCREEN.

We have now completed the instructions. Again, it is important that you remain silent and do not look on other participant's work. At the end of the experiment you will be paid your ENDING BALANCE, the value of your BALANCE at the end of PERIOD forty (40) minus the $5.0000 interest free loan, in cash.

If you have a question, please raise your hand.

Appendix B

TABLE B1

DATA

$f(e^i) = (90 - e^i)/89$

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Median
			Session 1			
1	35	44	35	52	75	44
2	35	40	39	37	40	39
3	35	38	40	37	39	38
4	90	36	41	37	37	37
5	36	37	38	37	38	37
6	38	37	38	37	37	37
7	37	37	37	37	37	37*
8	37	37	37	37	37	37*
9	37	37	37	37	37	37*
10	37	37	37	37	37	37*
11	37	37	37	37	37	37*
12	37	37	37	37	37	37*
13	37	37	37	37	37	37*
14	37	37	37	37	37	37*
15	37	37	37	37	37	37*
16	37	37	37	37	37	37*
17	37	37	37	37	37	37*
18	37	37	37	37	37	37*
19	37	37	37	37	37	37*
20	37	37	37	37	37	37*
21	37	37	37	37	37	37*
22	37	37	37	37	37	37*
23	37	37	37	37	37	37*
24	37	37	37	37	37	37*
25	37	37	37	37	37	37*
26	37	37	37	37	37	37*
27	37	37	37	37	37	37*
28	37	37	37	37	37	37*
29	37	37	37	37	37	37*
30	37	37	37	37	37	37*
31	37	37	37	37	37	37*
32	37	37	37	37	37	37*
33	37	37	37	37	37	37*
34	37	37	37	37	37	37*

TABLE B1 (*Continued*)

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Median
35	37	37	37	37	37	37*
36	37	37	37	37	37	37*
37	37	37	37	37	37	37*
38	37	37	37	37	37	37*
39	37	37	37	37	37	37*
40	37	37	37	37	37	37*
			Session 2			
1	42	41	90	53	45	45
2	35	35	48	42	45	42
3	36	37	35	34	45	36
4	37	38	35	40	45	38
5	37	38	37	41	45	38
6	35	38	37	37	45	37
7	37	37	37	39	44	37
8	37	37	37	35	37	37
9	37	37	37	38	37	37
10	90	37	37	31	37	37
11	37	37	37	45	37	37
12	37	38	37	37	37	37
13	37	37	37	37	37	37*
14	37	37	37	43	37	37
15	37	37	37	31	37	37
16	37	37	37	52	37	37
17	37	37	37	47	37	37
18	37	37	37	39	37	37
19	37	37	37	71	37	37
20	37	37	37	37	37	37*
21	37	37	37	32	37	37
22	37	37	37	37	37	37*
23	37	37	37	40	37	37
24	37	37	37	61	37	37
25	37	37	37	55	37	37
26	37	37	37	37	37	37*
27	37	37	37	11	37	37
28	37	37	37	37	37	37*
29	37	37	37	81	37	37
30	37	37	37	39	37	37
31	37	37	37	47	37	37
32	37	37	37	37	37	37*
33	37	37	37	38	37	37
34	37	37	37	37	37	37*
35	37	37	37	37	37	37*
36	37	37	37	33	37	37
37	37	37	37	83	37	37
38	37	37	37	49	37	37
39	37	37	37	42	37	37
40	37	37	37	65	37	37

TABLE B1 (*Continued*)

