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EFFECT OF MARKET ORGANIZATION ON COMPETITIVE EQUILIBRIUM *

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I. INTRODUCTION

Since the use of laboratory experimental techniques in the testing of economic hypotheses seems to have become reasonably well established in recent years, this paper will not attempt to provide any general methodological justification for its existence.[1] A previous study [2] presented the results of ten exploratory pilot experiments in competitive (multitrader auction) market behavior. The major methodological purposes of that study were to (i) test the feasibility of experimental techniques, (ii) synthesize one or more standard experimental designs, and (iii) provide the foundation for a more rigorous empirical examination of several specific hypotheses. The conclusions suggested in that paper were based in part upon hypotheses whose tests and theoretical rationalization were developed after the experimental data had been obtained. The results of such a posteriori testing and theorizing based largely upon un-

* The research reported in this paper was supported by National Science Foundation Grant No. G-24199 to Purdue University. I am indebted to Richard Swensson and John Wertz for computational assistance, and William Starbuck for valuable suggestions and comments.

1. S. Siegel and L. E. Fouraker, *Bargaining and Group Decision Making* (New York: McGraw-Hill, 1960). L. E. Fouraker and S. Siegel, *Bargaining Behavior* (New York: McGraw-Hill, 1963). P. Suppes and J. M. Carlsmith, "Experimental Analysis of a Duopoly Situation. . . ," *International Economic Review*, Vol. 3 (Jan. 1962).

2. V. L. Smith, "An Experimental Study of Competitive Market Behavior," *Journal of Political Economy*, LXX (April 1962).

replicated experiments should be considered highly tentative until such results have been confirmed by further experiments designed specifically to test the particular hypotheses in question.

This paper will report on the results of a series of experiments designed exclusively for the purpose of testing various hypotheses concerning the price equilibrium and adjustment behavior of markets whose organization permits either sellers or buyers, but not both, to engage actively in the higgling and bargaining process. Most retail markets, at least in this country, are characterized by an organization in which sellers post their offers competitively while buyers passively choose among such offers to form exchange contracts. With minor exceptions in such markets, custom precludes buyers from making counter bids in establishing contract prices.

The motivation for the present study stems largely from a pilot experiment [3] in which sellers only were permitted to make quotations. The results of that experiment suggested that although the initial contracts tended to be above the theoretical equilibrium price, all subsequent contracts tended to be executed at prices persistently below the theoretical equilibrium. After the fact, such behavior appeared reasonable. Sellers, desiring to maximize trading profit, should offer to sell at the highest prices they might hope to obtain. Buyers, perhaps fearing that they may be unable to do better, might be expected to accept some of these initial high offers. But as trading proceeds, and buyers learn that by waiting they can take advantage of the competitive pressure on sellers, contract prices may be lowered, possibly to a stable level below the theoretical equilibrium. Such a market has two forms of asymmetry which may operate to the benefit of buyers: the active competitive pressure is on the offers being quoted by sellers, and, in this process, sellers are revealing more information about the prices at which they are willing to sell than are buyers.[4] Buyers either remain silent or passively accept certain of the offers that are tendered. If this reasoning is correct, the results of the pilot experiment should be confirmed by replication of an experiment designed for this purpose. Furthermore, it is to be expected, a priori, that these results will be reversed in an experimental market in which buyers only are allowed to make price quotations.

3. *Ibid.*, pp. 124, 125, 134.

4. My referee notes that the reasoning in this paragraph contradicts the common assertion that administered pricing reacts to the disadvantage of the buyer and that administered purchase pricing (as in labor markets) reacts to the disadvantage of the seller. Also, that the reasoning is reminiscent of the idea that, in von Neuman-Morgenstern terminology, the price-quoter plays a minorant game.

II. The Hypotheses

We define the following trading rule conditions for subsequent reference:

R_S — Sellers are permitted to make offers; buyers are free to accept offers, but are not permitted to make bids.

R_{SB} — Sellers are permitted to make offers and are free to accept bids; buyers are permitted to make bids and are free to accept offers.

R_B — Buyers are permitted to make bids; sellers are free to accept bids, but are not permitted to make offers.

As a consequence of the pilot experimental outcome and speculation thereon, but prior to performing the experiments to be reported here, the following hypotheses were formulated:

H_1: Initial contract prices (defined in advance as those contracts executed in the first trading period) will tend to be ordered

$$P_t^S > P_t^{SB} > P_t^B,\ t = 1,2,\ \ldots\ ,N_1$$

H_2: All remaining contract prices (defined in advance as those contracts executed in trading periods 2, 3, 4 and 5) will tend to be ordered

$$P_t^S < P_t^{SB} < P_t^B,\ t = N_1 + 1,\ N_1 + 2,\ \ldots\ , N$$

where N_1 = number of contracts in trading period 1,

$N - N_1$ = number of contracts in trading periods 2, 3, 4 and 5 combined,

P_t^S = contract price on the t^{th} transaction under condition R_S,

P_t^{SB} = contract price on the t^{th} transaction under condition R_{SB},

P_t^B = contract price on the t^{th} transaction under condition R_B.

Observe that H_1 and H_2 refer to an ordering of the individual contract prices. A third hypotheses, not entirely independent of H_1 and H_2, refers to the ordering of expected equilibrium prices.

H_3: Expected prices in equilibrium will be ordered

$$E(P^S) < E(P^{SB}) < E(P^B).$$

As before, the superscripts refer to the three indicated trading rule conditions. The hypothesis H_3 will be tested in two ways. An ordinary analysis of variance test will be applied to the contract prices in trading periods 4 and 5 (defined in advance as the equilibrium trading periods). Then a first order stochastic difference equation will be estimated for each trading rule condition, using all contract prices occurring under each condition. The expected equilibrium prices implied by these difference equations will then

be used to test H_3. The regression coefficients in these equations will provide a measure of the speed at which contract prices converge, and will be employed to test H_4.

