Golden Eggs and Hyperbolic Discounting

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GOLDEN EGGS AND HYPERBOLIC DISCOUNTING*

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Hyperbolic discount functions induce dynamically inconsistent preferences, implying a motive for consumers to constrain their own future choices. This paper analyzes the decisions of a hyperbolic consumer who has access to an imperfect commitment technology: an illiquid asset whose sale must be initiated one period before the sale proceeds are received. The model predicts that consumption tracks income, and the model explains why consumers have asset-specific marginal propensities to consume. The model suggests that financial innovation may have caused the ongoing decline in U. S. savings rates, since financial innovation increases liquidity, eliminating commitment opportunities. Finally, the model implies that financial market innovation may reduce welfare by providing "too much" liquidity.

I. INTRODUCTION

"Use whatever means possible to remove a set amount of money from your bank account each month before you have a chance to spend it"

—advice in *New York Times* "Your Money" column [1993].

Many people place a premium on the attribute of self-control. Individuals who have this capacity are able to stay on diets, carry through exercise regimens, show up to work on time, and live within their means. Self-control is so desirable that most of us complain that we do not have enough of it. Fortunately, there are ways to compensate for this shortfall. One of the most widely used techniques is commitment. For example, signing up to give a seminar is an easy way to commit oneself to write a paper. Such commitments matter since they create constraints (e.g., deadlines) that generally end up being binding.

Strotz [1956] was the first economist to formalize a theory of commitment and to show that commitment mechanisms could be potentially important determinants of economic outcomes. He

showed that when individuals' discount functions are nonexpo-
nential, they will prefer to constrain their own future choices.
Strotz noted that costly commitment decisions are commonly
observed:

> ... we are often willing even to pay a price to precommit future
> actions (and to avoid temptation). Evidence of this in economic and
> other social behaviour is not difficult to find. It varies from the gra-
> tuitous promise, from the familiar phrase "Give me a good kick if I
> don't do such and such" to savings plans such as insurance policies
> and Christmas Clubs which may often be hard to justify in view of
> the low rates of return. (I select the option of having my annual
> salary dispersed to me on a twelve- rather than on a nine-month
> basis, although I could use the interest!) Personal financial manage-
> ment firms, such as are sometimes employed by high-income profes-
> sional people (e.g. actors), while having many other and perhaps
> more important functions, represent the logical conclusion of the
> desire to precommit one's future economic activity. Joining the army
> is perhaps the supreme device open to most people, unless it be
> marriage for the sake of "settling down." The worker whose income
> is garnished *chronically* or who is continually harassed by creditors,
> and who, when one oppressive debt is paid, immediately incurs an-
> other is commonly precommiting. There is nothing irrational about
> such behavior (quite the contrary) and attempts to default on debts
> are simply the later consequences which are to be expected. Inabil-
> ity to default is the force of the commitment.

Strotz's list is clearly not exhaustive. In general, all illiquid
assets provide a form of commitment, though there are some-
times additional reasons that consumers might hold such assets
(e.g., high expected returns and diversification). A pension or re-
tirement plan is the clearest example of such an asset. Many of
these plans benefit from favorable tax treatment, and most of
them effectively bar consumers from using their savings before
retirement. For IRAs, Keogh plans, and 401(K) plans, consumers
can access their assets, but they must pay an early withdrawal
penalty. Moreover, borrowing against some of these assets is le-
gally treated as an early withdrawal, and hence also subject to
penalty. A less transparent instrument for commitment is an in-
vestment in an illiquid asset that generates a steady stream of
benefits, but that is hard to sell due to substantial transactions
costs, informational problems, or incomplete markets. Examples

include purchasing a home, buying consumer durables, and building up equity in a personal business. Finally, there exists a class of assets that provide a store of illiquid value, like savings bonds, and certificates of deposit. All of the illiquid assets discussed above have the same property as the goose that laid golden eggs. The asset promises to generate substantial benefits in the long run, but these benefits are difficult, if not impossible, to realize immediately. Trying to do so will result in a substantial capital loss.

Instruments with these *golden eggs* properties make up the overwhelming majority of assets held by the U. S. household sector. For example, the Federal Reserve System publication *Balance Sheets for the U. S. Economy 1945–94* reports that the household sector held domestic assets of $28.5 trillion at year-end 1994. Over two-thirds of these assets were illiquid, including $5.5 trillion of pension fund and life insurance reserves, $4.5 trillion of residential structures, $3.0 trillion of land, $2.5 trillion of equity in noncorporate business, $2.5 trillion of consumer durables, and at least $1 trillion of other miscellaneous categories. Finally, note that social security wealth and human capital, two relatively large components of illiquid wealth, are not included in the Federal Reserve Balance Sheets.

Despite the abundance of commitment mechanisms, and Strotz's well-known theoretical work, intrapersonal commitment phenomena have generally received little attention from economists. This deficit is probably explained by the fact that commitment will only be chosen by decision-makers whose preferences are dynamically inconsistent, and most economists have avoided studying such problematic preferences. However, there is a substantial body of evidence that preferences are dynamically inconsistent. Research on animal and human behavior has led psychologists to conclude that discount functions are approximately hyperbolic [Ainslie 1992].