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Median
			Session 3			
1	90	65	59	83	35	65
2	90	20	30	20	46	30
3	43	9	30	24	4	24
4	1	14	24	37	53	24
5	24	15	24	24	40	24
6	24	24	24	26	37	24
7	24	24	24	24	15	24
8	24	24	24	30	24	24
9	24	24	24	24	24	24*
10	24	24	24	25	24	24
11	24	24	24	32	23	24
12	24	24	24	25	24	24
13	24	24	24	17	24	24
14	24	24	24	24	24	24*
15	24	24	24	24	24	24*
16	24	24	24	24	24	24*
17	24	24	24	24	24	24*
18	24	24	24	25	24	24
19	24	24	24	23	24	24
20	24	24	24	23	24	24
21	24	24	24	23	24	24
22	24	24	24	24	24	24*
23	24	24	24	24	24	24*
24	24	24	24	24	24	24*
25	24	24	24	24	24	24*
26	24	24	24	24	24	24*
27	24	24	24	24	24	24*
28	24	24	24	24	24	24*
29	24	24	24	24	24	24*
30	24	24	24	24	24	24*
31	24	24	24	24	24	24*
32	24	24	24	24	24	24*
33	24	24	24	24	24	24*
34	24	24	24	24	24	24*
35	24	24	24	24	24	24*
36	24	24	24	24	24	24*
37	24	24	24	24	24	24*
38	24	24	24	24	24	24*
39	24	24	24	24	24	24*
40	24	24	24	24	24	24*
			Session 4			
1	25	69	41	40	48	41
2	35	32	20	45	1	32
3	11	29	20	45	12	20
4	33	20	17	45	32	32
5	12	32	16	12	12	12
6	53	29	20	45	6	29
7	25	20	20	15	42	20
8	30	32	32	90	30	32
9	25	32	19	45	12	25
10	22	32	22	45	28	28

TABLE B1 (*Continued*)

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Median
11	20	20	17	90	28	20
12	12	20	20	36	26	20
13	25	32	30	32	26	30
14	25	14	33	14	22	22
15	17	39	24	15	24	24
16	24	24	24	24	24	24*
17	24	24	24	24	24	24*
18	24	24	24	24	24	24*
19	24	24	24	24	24	24*
20	24	24	24	24	24	24*
21	24	24	24	24	24	24*
22	24	24	24	24	24	24*
23	24	24	24	24	24	24*
24	24	24	24	24	24	24*
25	24	24	24	24	24	24*
26	24	24	24	24	24	24*
27	24	24	24	24	24	24*
28	24	24	24	24	24	24*
29	24	24	24	24	24	24*
30	24	24	24	24	24	24*
31	24	24	24	24	24	24*
32	24	24	24	24	24	24*
33	24	24	24	24	24	24*
34	24	24	24	24	24	24*
35	24	24	24	24	24	24*
36	24	24	24	24	24	24*
37	24	24	24	24	24	24*
38	24	24	24	24	24	24*
39	24	24	24	24	24	24*
40	24	24	24	24	24	24*
			Session 5			
1	11	45	20	40	13	20
2	11	30	50	20	20	20
3	20	32	17	10	32	20
4	90	32	28	32	32	32
5	35	22	33	9	51	33
6	11	11	90	13	41	13
7	28	35	54	9	50	35
8	21	49	18	22	38	22
9	25	17	29	17	36	25
10	23	28	24	28	19	24
11	20	24	26	24	19	24
12	21	24	24	28	22	24
13	24	24	25	24	24	24
14	24	24	24	24	24	24*
15	24	24	24	24	24	24*
16	24	24	24	24	24	24*
17	24	24	65	24	24	24
18	24	24	24	24	24	24*
19	24	24	24	24	24	24*
20	24	24	24	24	24	24*

TABLE B1 (*Continued*)

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Median
21	24	24	24	24	24	24*
22	24	24	24	24	24	24*
23	24	24	24	24	24	24*
24	24	24	24	24	24	24*
25	24	24	24	24	24	24*
26	24	24	24	24	24	24*
27	24	24	24	24	24	24*
28	24	24	24	24	24	24*
29	24	24	24	24	24	24*
30	24	24	24	24	24	24*
31	24	24	24	24	24	24*
32	24	24	24	24	24	24*
33	24	24	24	24	24	24*
34	24	24	24	24	24	24*
35	24	24	24	24	24	24*
36	24	24	24	24	24	24*
37	24	24	24	24	24	24*
38	24	24	24	24	24	24*
39	24	24	24	24	24	24*
40	24	24	24	24	24	24*
			Session 6			
1	21	40	22	61	90	40
2	1	41	32	4	40	32
3	21	46	50	7	9	21
4	21	85	7	37	17	21
5	31	21	28	24	13	24
6	90	24	6	16	13	16
7	31	21	24	34	20	24
8	22	30	19	22	20	22
9	31	37	25	24	24	25
10	22	30	30	26	23	26
11	24	28	19	22	17	22
12	24	24	26	30	23	24
13	26	28	26	26	21	26
14	24	26	24	22	26	24
15	22	28	23	22	26	23
16	20	30	30	22	22	22
17	24	24	26	25	22	24
18	22	26	25	28	24	25
19	26	24	22	24	21	24
20	26	29	24	24	26	26
21	24	24	22	22	22	22
22	24	23	24	24	28	24
23	24	24	26	24	24	24
24	24	24	24	24	24	24*
25	24	15	23	24	26	24
26	24	24	24	24	24	24*
27	24	27	24	24	24	24
28	24	24	24	24	24	24*
29	24	34	24	24	24	24
30	24	24	24	24	24	24*