H_4: The speed of convergence to equilibrium will be greater under condition R^{SB} than either R^S or R^B.

Hypotheses H_1 through H_4 will be tested by classical statistical procedures in Section IV. In Section V a Bayesian subjective probability analysis will be used to determine the degree of confidence to be attached to the hypotheses that the ordering relation $P_t^S < P_t^{SB} < P_t^B$ holds for the contract prices in trading periods 4 and 5 (equilibrium).

III. Subjects and Experimental Procedure

The above hypotheses will be tested using data from six experimental sessions consisting of two sessions under each of the three conditions R_S, R_{SB} and R_B. The two replications under R_{SB} served as controls on the R_S and R_B sessions. A total of 144 male students enrolled in three sections of each of two sophomore level courses in economics (which we will call course A and course B) provided the subjects for these sessions.[5] Table I illustrates the over-all experimental design and indicates the combination of experimental condition, course, and number of subjects associated with each experiment session. No subject participated in more than one of the experimental sessions. The sessions were run separately in each of two series separated by several months (one semester).

As a means of controlling on information transfer from earlier to later sessions the following procedures were employed:

1. Every session was performed with "captive" subjects. I never used volunteers. Volunteers were more likely to have heard something about "those experiments conducted by the economics department," and were more likely to have superior motivation, which was not necessary for these experiments.

2. The subjects were given no advance warning of an experimental session. I cleared with the instructor in charge and then appeared on a specified date with equipment, payoff money, and

5. Experimental session 2, shown in Table I, had to be repeated on a second group of Course B subjects at a later date to obtain data under adequate controls. In the first run of session 2 a subject executed a contract in violation of the limit price rules specified in the instructions below. This provided false, uncontrolled public information to the experimental market, and it was decided to invalidate the session. Detailed tests were not performed on the data from this invalidated session, but casual examination revealed that the general results were similar to those obtained from the validated session.

TABLE I

NUMBER OF SUBJECTS AND EXPERIMENTAL CONDITION FOR EACH SESSION

Experimental Session Number	Condition			
	R_S	R_{SB}	R_B	Total Subjects
Course A (20 subjects)	1	3	5	60
Course B (28 subjects)	2	4	6	84
Total Subjects	48	48	48	144

materials prepared. The objective was to control as much as possible the amount of pre-game speculation and information-seeking that could occur. This was important where substantial cash payoffs were employed. (The individual subject payoffs in these experiments ran as high as $6.50 for about 40 minutes of participation).

3. The results and all information concerning the constants of the experiments were suppressed until the design block of six sessions was completed.

4. The experimental sessions discussed in this paper were intermingled with sessions for two entirely different studies involving a variety of different experimental designs, conditions, and information. In this way even if a subject had heard something about a previous session, there was very little chance that the session was identical to the one in which he was to participate.

Each session was begun with a general statement that the group was being asked to participate in a decision-making experiment; that they would not be subjected to any unpleasant stimuli or experiences; and furthermore, that they would have an opportunity to earn real money during their participation. Copies of instructions, printed as an appendix to this paper, were then passed out, and read out loud to the entire group.

After reading the second paragraph of these instructions an assistant passed out the indicated yellow and white limit price cards. Then the remaining instructions were read. Paragraphs 5S and 6S were read in the sessions under condition R_S, 5SB and 6SB under

conditions R_{SB}, and 5B and 6B under condition R_B. Each session consisted of a series of five trading periods. In order to provide some control over end effects, this information was not given to the subjects. In one session — number 5 — a sixth trading period was run as an additional check on the assumption that periods 4 and 5 represented equilibrium behavior. As a means of assuring an orderly trading process, the subjects were asked to raise their hand when they desired to make a bid or offer. I would then skip around calling upon those with raised hands for their bids or offers. I would then repeat each price bid or offer before calling for another. Each quotation was an outstanding bid or offer that could not be withdrawn, until a new quotation had been made, at which time the previous quotation was no longer outstanding. In this way, one quotation at a time was before the group. The subjects were free to alter previous bids and offers in any way they pleased [6] provided that their limit price conditions were not violated. The subjects were given no information as to the number of buyers and sellers, possible prices at which the commodity might, should, or could sell, and so on — no information beyond that provided by the instructions and the limit prices.

Unknown to the subjects the limit buy prices generated the demand schedules, while the limit sell prices formed the supply schedules shown in Figure I. In experiments 1, 3 and 5, 20 subjects were available in each of three sections of course A (Table I). The supply and demand designs are given by SS' (10 sellers) and DD' (10 buyers). In experiments 2, 4 and 6, 28 subjects were available, and the supply and demand designs are given by SS (14 sellers) and DD (14 buyers). Symmetrical demand and supply designs were used throughout the six experiments as a control on other variables, not of interest in the present investigation, that might explain the expected equilibrium biases under the study conditions R_S and R_B. Thus, in each session:

1. The number of buyers is equal to the number of sellers.

2. Equilibrium buyer's rent (consumer surplus) equals equilibrium seller's rent (producer surplus). Differences in these rents

6. In their bilateral bargaining experiments, Siegel and Fouraker require "bargaining in good faith." That is, any bid turned down by a rival may be subsequently accepted by him (see instruction 5, p. 20, in *Bargaining and Group Decision Making, op. cit.*). I have elected to give the trading subjects in these experiments the greatest possible freedom to alter previous bids and offers that were not accepted. This is another aspect of market organization — whether bargaining in good faith, in the above sense, is or is not required — that would be of interest to investigate experimentally.

might affect either the equilibrium level of contract prices or the convergence process.[7]