Hyperbolic discount functions are characterized by a relatively high discount rate over short horizons and a relatively low discount rate over long horizons. This discount structure sets up a conflict between today's preferences, and the preferences that will be held in the future. For example, from today's perspective, the discount rate between two far-off periods, t and $t + 1$, is the long-term low discount rate. However, from the time t perspective, the discount rate between t and $t + 1$ is the short-term high

discount rate. This type of preference change is reflected in many common experiences. For example, this year I may desire to start an aggressive savings plan next year, but when next year actually rolls around, my taste at that time will be to postpone any sacrifices another year. In the analysis that follows, the decision-maker foresees these conflicts and uses a stylized commitment technology to partially limit the options available in the future.

This framework predicts that consumption will track income. Second, the model explains why consumers have a different propensity to consume out of wealth than they do out of labor income. Third, the model explains why Ricardian equivalence should not hold even in an economy characterized by an infinitely lived representative agent. Fourth, the model suggests that financial innovation may have caused the ongoing decline in U. S. savings rates, since financial innovation increases liquidity and eliminates implicit commitment opportunities. Finally, the model provides a formal framework for considering the proposition that financial market innovation reduces welfare by providing "too much" liquidity.

The body of this essay formalizes these claims. Section II lays out the model. Equilibrium outcomes are characterized in Section III. Section IV considers the implications of the model for the macroeconomic issues highlighted above. Section V concludes with a discussion of ongoing work.

II. The Consumption Decision

The large number of commitment devices, discussed above, is good news for consumers. They have access to a wide array of assets that effectively enable them to achieve many forms of commitment. However, from the perspective of an economist, the abundance poses a challenge. It is hard to model the institutional richness in a realistic way without generating an extremely burdensome number of state variables.

I consider a highly stylized commitment technology that is amenable to an analytic treatment. Specifically, I assume that consumers may invest in two instruments: a liquid asset x and an illiquid asset z. Instrument z is illiquid in the sense that a sale of this asset has to be initiated one period before the actual proceeds are received. So a current decision to liquidate part or all of an individual's z holding will generate cash flow that can be

consumed no earlier than next period.[1] By contrast, agents can always immediately consume their x holdings.

Consumers in this model may borrow against their holdings of asset z. Like asset sales, such borrowing takes one period to implement. If a consumer applies for a loan at time period t, the associated cash flow will not be available for consumption until time period $t + 1$.

In later sections I embed consumers in a general equilibrium model in which prices will be endogenous. Now, however, I consider the consumer in isolation, and assume that the consumer faces a deterministic sequence of interest rates and wages. For simplicity, I assume that asset z and asset x have the same rate of return.[2]

The consumer makes consumption/savings decisions in discrete time $t \in \{1, 2, \ldots, T\}$. Every time period t is divided into four subperiods. In the first subperiod, production takes place. The consumer's liquid assets x_{t-1} and nonliquid assets z_{t-1}—both chosen at time period $t - 1$—yield a gross return of $R_t = 1 + r_t$, and the consumer inelastically supplies one unit of labor. In the second subperiod the consumer receives deterministic labor income y_t and gets access to her liquid savings, $R_t \cdot x_{t-1}$. In the third subperiod the consumer chooses current consumption,

$$c_t \leq y_t + R_t x_{t-1}.$$

In the fourth subperiod the consumer chooses her new asset allocations, x_t and z_t, subject to the constraints,

$$y_t + R_t(z_{t-1} + x_{t-1}) - c_t = z_t + x_t,$$
$$x_t, z_t \geq 0.$$

The consumer begins life with exogenous endowments $x_0, z_0 \geq 0$.

The consumer may borrow against her illiquid assets by giving a creditor a contingent control right over some of those assets. In exchange, the consumer receives liquidity that can be consumed. Such a loan is formally represented as a reallocation of assets from the illiquid account to the liquid account. I assume that a loan, i.e., asset reallocation, which generates consumable liquidity in period $t + 1$ must be initiated in period t. Specifically,

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1. One could alternatively assume that instantaneous access to asset z is possible with a sufficiently high transaction cost.
2. The qualitative results do not hinge on the identical returns assumption.

the asset reallocation occurs in subperiod 4 of period t, thereby providing consumable liquidity in period $t + 1$. Such asset reallocations are subsumed in the consumer's choice of x_t and z_t in subperiod 4.

In the framework introduced above, an uncollateralized loan has occurred if an asset reallocation leaves the illiquid account negative. Creditors are unwilling to make such loans because a consumer who received such a loan would not have an incentive to repay. Hence, I assume that $z_t \geq 0$.

Finally, the constraint $x_t \geq 0$ rules out forced savings contracts. If the consumer could set x_t to any negative value, then she could perfectly commit her future savings behavior and hence her consumption level (or at least commit to any upper bound on tomorrow's consumption level). For example, if she foresaw a high level of labor income next period, she could set x_t negative to force tomorrow's self to save some of that income (recall that $c_{t+1} \leq y_{t+1} + R_{t+1}x_t$). A negative x_t value would be interpreted as a contract with an outside agent requiring the consumer to transfer funds to the outside agent, which the outside agent would then deposit in an illiquid account of the consumer.[3] The constraint $x_t \geq 0$ effectively rules out such contracts. Two arguments support this implicit assumption against forced savings contracts.