TABLE B1 (*Continued*)

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Median
31	24	24	24	24	24	24*
32	24	26	24	24	24	24
33	24	24	24	24	24	24*
34	24	24	24	24	24	24*
35	24	25	24	24	24	24
36	24	24	24	24	24	24*
37	24	25	24	24	24	24
38	24	23	24	24	24	24
39	24	22	24	24	24	24
40	24	26	24	24	24	24
			Session 7			
1	39	45	5	24	4	24
2	27	10	28	24	41	27
3	19	13	18	24	89	19
4	34	48	21	24	25	25
5	22	15	17	22	25	22
6	28	17	17	24	23	23
7	22	17	26	20	21	21
8	23	26	19	24	23	23
9	25	22	29	24	23	24
10	25	32	17	24	23	24
11	24	20	27	24	23	24
12	24	22	24	24	24	24
13	24	24	24	24	24	24*
14	24	24	24	24	24	24*
15	24	24	24	24	25	24
16	24	24	24	24	24	24*
17	24	24	24	24	24	24*
18	24	24	24	24	23	24
19	24	24	24	24	24	24*
20	24	24	24	24	25	24
21	24	24	24	24	25	24
22	24	24	24	24	24	24*
23	24	24	24	24	30	24
24	24	24	24	24	24	24*
25	24	24	24	24	24	24*
26	24	24	24	24	25	24
27	24	24	24	24	23	24
28	24	24	24	24	24	24*
29	24	24	24	24	24	24*
30	24	24	24	24	24	24*
31	24	24	24	24	24	24*
32	24	24	24	24	24	24*
33	24	24	24	24	24	24*
34	24	24	24	24	24	24*
35	24	24	24	24	24	24*
36	24	24	24	24	24	24*
37	24	24	24	24	24	24*
38	24	24	24	24	24	24*
39	24	24	24	24	24	24*
40	24	24	24	24	24	24*

TABLE B1 (*Continued*)

Period	Subject 1	Subject 2	Subject 3	Subject 4	Subject 5	Median
			Session 8			
1	4	87	48	7	90	48
2	8	3	8	22	55	8
3	26	21	40	24	12	24
4	26	24	40	24	9	24
5	30	27	30	24	4	27
6	15	30	21	24	24	24
7	17	24	27	24	24	24
8	24	25	50	11	24	24
9	24	24	10	24	24	24
10	24	24	45	24	24	24
11	24	24	27	24	24	24
12	24	24	23	24	24	24
13	24	24	24	24	24	24*
14	24	24	24	24	24	24*
15	24	24	24	24	24	24*
16	24	24	24	24	24	24*
17	24	24	24	24	24	24*
18	24	24	24	24	24	24*
19	24	24	24	24	24	24*
20	24	24	24	24	24	24*
21	24	24	24	24	24	24*
22	24	24	24	24	24	24*
23	24	24	24	24	24	24*
24	24	24	24	24	24	24*
25	24	24	24	24	24	24*
26	24	24	24	24	24	24*
27	24	24	24	24	24	24*
28	24	24	24	24	24	24*
29	24	24	24	24	24	24*
30	24	24	24	24	24	24*
31	24	24	24	24	24	24*
32	24	24	24	24	24	24*
33	24	24	24	24	24	24*
34	24	24	24	24	24	24*
35	24	24	24	24	24	24*
36	24	24	24	24	24	24*
37	24	24	24	24	24	24*
38	24	24	24	24	24	24*
39	24	24	24	24	24	24*
40	24	24	24	24	24	24*

* Denotes a mutual best-response outcome.

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