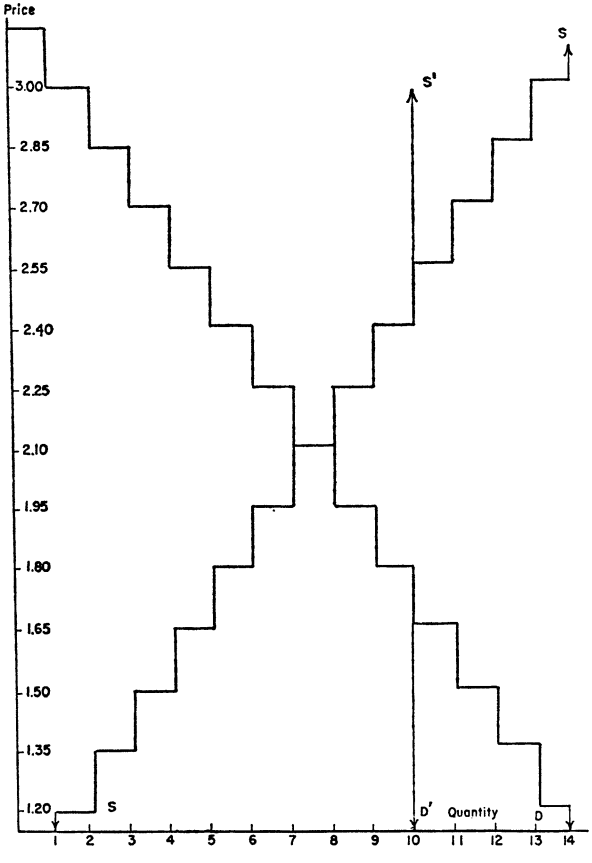

FIGURE I
Experimental Supply and Demand Schedules

IV. EXPERIMENTAL RESULTS AND CLASSICAL TESTS OF HYPOTHESES

Charts 1 through 6 provide a complete series of contract prices in the order in which they were executed in the five trading periods of each session (six trading periods of session 5).

(a) Test of H_1

H_1 will be tested by applying the Jonckheere k-sample test [8]

7. See Smith, *op. cit.*, pp. 119, 120, 130, 134.
8. A. R. Jonckheere, "A Test of Significance for the Relation Between m

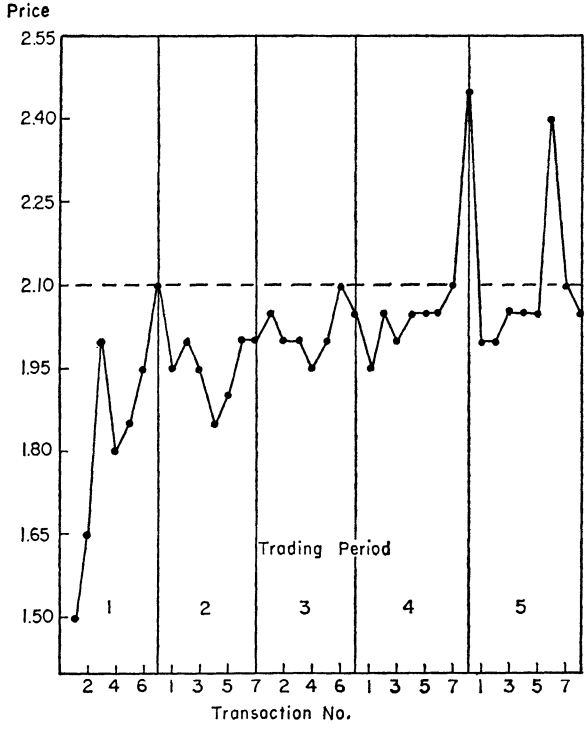

CHART 1
Condition: R_S

against ordered alternatives to the first period contract prices under the three trading rule conditions. The Jonckheere procedure provides a nonparametric test of the null hypotheses that the sample observations $\hat{P}_t^B$, $\hat{P}_t^{SB}$, and $\hat{P}_t^S$ were drawn from three identical populations against the alternative that they came from populations that are in an expected order of increasing value. In this application we have three categories, with 14 observations in the first category (the 14 contract prices in trading period 1 of sessions 5 and 6 under R_B), 15 in the second and 16 in the third. The results are not significant.[9] Hence, we are unable to reject the null hypotheses

Rankings and k Ranked Categories," *The British Journal of Statistical Psychology*, VII, Part II (Nov. 1954), 93–100. Also see A. R. Jonckheere, "A Distribution — Free k-Sample Test Against Ordered Alternatives," *Biometrika*, Vol. 41 (June 1954), pp. 133–45.

9. Using the notation in Jonckheere, "A Test of Significance for the Relation. . .," pp. 94–97, we have $n = 45$, $m = 1$, $k = 3$, $l_1 = 14$, $l_2 = 15$ and $l_3 = 16$. The test statistic is $P = 268$, with mean $\chi_1(P) = 337$ and variance

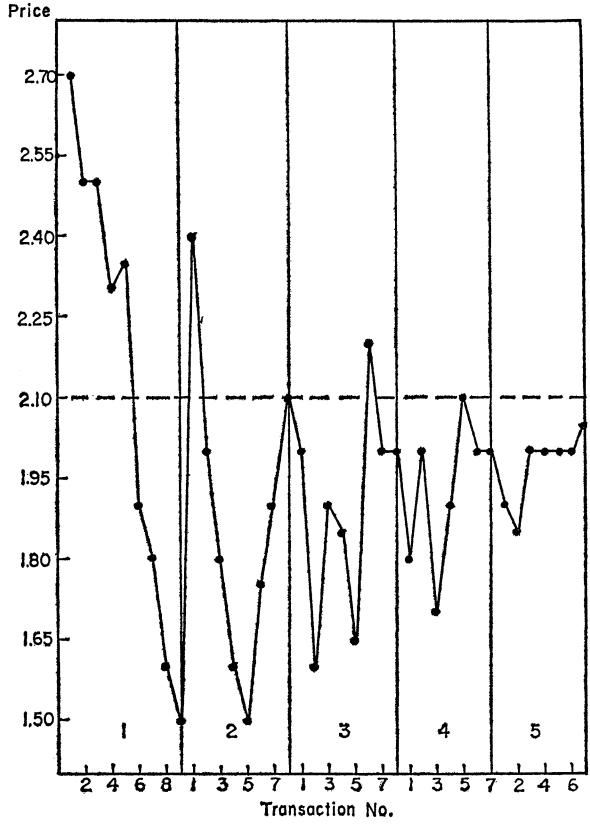

CHART 2
Condition: R_S

that the first period transactions under condition R_S, R_{SB}, and R_B came from the same or identical populations. The failure of the data to support H_1 is evident in sessions 1 and 2, where it would appear that under R_S the initial contracts are as likely to be below as above the theoretical equilibrium.