First, such contracts are susceptible to renegotiation by tomorrow's self, and in any finite-horizon environment, the contract would unwind. (In the second to last period renegotiation would occur, implying renegotiation in the third to last period, etc.) Second, such contracts are generally unenforceable in the United States.[4] To make such a contract work, tomorrow's self must be forced to pay the specified funds to the outside agent or be penalized for not doing so (note that the transfer is not in the interest of tomorrow's self). However, U. S. courts will generally not enforce contracts with a penalty of this kind.[5]

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3. Mortgage payments are an example of a contract that $x_t \geq 0$ rules out. However, even though mortgage payments may be interpreted as forced savings contracts, they do not have the necessary flexibility to achieve the full commitment solution. Mortgage contracts generally do not make mortgage payments contingent on the level of labor income flows.

4. I am indebted to Robert Hall for pointing out this fact to me.

5. U. S. contract law is based around the "fundamental principle that the law's goal on breach of contract is not to deter breach by compelling the promisor to perform, but rather to redress breach by compensating the promisee" [Farnsworth 1990, p. 935]. Hence, courts allow contracts to specify "liquidated damages" which reflect losses likely to be experienced by the promisee, but courts do not allow "penalties" which do not reflect such losses.

At time t, the consumer has a time-additive utility function U_t with an instantaneous utility function characterized by constant relative risk aversion ρ. Consumers are assumed to have a discount function of the type proposed by Phelps and Pollak [1968] in a model of intergenerational altruism, and which is used here to model intrapersonal dynamic conflict:[6]

$$(1) \qquad U_t = E_t \left[u(c_t) + \beta \sum_{\tau=1}^{T-t} \delta^\tau u(c_{t+\tau}) \right].$$

I adopt equation (1) to capture the qualitative properties of a generalized hyperbolic discount function: events τ periods away are discounted with factor $(1 + \alpha\tau)^{-\gamma/\alpha}$, with $\alpha, \gamma > 0$.[7] This class of discount functions was first proposed by Chung and Herrnstein [1961] to characterize the results of animal behavior experiments.[8] Their conclusions were later shown to apply to human subjects as well (see Ainslie [1992] for a survey).

Hyperbolic discount *functions* imply discount *rates* that decline as the discounted event is moved further away in time [Loewenstein and Prelec 1992]. Events in the near future are discounted at a higher implicit discount rate than events in the distant future.

Given a discount function $f(\tau)$, the instantaneous discount rate at time τ is defined as

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Applying the principle of 'just compensation for the loss or injury actually sustained' to liquidated damage provisions, courts have . . . refused enforcement where the clause agreed upon is held to be *in terrorem*—a sum fixed as a deterrent to breach or as security for full performance by the promisor, not as a realistic assessment of the provable damage. Thus, attempts to secure performance through *in terrorem* clauses are currently declared unenforceable even where the evidence shows a voluntary, fairly bargained exchange [Goetz and Scott 1977, p. 555].

In our case, the promisee—the outside agent—experiences no loss if the consumer fails to make the payment. Hence, penalties or liquidated damages specified in such contracts are not enforceable, so the contract is incapable of compelling tomorrow's self to make the payment. For a more extensive discussion of these issues, see Farnsworth [1990, pp. 935–46], Goetz and Scott [1977], and Rea [1984].

6. Zeckhauser and Fels [1968] provide an altruism-based microfoundation for the Phelps and Pollak preferences. Akerlof [1991] analyzes a special case of the Phelps and Pollak preferences ($\delta = 1$). Akerlof assumes consumer myopia, while my analysis assumes that consumers foresee their future preference reversals.

7. See Loewenstein and Prelec [1992] for an axiomatic derivation of this discount function.

8. Chung and Herrnstein claimed that the appropriate discount function is an exact hyperbola: events τ periods away are discounted with factor $1/\tau$. This corresponds to the limiting case $\alpha = \gamma \to \infty$.

$$-f'(\tau)/f(\tau).$$

Hence, an exponential discount function, δ^τ is characterized by a constant discount rate, $\log(1/\delta)$, while the generalized hyperbolic discount function is characterized by an instantaneous discount rate that falls as τ rises:

$$\gamma/(1 + \alpha\tau).$$

Psychologists and economists—notably Ainslie [1975, 1986, 1992], Prelec [1989], and Loewenstein and Prelec [1992]—have argued that such declining discount rates play an important role in generating problems of self-regulation.

When $0 < \beta < 1$, the discount structure in equation (1) mimics the qualitative property of the hyperbolic discount function, while maintaining most of the analytical tractibility of the exponential discount function. I call the discount structure in equation (1) "quasi-hyperbolic." Note that the quasi-hyperbolic discount function is a discrete time function with values $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots \}$. Figure I graphs the exponential discount function (assuming that $\delta = 0.97$), the generalized hyperbolic discount function (assuming that $\alpha = 10^5$, and $\gamma = 5 \cdot 10^3$), and the quasi-hyperbolic discount function (with $\beta = 0.6$ and $\delta = 0.99$). The
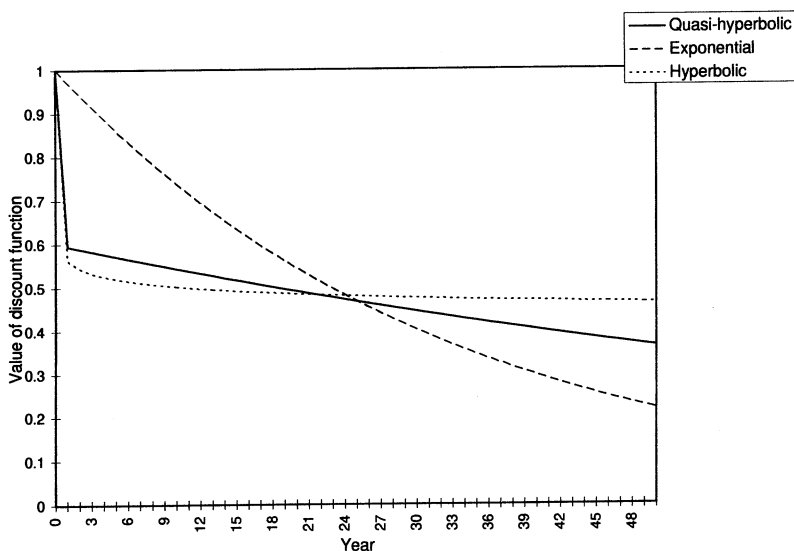

FIGURE I
Discount Functions

points of the discrete-time quasi-hyperbolic function have been connected to generate the curve in Figure I.

The preferences given by equation (1) are dynamically inconsistent, in the sense that preferences at date t are inconsistent with preferences at date $t + 1$. To see this, note that the marginal rate of substitution between periods $t + 1$ and $t + 2$ from the perspective of the decision-maker at time t is given by $u'(c_{t+1})/(\delta u'(c_{t+2}))$, which is not equal to the marginal rate of substitution between those same periods from the perspective of the decision-maker at $t + 1$: $u'(c_{t+1})/(\beta\delta u'(c_{t+2}))$.

To analyze equilibrium behavior when preferences are dynamically inconsistent, it is standard practice to formally model a consumer as a sequence of temporal selves making choices in a dynamic game (e.g., Pollak [1968], Peleg and Yaari [1973], and Goldman [1980]). Hence, a T-period consumption problem translates into a T-period game, with T players, or "selves," indexed by their respective periods of control over the consumption decision. (Note that self t is in control during all of the subperiods at time t.) I look for subgame perfect equilibrium (SPE) strategies of this game.

It is helpful to introduce some standard notation that will be used in the analysis which follows. Let h_t represent a (feasible) history at time t, so h_t represents all the moves that have been made from time 0 to time $t - 1$: $\{x_0, z_0, (c_\tau, x_\tau, z_\tau)_{\tau=1}^{t-1}\}$. Let S_t represent the set of feasible strategies for self t. Let $S = \Pi_{t=1}^{T} S_t$ represent the joint strategy space of all selves. If $s \in S$, let $s \mid h_t$ represent the path of consumption and asset allocation levels from t to T which would arise if history h_t were realized, and selves t to T played the strategies given by s. Finally, let $U_t(s \mid h_t)$ represent the continuation payoff to self t if self t expects the consumption and asset allocation levels from t to T to be given by $s \mid h_t$.

III. EQUILIBRIUM STRATEGIES

This section characterizes the equilibrium strategies of the game described above. Recall that the agent faces a deterministic (time-varying) sequence of interest rates and a deterministic (time-varying) labor income sequence. Unfortunately, for general interest rate and labor income sequences, it is not possible to use marginal conditions to characterize the equilibrium strategies. This nonmarginality property is related to the fact that selves who make choices at least two periods from the end of the game

face a nonconvex reduced-form choice set, where the reduced-form choice set is defined as the consumption vectors which are attainable, assuming that all future selves play equilibrium strategies. The nonconvexity in the reduced-form choice set of self $T - 2$ generates discontinuous equilibrium strategies for self $T - 2$, which in turn generate discontinuities in the equilibrium payoff map of self $T - 3$. This implies that marginal conditions cannot be used to characterize the equilibrium choices of selves at least three periods from the end of the game.[9]

I have found a restriction on the labor income process that eliminates these problems:

$$(A1) \qquad u'(y_t) \geq \beta \delta^\tau \left(\prod_{i=1}^{\tau} R_{t+i} \right) u'(y_{t+\tau}) \qquad \forall t, \tau \geq 1.$$

This restriction constrains the sequence $\{y_t\}_{t=1}^{t=T}$ to lie in a band whose thickness is parameterized by the value of β; the closer β is to zero, the wider the band. Calibration of the model reveals that A1 allows for substantial flexibility in the deterministic income process. Ainslie [1992] reviews evidence that the one-year discount rate is at least 1/3. This suggests that β should be calibrated in the interval (0, 2/3) (assuming that δ is close to unity). To see what this implies, consider the following example. Assume that $R_t = R$ for all t, $\delta R = 1$, and $u(\cdot) = \ln(\cdot)$. Then A1 is satisfied if, for all t, $y_t \in [\underline{y}, (1/\beta)\underline{y}]$. If $\beta = 2/3$, this interval becomes $[\underline{y}, (3/2)\underline{y}]$, and as β falls the interval grows even larger.