(b) Test of H_2

H_2 was tested by applying the Jonckheere test to the contract prices in all trading periods beyond the first under the three trading rule conditions. The test is highly significant.[1] The null hypotheses

$\chi_2(P) = 2304$. The unit normal deviate $Z = \dfrac{P - \frac{1}{2} - \chi_1(P)}{\sqrt{\chi_2(P)}} = 1.45$ is definitely not significant.

1. Continuing with the notation of the previous footnote, we have $n =$

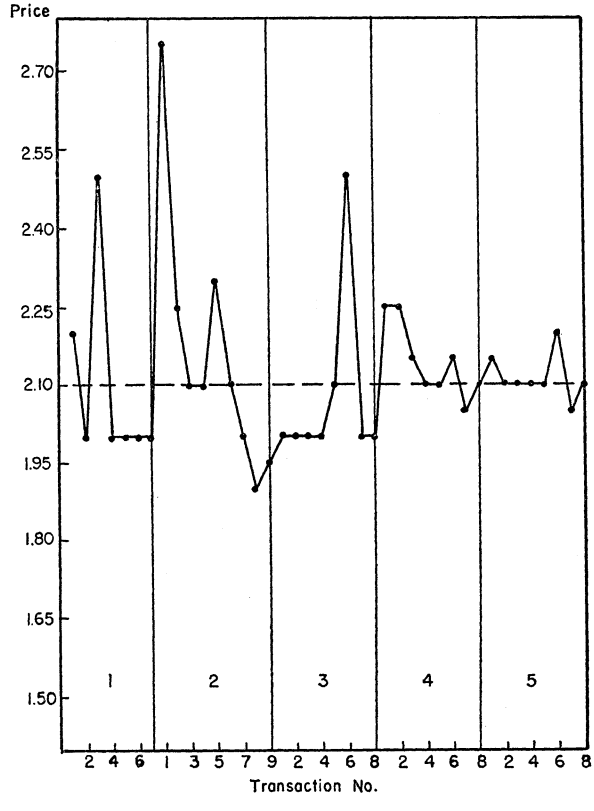

CHART 3
Condition: R_{SB}

that these contract prices came from the same population is rejected at $\alpha < 0.001$.

Since the null complement of H_1 failed to be rejected while the null complement of H_2 was rejected at a very high level of significance, a more general hypothesis than H_2 is suggested by the data, viz,

$$P_t^S < P_t^{SB} < P_t^B, \; t = 1,2, \; . \; . \; . \; ,N.$$

That is, the ordering relationship is expected to hold for all contract prices in all trading periods, with an extremely low probability that its null complement would be rejected if it is true.

It might be reasonable to conjecture that the tendencies implied by this ordering relationship are influenced by the first

190, $m = 1$, $k = 3$, $l_1 = 59$, $l_2 = 62$, $l_3 = 69$, $P = 8731$, $\chi_1(P) = 6003$, and $\chi_2(P) = 17000$. Hence, $Z = 20.9$, which is significant at $\alpha < 0.001$.

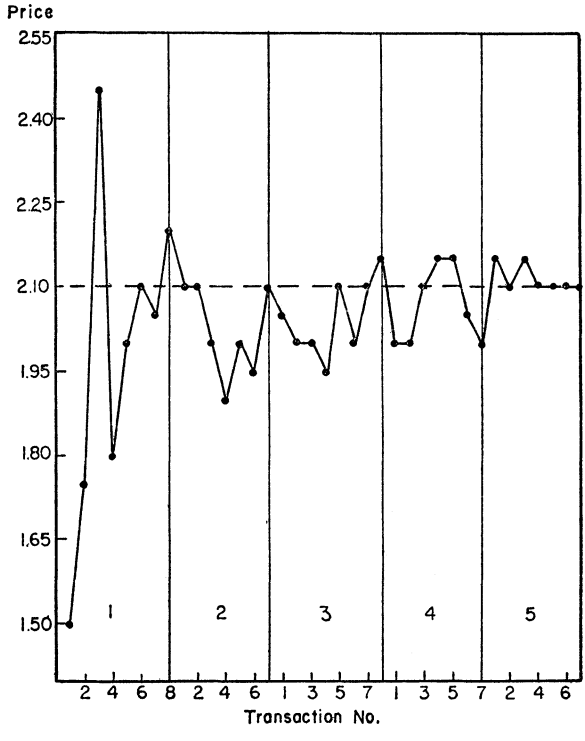

CHART 4
Condition: R_{SB}

contract. For example, if the first contract is below equilibrium, prices might tend to remain below equilibrium. These effects would of course be random in these experiments since no attempt was made to control on the first contract. However, it is of interest to note that the first contract in sessions 1, 4 and 6 happens to have been executed at $1.50 (See Charts 1, 4 and 6). Thus the first contract was the same in one session under each trading condition, yet this fact clearly did not disturb the fundamental tendencies expected under this ordering hypothesis.