Before characterizing the equilibria of the game, it is helpful to introduce the following definitions. First, we will say that a joint strategy, s, is *resource exhausting* if $s \mid h_{T-1}$ is characterized by $z_T = x_t = 0$, for all feasible h_{T-1}. Second, we will say that a sequence of feasible consumption/savings actions, $\{c_{\hat{t}}, x_{\hat{t}}, z_{\hat{t}}, \ldots, c_T, x_T, z_T\}$ satisfies P1–P4 if $\forall t \geq \hat{t}$,

$$P1 \quad u'(c_t) \geq \max_{\tau \in \{1, \ldots, T-t\}} \beta \delta^\tau \left(\prod_{i=1}^{\tau} R_{t+i} \right) u'(c_{t+\tau})$$

$$P2 \quad u'(c_t) > \max_{\tau \in \{1, \ldots, T-t\}} \beta \delta^\tau \left(\prod_{i=1}^{\tau} R_{t+i} \right) u'(c_{t+\tau}) \Rightarrow c_t = y_t + R_t x_{t-1}$$

$$P3 \quad u'(c_{t+1}) < \max_{\tau \in \{1, \ldots, T-t-1\}} \delta^\tau \left(\prod_{i=1}^{\tau} R_{t+i} \right) u'(c_{t+1+\tau}) \Rightarrow x_t = 0$$

$$P4 \quad u'(c_{t+1}) > \max_{\tau \in \{1, \ldots, T-t-1\}} \delta^\tau \left(\prod_{i=1}^{\tau} R_{t+i} \right) u'(c_{t+1+\tau}) \Rightarrow z_t = 0.$$

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9. For an exposition of these problems see Laibson [1993]. Related issues are also discussed in Peleg and Yaari [1973] and Goldman [1980].

Finally, we will say that a joint strategy $s \in S$ *satisfies* P1–P4 if for any feasible history h_i, $s \mid h_i$ satisfies P1–P4.

It is now possible to state the main theorem of the paper. This theorem establishes that the consumption game has a unique equilibrium, and the theorem characterizes this equilibrium.

THEOREM 1. Fix any T-period consumption game with exogenous variables satisfying A1. There exists a unique resource-exhausting joint strategy, $s^* \in S$, that satisfies P1–P4, and this strategy is the unique subgame perfect equilibrium strategy of this game.

(All proofs appear in the Appendix.) Theorem 1 implies that the equilibrium consumption path is resource exhausting and satisfies P1–P4. It is straightforward to see why the equilibrium path is resource exhausting: the final self—self T—consumes all liquid resources in period T, and self $T - 1$ makes certain that all wealth is liquid in period T (i.e., $z_{T-1} = 0$). Hence, no wealth goes unconsumed in equilibrium.

Properties P1–P4 are also intuitive. It is important to interpret them in light of the strategic self-control behavior that arises in the intrapersonal consumption game. In this game, early selves prevent late selves from splurging. Self $t - 1$ uses the illiquid asset z_{t-1} to limit the liquidity available for consumption in period t. Note that self t cannot consume the illiquid asset during its period of control, $c_t \le y_t + R_t x_{t-1}$. On the equilibrium path, each self is endogenously liquidity constrained by the allocation choices of earlier selves. Property P1 is simply a standard Euler equation relation for an environment in which liquidity constraints exist. The inequality arises because marginal utility can be too high relative to future marginal utilities, but it cannot be too low since consumers always have the option to save. Property P2 reflects another standard Euler equation intuition: when marginal utility is strictly too high, the liquidity constraint must be binding. Properties P3 and P4 reflect the strategic decisions that self t makes when it chooses asset allocation levels (x_t and z_t). P3 implies that self t will limit self $t + 1$'s liquidity as much as possible ($x_t = 0$) if consumption at time $t + 1$ is expected to be high relative to what self t would prefer it to be. P4 implies that self t will not limit self $t + 1$'s liquidity at all ($z_t = 0$) if consumption at time $t + 1$ is expected to be low relative to what self t would prefer it to be. Note that the equations associated with P3 and P4 do not contain the β term. This omission arises because, from the

perspective of self t, utility trade-offs between period $t + 1$ and any period after $t + 1$ are independent of the value of β.

IV. ANALYSIS

In the following subsections I discuss several implications of the golden eggs model. Some of the applications consider the infinite-horizon game that is analogous to the finite-horizon game discussed above. When doing so, I will focus consideration on the equilibrium that is the limit (as the horizon goes to infinity) of the unique finite-horizon equilibrium.[10]

A. Comovement of Consumption and Income

There is a growing body of evidence that household consumption flows track corresponding household income flows "too" closely, generating violations of the life-cycle/permanent-income consumption model. In particular, household consumption is sensitive to expected movements in household income: e.g., Hall and Mishkin [1982], Zeldes [1989], Carroll and Summers [1991], Flavin [1991], Carroll [1992], Shea [1995], and Souleles [1995].[11] Many of these authors find that consumption tracks expected income changes even when consumers have large stocks of accumulated assets.

Several models have been proposed to explain the consump-

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10. For the infinite horizon game, a joint strategy, s, is *resource exhausting* if the continuation paths after all histories imply that the intertemporal budget constraint is exactly satisfied:

$$z_0 + x_0 + \sum_{t=1}^{\infty}\left(\prod_{i=1}^{t}R_i\right)^{-1}y_t = \sum_{t=1}^{\infty}\left(\prod_{i=1}^{t}R_i\right)^{-1}c_t.$$

For the infinite-horizon game, I will focus on the equilibrium that satisfies the following infinite-horizon analogs of P1–P4:

$$P1 \quad u'(c_t) \geq \sup_{\tau \geq 1}\beta\delta^{\tau}\left(\prod_{i=1}^{\tau}R_{t+i}\right)u'(c_{t+\tau})$$

$$P2 \quad u'(c_t) > \sup_{\tau \geq 1}\beta\delta^{\tau}\left(\prod_{i=1}^{\tau}R_{t+i}\right)u'(c_{t+\tau}) \Rightarrow c_t = y_t + R_t x_{t-1}$$

$$P3 \quad u'(c_{t+1}) < \sup_{\tau \geq 1}\delta^{\tau}\left(\prod_{i=1}^{\tau}R_{t+i}\right)u'(c_{t+1+\tau}) \Rightarrow x_t = 0$$

$$P4 \quad u'(c_{t+1}) > \sup_{\tau \geq 1}\delta^{\tau}\left(\prod_{i=1}^{\tau}R_{t+i}\right)u'(c_{t+1+\tau}) \Rightarrow z_t = 0.$$

11. Although Runkle [1989] is unable to reject the permanent income hypothesis, there are reasons to believe his test lacks power (see Shea [1995]).

tion-income comovement. Carroll [1992] proposes a buffer-stock theory of savings in which impatient consumers with a precautionary savings motive hold little wealth and choose optimal consumption policies in which consumption and income move together over the life-cycle. Gourinchas and Parker [1995] simulate an extended version of this model. Attanasio and Weber [1993] argue that demographic dynamics explain much of the consumption-income comovement.

The golden eggs model provides a new explanation for the observed comovement in consumption and income. In the model, self $t - 1$ chooses x_{t-1} to constrain the consumption of self t. In this way "early" selves manipulate the cash flow process by keeping most assets in the illiquid instrument. Hence, at any given moment the consumer is effectively liquidity constrained, though the constraint is self-imposed. In equilibrium consumption is exactly equal to the current level of cash flow: $c_t = y_t + R_t x_{t-1}$ (see Lemma 3 in the Appendix for a formal proof). However, this does not imply by itself that consumption will track labor income. Note that x_{t-1} is endogenous, and in equilibrium x_{t-1} covaries negatively with labor income. Self $t - 1$ varies x_{t-1} to try to offset the predictable fluctuations in y_t. When y_t is large, self $t - 1$ will make x_{t-1} small in an effort to prevent self t from overconsuming.

However, there are limits to the ways in which "early" selves can constrain the choices of "later" selves. Self $t - 1$ can only deny self t access to assets that have been accumulated in the past. Self $t - 1$ cannot deny self t access to y_t, labor income at time t. So when y_t is particularly high (i.e., cash flow at time t is particularly high), consumption at time t will also be high. This implies that on the equilibrium path, predictable movements in income will tend to be reflected in movements in consumption.

An example may help to make this more concrete. Let the horizon be infinite. Assume that labor income follows a trending high-low process: $y_t = \bar{y}e^{gt}$ when t is odd, $y_t = \underline{y}e^{gt}$ when t is even. Assume that the interest rate is constant and $\exp(\rho g) = \delta R$. (This last relationship is motivated by the steady state results below.) Assume that $\{y_t\}_{t=1}^{\infty}$ satisfies A1. Finally, assume that $x_0 = 0, z_0 \geq 0$, and z_0 not be "too" large relative to the labor income variability. Specifically, z_0 must satisfy the relationship,

$$u'(\bar{y}) \leq \delta R u'(\underline{y}e^g + z_0(1 - e^{2g}/R^2)e^g).$$

Then the equilibrium consumption path is

$$c_t = \begin{cases} y_t & \text{if } t \text{ odd} \\ \\ y_t + z_0(1 - e^{2g}/R^2)e^{gt} & \text{if } t \text{ even}. \end{cases}$$

Figure II graphs the labor income path and equilibrium consumption path, using parameter values, $\beta = 0.6$, $R = 1.04$, $g = 0.02$, $\bar{y} = z_0/3 = 1$, and $\underline{y} = 0.8$.[12] Two properties stand out. First, the illiquid asset is exclusively used to augment consumption in the even periods, i.e., in the periods with relatively low labor income. However, this increase is not sufficient to smooth consumption. A regression of $\Delta \ln c_t$ on $\Delta \ln y_t$ yields a coefficient of .40. Since the income process is completely deterministic, this implies that predictable changes in income are associated with changes in consumption. Hence, consumption tracks income despite the fact that the consumer in this example controls a substantial asset stock ($K/Y \approx 3$).