(c) Tests of H_3

In testing H_3 by an analysis of variance applied to the contract prices of trading periods 4 and 5, we note from Table I, that a 2 x 3 factorial design is appropriate. Our primary a priori interest is in testing for the effect of trading condition. However, because of differences in the number of subjects available, the two replications

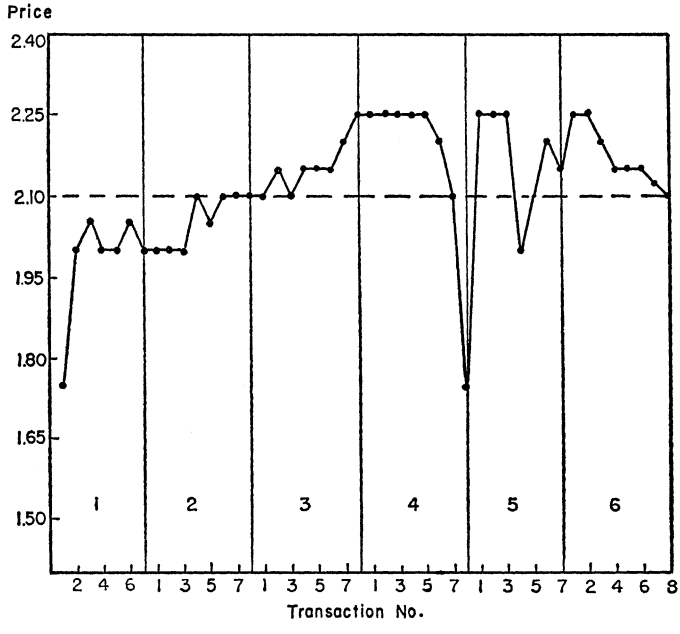

CHART 5
Condition: R_B

under each condition could not be matched. Therefore, the experimental design was deliberately balanced in a way that would permit us also to test for the effect of subject group. The result of our analysis of variance is summarized in the standard form for 2 x 3 designs shown in Table II.

TABLE II

ANALYSIS OF VARIANCE

Source of Variance	Sums of Squares	Degrees of Freedom	Mean Square	F Value
Subject groups	1098.065	1	1098.065	10.894**
Trading rule effect	2706.666	2	1353.333	13.427**
Interaction	465.616	2	232.808	2.310*
Error	8466.637	84	100.793	

** Significant at $\alpha < 0.001$. * Not significant.

Table III shows the mean contract price in trading periods 4 and 5 for each subject group and trading condition. From the components

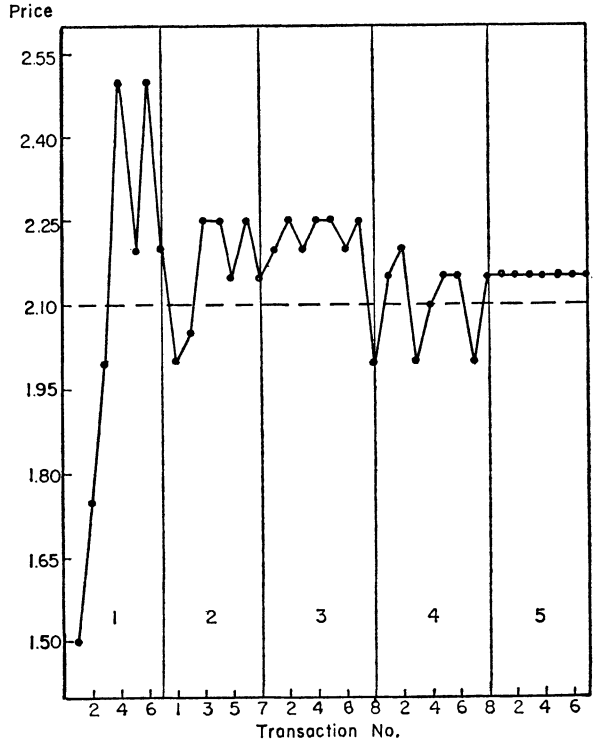

CHART 6
Condition: R_B

of variance shown in Table II, we conclude that there is no significant interaction between the subject group variable and the trading condition variable. Their separate effects are additive. However, both subject group and trading condition are highly significant variables

TABLE III

MEAN CONTRACT PRICE IN TRADING PERIODS 4 AND 5 BY SUBJECT GROUP AND TRADING CONDITION

	R_S	R_{SB}	R_B	Marginal Means
Group A				
20 Subjects	208	213	217	213
Group B				
28 Subjects	195	209	213	206
Marginal means	202	211	215	209

($a < 0.001$). The effect of trading conditions H_S, R_{SB} and R_B are in the direction indicated a priori by H_3 (Table III). We therefore conclude in favor of H_3 by the analysis of variance test. However, from Table III we also note that the mean equilibrium contract prices were lower for the 28 member subject groups than for the 20 member subject groups. This consistent difference is highly significant, by the analysis in Table II, but neither the difference nor its direction were predicted a priori. Since I have no explanation as to why equilibrium contract prices under all trading conditions should be lowered by the addition of 4 submarginal sellers and 4 submarginal buyers (See Figure I), I attribute this result to unanticipated differences in the subject groups.

The second test of H_3 was obtained from least square estimates of the first order stochastic difference equation:

$$(1) \qquad p_{t+1} = ap_t + a_0 + \epsilon_{t+1}, \ |a| < 1, \ t = 0,1,2. \ . \ .$$

where $p_t = P_t - P^0$, P_t is the t^{th} contract price, and P^0 is the theoretical equilibrium price. The general expression for p_t is given by

$$(2) \qquad p_t = a^t p_0 + \frac{a_0(1 - a^t)}{1 - a} + \sum_{s=1}^{t} a^{t-s} \epsilon_s .$$

If ϵ_t has mean zero and variance σ^2, then the mean and variance of p_t is given by

$$(3) \qquad E(p_t) = \frac{a_0(1 - a^{t+1})}{1 - a}$$

$$(4) \qquad S^2(p_t) = \frac{1 - a^{2t}}{1 - a^2} \sigma^2 .$$

If we define $\lim_{t \to \infty} E(P_t)$ as the equilibrium price implied by equation (1), then the expected deviation of this empirical equilibrium from the theoretical equilibrium is

$$(4) \qquad E(p_\infty) = \lim_{t \to \infty} E(p_t) = \frac{a_0}{1 - a}$$

with variance

$$(5) \qquad S^2(p_\infty) = \lim_{t \to \infty} V(p_t) = \frac{\sigma^2}{1 - a^2} .$$

The least squares estimates of a_0 and a in equation (1) for the combined experiments under each trading condition are shown in Table IV. The standard errors are shown in parenthesis. All price deviations are measured in cents.