B. Aggregate Saving

In most intertemporal rational choice models, high discount rates are a necessary condition for consumption-income comovement. Such relatively high discount rates, however, tend to imply relatively low levels of capital accumulation in general equilibrium (see Aiyagari [1992]). The golden eggs model generates consumption-income comovement even when actors are wealthy. This is because in equilibrium decisions to dissave out of the illiquid asset stock do not depend on β. Self t is not able to consume the illiquid asset immediately, so self t does not consider trade-offs between consumption today and consumption tomorrow when dissaving from the illiquid instrument. Instead self t considers trade-offs between consumption at $t + 1$ and consumption at periods after $t + 1$. The value of β is superfluous for such a decision—from self t's perspective—and hence the steady state capital stock is independent of β.

The following general equilibrium analysis formalizes this intuition. Assume that there exists a continuum of individual agents indexed by the unit interval. Individual decision and state variables are represented with an i index (e.g., $c_t(i)$). Consider a standard Cobb-Douglas production function with aggregate capital K_t, aggregate labor L_t, and exogenous productivity A_t:
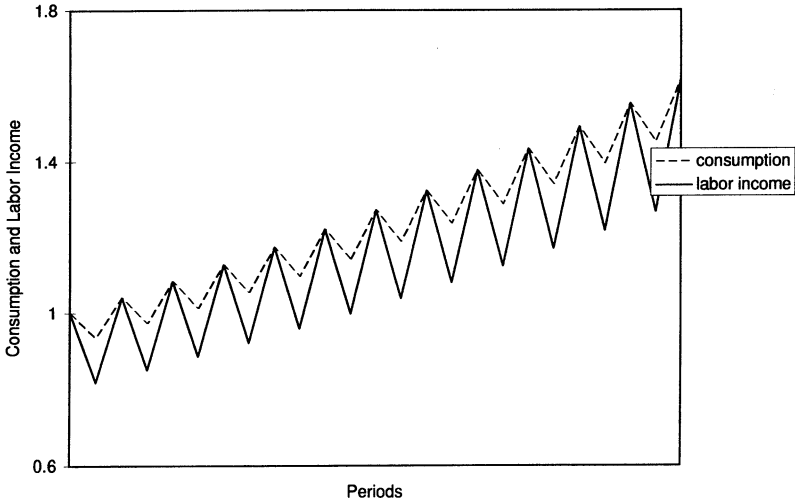
FIGURE II
Consumption and Labor Income

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}.$$

Aggregate capital is composed of the liquid and illiquid capital holdings of individual agents in the economy:

$$K_t \equiv \int_0^1 [x_{t-1}(i) + z_{t-1}(i)]di.$$

Recall that labor is assumed to be supplied inelastically, so $L(i) = 1$, and

$$L_t \equiv \int_0^1 L_t(i)di = 1.$$

In competitive equilibrium labor receives its marginal product, so labor income of agent i at time period t is given by $y_t(i) = (1 - \alpha)Y_t$. Competitive equilibrium also implies that capital receive its marginal product, so $R_t = 1 + \alpha Y_t/K_t - d$, where d is the rate of depreciation. Liquid and illiquid gross asset returns of agent i at time period t are, respectively, $R_t x_{t-1}$ and $R_t z_{t-1}$. Finally, A_t is assumed to grow exogenously at rate g_A, so in steady state, capital and output must grow at rate $g_A/(1-\alpha) \equiv g$.

PROPOSITION 1. In the economy described above there exists a unique steady state that satisfies A1. In that steady state

(2) $\exp(\rho g) = R\delta.$

The important property of the steady state identified in Proposition 1 is that the parameter β does not appear in the equation relating the discount rate and the growth rate. So β can be calibrated to generate excess sensitivity (i.e., consumption-income comovement), while δ can be calibrated to match the historical capital-output ratio of three. If β is in the interior of the unit interval, then the equilibrium path will exhibit consumption-income tracking (e.g., see the example in previous subsection). Meanwhile, δ can be chosen to satisfy the equation,

$$(3) \qquad \rho g \approx r - (1 - \delta) = \alpha(Y/K) - d - (1 - \delta),$$

which is a log-linearized version of equation (2). Setting $\delta = 0.98$ rationalizes $K/Y = 3$, assuming that the other parameters in the equation take standard values: $\rho = 1$, $g = 0.02$, $\alpha = 0.36$, $d = 0.08$.

C. Asset-Specific MPCs

Thaler [1990] argues that consumers have different marginal propensities to consume for different categories of assets. For example, he presents evidence that an unexpected increase in the value of an equity portfolio will have a very small effect on consumption, while an unexpected job-related bonus will be immediately consumed. Thaler divides consumer wealth into three categories: current income, net assets, and future income. He cites a wide body of evidence which suggests that "the MPC from [current income] is close to unity, the MPC from [future income] is close to zero, and the MPC from [net assets] is somewhere in between." Thaler explains this behavior by postulating that consumers use a system of nonfungible mental accounts to guide rule-of-thumb decision-making. By contrast, the golden eggs model predicts that even fully rational consumers will exhibit asset-specific MPCs.[13]

In the golden eggs model the current self is always endogenously liquidity constrained on the equilibrium path. So the MPC out of current cash flow is one. Proposition 2 formalizes this claim.