TABLE IV

STOCHASTIC DIFFERENCE EQUATION PARAMETERS FOR EXPERIMENTAL SESSIONS

Trading Condition	Experimental Session	a	a_0	Correlation Coefficient	Standard Error of Estimate	Number of Observations
R_S	1	0.38	−4.8	0.45	12.3	36
		(0.13)				
	2	0.47	−8.9	0.52	21.5	38
		(0.13)				
R_{SB}	3	0.03	+1.7	0.03	16.8	39
		(0.16)				
	4	0.00	−4.3	0.00	11.9	36
		(0.14)				
R_B	5	0.42	+1.8	0.48	9.3	45
		(0.12)				
	6	0.34	+4.6	0.44	12.0	36
		(0.12)				
R_S	1 and 2	0.46	−6.6	0.51	17.6	74
		(0.09)	(2.3)			
R_{SB}	3 and 4	0.06	−1.0	0.07	14.7	75
		(0.11)	(1.7)			
R_B	5 and 6	0.38	+3.1	0.46	10.6	81
		(0.08)	(1.2)			

The estimates of $E(p_\infty)$ and $S(p_\infty)$ from (4) and (5) are

$$E(p^S_\infty) = -12.2,\ S(p^S_\infty) = 19.8$$
$$E(p^{SB}_\infty) = -1.1,\ S(p^{SB}_\infty) = 14.8$$
$$E(p^B_\infty) = +5.0,\ S(p^B_\infty) = 11.4\ .$$

From these estimates of $E(p_\infty)$ it is clear that $E(p^S_\infty) < E(p^{SB}_\infty) < E(p^B_\infty)$. The null hypothesis that these means came from a common population is rejected by an F test at $\alpha < 0.001$, which supports H_3. The importance of this test is that the estimate of $E(p_\infty)$ under each trading condition utilizes all the information in the samples, as opposed to the previous test which utilized only the a priori assumed equilibrium transactions in periods 4 and 5. The estimates of $E(p_\infty)$ provide predictions of the deviations from the theoretical equilibrium based upon the convergence tendencies reflected in the entire data. If H_3 had not been supported by this calculation then there would be serious question as to the validity of the assumption that trading periods 4 and 5 represented a sufficiently close approximation to equilibrium.

(d) Test of H_4

Examination of the a coefficients in Table IV reveals that a is greater under conditions R_S and R_B than under R_{SB}, as predicted. An F test of the null hypothesis that the three a coefficients shown in the last three rows of column 3, Table IV, came from a common population is rejected at $a < 0.001$. Rejection of the null hypothesis in favor of differences as predicted confirms hypothesis H_4.

The distinct effect of trading condition on the parameters of the stochastic difference equations summarized in Table IV, suggests the possibility of comprehending these results and those of the previous analysis of variance in a more general nonlinear regression hypothesis in which the trading condition variables are treated explicitly. For this purpose we introduce the binary variables S, B and SB, where

$$S = \begin{cases} 1, & \text{Sellers making quotations} \\ 0, & \text{Sellers not making quotations} \end{cases}$$

$$B = \begin{cases} 1, & \text{Buyers making quotations} \\ 0, & \text{Buyers not making quotations.} \end{cases}$$

Thus, when both sellers and buyers are making quotations $SB = 1$, otherwise $SB = 0$. The general nonlinear (in B, S and p_t) or "interaction" hypothesis is

$$(1') \qquad p_{t+1} = a_{00} + a_{0S}S + a_{0B}B + (a_S S + a_B B + a_{SB}SB)\, p_t + \epsilon_{t+1}$$

in which $a_0 = a_{00} + a_{0S}S + a_{0B}B$ and $a = a_S S + a_B B + a_{SB}SB$, from (1) and (1'). The empirical results of this regression are contained in Table V.

TABLE V

Coefficient	Least Squares Estimate	Standard Error	t Ratio	F Ratio
a_{00}	−2.5	0.63	−4.01	16.0
a_{0S}	−4.1	2.35	−1.73	3.0
a_{0B}	5.6	2.54	2.20	4.8
a_S	0.46	0.07	6.15	37.8
a_B	0.38	0.11	3.37	11.4
a_{SB}	−0.77	0.17	−4.55	20.7

a_{0S} is significant at approximately the .04 level. The remaining coefficients are significant at the .01 level or lower. The significance of a_{0S} and a_{0B} implies that the organization variables B and S have an important effect on the *equilibrium states* toward which these markets are tending. The high significance levels of a_S, a_B and a_{SB}

implies that these organization variables have an even more reliable effect on the *speed* with which our experimental markets converge.

V. A Bayesian Subjective Probability Analysis

In this section a Bayesian analysis [2] will be developed for the following hypothesis:

H: In the population of competive market equilibrium transactions (defined as trading periods 4 and 5) contract prices are ordered $P_t^S < P_t^{SB} < P_t^B$. Based upon certain a priori specified probabilities, and the outcome of the experiments, the objective will be to compute the posterior probability that H is true.

In comparing corresponding equilibrium transactions under the conditions R_S and R_{SB}, let O_S be the sample outcome $\hat{P}_t^S < \hat{P}_t^{SB}$ on the t^{th} transaction, and $\bar{O}_S$ be the outcome $\hat{P}_t^S \geqq \hat{P}_t^{SB}$. Similarly under the conditions R_{SB} and R_B, let O_B be the sample outcome $\hat{P}_t^{SB} < \hat{P}_t^B$, and $\bar{O}_B$ be the outcome $P_t^{SB} \geqq P_t^B$. Before performing the experiments two kinds of prior probability assignments must be, and were, specified:

(i) The a priori degree of belief or probability that the hypothesis is true, $P(H)$, and the probability that the hypothesis is false $P(\bar{H}) = 1 - P(H)$. My assignments were

$$P(H) = 0.6$$

$$P(\bar{H}) = 0.4.$$

Thus, from the pilot experiment and my experience with these kinds of experiments generally, I was prepared to give odds of 1.5 to 1 that the ordering relation in H represented the true state of nature.

(ii) The a priori degree of confidence in the ability of observations from the six experimental sessions to confirm or disconfirm the hypothesis if it is true, or if it is false. These conditional prior probabilities can be written

$$P(O_S \cap O_B|H) = p_1 \qquad P(O_S \cap O_B|\bar{H}) = q_1$$

$$P(O_S \cap \bar{O}_B|H) = p_2 \qquad P(O_S \cap \bar{O}_B|\bar{H}) = q_2$$

$$P(\bar{O}_S \cap O_B|H) = p_3 \qquad P(\bar{O}_S \cap O_B|\bar{H}) = q_3$$

$$P(\bar{O}_S \cap \bar{O}_B|H) = 1 - p_1 - p_2 - p_3 \qquad P(\bar{O}_S \cap \bar{O}_B|\bar{H}) = 1 - q_1 - q_2 - q_3$$

2. See e.g., L. J. Savage, "Bayesian Statistics" in R. E. Machol and Paul Grey (eds.), *Recent Developments in Information and Decisions Processes* (New York: MacMillan, 1962).

where $P(O_S \cap O_B|H)$ is the a priori probability that the relation $\hat{P}^S_t < \hat{P}^{SB}_t < \hat{P}^B_t$ would hold on the $\overset{th}{t}$— transaction if the hypothesis were known to be true. It represents the degree of confidence in the ability of the experimental sessions, under both the deviant conditions R_S and R_B, to confirm the hypothesis if it is true. Similarly, $P(O_S \cap \bar{O}_B|H)$, is the probability that the ordering relation $\hat{P}^S_t < \hat{P}^{SB}_t \geqq \hat{P}^B_t$ would hold on the $\overset{th}{t}$— transaction if H were true, i.e., the left half of the ordering relation in H would be confirmed, but not the right half. Again, $P(O_S \cap O_B|\bar{H})$ is the probability that the ordering relation in H would be confirmed by the $\overset{th}{t}$— transaction though H were false. This is, of course, entirely likely due to experimental error — the effect of random elements not controlled in the experiment.

My prior assignments were

$$p_1 = 0.55 \qquad q_1 = 0.25$$
$$p_2 = 0.15 \qquad q_2 = 0.25$$
$$p_3 = 0.15 \qquad q_3 = 0.25$$
$$p_4 = 0.15 \qquad q_4 = 0.25.$$

I was prepared to believe that if H were true, a conservative estimate of 55 out of 100 trials of transactions under R_S, R_{SB} and R_B, would be consistent with H. I guessed that 15 per cent of the transactions would violate either the left, right, or both halves of the ordering relation, if H were true. If H were false, then I felt there was no reason to expect any sample outcome to be more likely than any other ($q_1 = q_2 = q_3 = q_4$). If, for example, the condition R_S does not introduce a downward bias in equilibrium prices then I would expect $\hat{P}^S_t$ to be as likely above as below $\hat{P}^{SB}_t$.

Now let E_{n,n_i} be the event that of the total number, n, of equilibrium transactions, $O_S \cap O_B$ (both the left and right halves of the ordering relation in H is confirmed by the sample outcome) occurs n_1 times, $O_S \cap \bar{O}_B$ occurs n_2 times, $\bar{O}_S \cap O_B$ occurs n_3 times, and $\bar{O}_S \cap \bar{O}_B$ occurs $n - n_1 - n_2 - n_3$ times. Hence, if the observations are independent (equilibrium), the conditional distribution of the n_i are given by the multinomials

$$(6) \qquad P(E_{n,n_i}|H) = \frac{n!p_1^{n_1}p_2^{n_2}p_3^{n_3}(1 - p_1 - p_2 - p_3)^{n-n_1-n_2-n_3}}{n_1!n_2!n_3!(n - n_1 - n_2 - n_3)!}$$

$$(7) \qquad P(E_{n,n_i}|\bar{H}) = \frac{n!q_1^{n_1}q_2^{n}q_3^{n}(1 - q_1 - q_2 - q_3)^{n-n-n-n}}{n_1!n_2!n_3!(n - n_1 - n_2 - n_3)!}.$$

From Bayes theorem, we can now write the posterior densities

$$(8)\qquad P(H|E_{n,n_i}) = \frac{P(E_{n,n_i}|H)\,P(H)}{P(E_{n,n_i}|H)\,P(H) + P(E_{n,n_i}|\bar{H})\,P(\bar{H})}$$

$$= \frac{p_1^{n_1} p_2^{n_2} p_3^{n_3} (1 - p_1 - p_2 - p_3)^{n-n_1-n_2-n_3} P(H)}{p_1^{n_1} p_2^{n_2} p_3^{n_3} (1 - p_1 - p_2 - p_3)^{n-n_1-n_2-n_3} P(H) + q_1^{n_1} q_2^{n_2} q_3^{n_3} (1 - q_1 - q_2 - q_3)^{n-n_1-n_2-n_3} P(\bar{H})}$$

$$(9)\qquad P(\bar{H}|E_{n,n_i}) = 1 - P(H|E_{n,n_i}).$$

From the data of our six experimental sessions (see Charts 1–6), we have [3] $n = 31$, $n_1 = 14$, $n_2 = 10$, $n_3 = 3$, $n_4 = 4$. Applying (8) and (9) to these results and the prior probabilities specified above we compute $P(H|E_{n,n_i}) = 0.94$, $P(\bar{H}|E_{n,n_i}) = 0.06$.

I now stand 94 per cent sure that H is true.

Appendix

Instructions for Market Experiment

1. This is an experiment in the economics of market decision-making. The National Science Foundation has provided funds for the conduct of this research. The instructions are simple, and if you follow them carefully and make good decisions you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment.

2. In this experiment we are going to simulate a market in which some of you will be buyers and some of you will be sellers in a sequence of trading periods or market days. Two kinds of cards will now be passed out — a set of white cards and a set of yellow cards. Those of you who receive a white card will be sellers, and only sellers. Those of you who receive a yellow card are buyers, and only buyers. These cards have an identification number, which you are to ignore, on the side facing up. On the side facing down appears a figure or price in dollars and cents. You are not to reveal this price to anyone. It is your own private information.

3. If you have received a white card you are a seller of at most one unit of the fictitious commodity being sold in each trading

3. In sessions 1, 3 and 5 forming one trial set of observations, there are 16 transactions in periods 4 and 5 in sessions 1 and 3, and 15 transactions in session 5. I considered this set as representing 16 "trials," and counted the "missing" observation in session 5 against the hypothesis. Similarly there are 14 transactions in sessions 2 and 4, and 15 in session 6, giving 15 more trials for a total of 31. The missing observation in sessions 2 and 4 were counted against the hypothesis.

period. The price on the underside of your white card is the lowest price at which you are to sell your unit of this commodity in any trading period. If you have received a yellow card, you are a buyer of at most one unit of the commodity being sold in each trading period. The price on your yellow card is the maximum price at which you are to buy a unit of this commodity per trading period.

4. The payoffs are as follows: If you are a seller, and you were able to make a sale, you will receive 5 cents for having made a sale plus the difference between the price at which you sold and the price on your white card. Think of the price on your white card as your cost of production. Your profits depend directly upon your ability to sell above this cost, but you should be prepared to sell at this cost, and receive your 5 cent commission, if you can do no better. If you are a buyer, and make a purchase, you will receive a 5 cent commission plus the difference between the price on your card and the price at which you bought. Think of the price on your yellow card as the price you can get by reselling the unit in an entirely separate market, while the price at which you buy in this market is your cost. Your profits depend directly upon your ability to buy at a cost below the price on your card, but you should be prepared to buy at that price, and collect your 5 cents, if you cannot do better. The payoffs for each subject will be accumulated over several trading periods, and the total amount paid in cash at the very end of the experiment. You are not to reveal your profits to anyone until the experiment is completed. There is no penalty except the profits you lose from failing to make a contract.

5S. The market for this commodity is organized as follows: We open the market for a trading day. Any seller is free at any time to raise his hand and make a verbal offer to sell at any price which is *not below* the price on his white card. Any buyer is free to accept the offer of any seller but no buyer is to buy at a price above the price on his yellow card. As soon as an offer is accepted, a binding contract has been closed and the buyer and seller making the deal are to drop out of the market, making no more offers or contracts for the remainder of that trading period. This process continues for a period of several minutes, depending upon the volume of trading. You will be warned when the market is to close and a few more offers will be called for before actually closing. This completes a trading "day." We will then reopen the market for a new trading period, and so on, for a sequence of several periods.

6S. Some of you may be unable to make a purchase or sale in any trading period. Some of you will be able to make a purchase or sale in some trading periods, but not in others. There are likely to be many offers that are not accepted. You are to keep trying and you are to feel free to earn as much cash as you can. Except for the offers you are not to speak to any other subject until the experiment is completed.

5SB. The market for this commodity is organized as follows: We open the market for a trading day. Any buyer is then free at any time to raise his hand and make a verbal bid to buy at any

price which *does not exceed* the price on his yellow card. Likewise, any seller is free at any time to raise his hand and make a verbal offer to sell at any price which is *not below* the price on his white card. Any seller is free to accept the bid of any buyer, and any buyer is free to accept the offer of any seller. As soon as a bid or offer is accepted, a binding contract has been closed and the buyer and seller making the deal are to drop out of the market, making no more bids, offers, or contracts for the remainder of that trading period. This process continues for a period of several minutes, depending upon the volume of trading. You will be warned when the market is to close and a few more bids and offers will be called for before actually closing. This completes a trading "day." We will then reopen the market for a new trading period, and so on, for a sequence of several periods.

6SB. Some of you may be unable to make a purchase or sale in any trading period. Some of you will be able to make a purchase or sale in some trading periods, but not in others. There are likely to be many bids and offers that are not accepted. You are to keep trying and you are to feel free to earn as much cash as you can. Except for the bids and offers you are not to speak to any other subject until the experiment is completed.

5B. The market for this commodity is organized as follows: We open the market for a trading day. Any buyer is free at any time to raise his hand and make a verbal bid to buy at any price which does *not exceed* the price on his yellow card. Any seller is free to accept the bid of any buyer but no seller is to sell at a price below the price on his white card. As soon as a bid is accepted, a binding contract has been closed and the buyer and seller making the deal are to drop out of the market, making no more bids or contracts for the remainder of that trading period. This process continues for a period of several minutes, depending upon the volume of trading. You will be warned when the market is to close and a few more bids will be called for before actually closing. This completes a trading "day." We will then reopen the market for a new trading period, and so on, for a sequence of several periods.

6B. Some of you may be unable to make a purchase or sale in any trading period. Some of you will be able to make a purchase or sale in some trading periods, but not in others. There are likely to be many bids that are not accepted. You are to keep trying and you are to feel free to earn as much cash as you can. Except for the bids you are not to speak to any other subject until the experiment is completed.

7. Are there any questions?

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