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13. Laibson [1994b] proposes another hyperbolic discounting model that generates some mental accounting behavior. In Laibson [1994b] rational consumers set up a system of self-rewards and self-punishments to motivate later selves to exert high effort. Laibson [1994b] discusses effort-related mental accounts, while the current paper discusses liquidity-related mental accounts.

PROPOSITION 2. Fix a consumption game in which inequality A1 is strictly satisfied. Let $c_t = c_t(R_t x_{t-1}, R_t z_{t-1})$ represent the equilibrium (Markov) consumption strategy of self t. Then,

$$\frac{\partial c_t}{\partial (R_t x_{t-1})} = 1 \quad \forall t \geq 2,$$

when the partial derivative is evaluated on the equilibrium path.

In this subsection I contrast this MPC with its analog for illiquid assets. At first glance it is not clear how to best make this comparison. I will consider two approaches.

PROPOSITION 3. Fix a consumption game in which inequality A1 is strictly satisfied. Let $c_t(R_t x_{t-1}, R_t z_{t-1})$ represent the equilibrium (Markov) consumption strategy of self t. Then,

$$\frac{\partial c_t}{\partial (R_t z_{t-1})} = 0 \quad \forall t \geq 2,$$

when the partial derivative is evaluated on the equilibrium path.

This result is not surprising, since on the equilibrium path the individual always faces a self-imposed liquidity constraint. Small perturbations to the illiquid asset stock are not sufficient to stop the current self's liquidity constraint from being binding. A more interesting question to ask is how a perturbation to z_{t-1} affects the choice of x_t. Recall that liquid assets set aside at time t will be completely consumed at time $t + 1$. Unfortunately, the value of $\partial x_t / \partial (R_t z_{t-1})$ can take on any value between zero and one. For example, the partial derivative is equal to zero if the equilibrium value of x_t is equal to zero. The partial derivative is equal to unity if t is the penultimate period of the game. It would be helpful to develop an MPC measure that provides a representative value of $\partial x_t / \partial (R_t z_{t-1})$. The following proposition introduces such a measure, by considering the geometric average of MPCs over a deterministic business cycle of duration τ.

PROPOSITION 4. Fix any ∞–horizon consumption game with $R_t = R \; \forall \; t$. Fix a particular value of $\tau \geq 1$. Assume that $\{y_t\}_{t=1}^{\infty}$ satisfies A1, and $y_{t+\tau} = \exp(\tau g) y_t, \; \forall \; t \geq 0$. Assume that $\exp(\rho g) = \delta R$. Let $x_t = x_t(R \cdot x_{t-1}, R \cdot z_{t-1})$ represent the equilibrium (Markov) consumption strategy of self t. Let

$$1 - MPC_t^z \equiv \left[\prod_{i=0}^{\tau-1} \left(1 - \frac{\partial x_{t+i}}{\partial (R z_{t+i-1})} \right) \right]^{1/\tau}$$

evaluated on the equilibrium path. Then,
$$MPC_t^z = MPC^z \equiv 1 - (\delta R^{1-\rho})^{1/\rho} \qquad \forall t \geq 2.$$

Note that Proposition 4 assumes that the growth rate of labor income is related to the return on capital by the steady state equation in Proposition 2: $\exp(\rho g) = \delta R$. Note also that the resulting measure of the marginal propensity to consume, $1 - (\delta R^{1-\rho})^{1/\rho}$, is equivalent to the marginal propensity to consume in the standard Ramsey model with no liquidity constraints and exponential discount function δ^t. For all reasonable parameter values MPC^z is close to zero. Recall that the first proposed measure of the MPC out of illiquid assets (i.e., the MPC measure introduced in Proposition 3) was exactly equal to zero. Finally, contrast these proposed measures of the MPC out of illiquid assets (which take values close to or exactly equal to zero) with the unity marginal propensity to consume out of liquid assets.

D. Ricardian Equivalence

In the economy analyzed in this paper, the sequence of exogenous cash flows matters, in a way that is independent of the present value of those cash flows. This is immediately apparent from Figure II. Because taxation schemes affect these exogenous cash flows, Ricardian equivalence will be violated. Moreover, the model generates such violations even when the consumer has a large asset stock at all times. Hence, Ricardian equivalence is violated for all agents, whether or not they hold substantial wealth.

E. Declining Savings Rates in the 1980s

The golden eggs model may help to explain the decline in U. S. savings rates during the 1980s. I pursue two approaches in this subsection. The first explanation is driven by the fact that during the 1980s a relatively large proportion of national income was realized as cash flow to consumers. However, I am unsatisfied with this first story for reasons that I describe below. Hence, I focus most of my attention on a second explanation that is driven by developments in the consumer credit market.

Hatsopoulos, Krugman, and Poterba [1989] document the ob-

servation that cash flow to consumers (as a percentage of NNP) was high during the 1980s relative to the 1970s. They report that from 1970–1979 cash flow averaged 77.9 percent of NNP, while the corresponding number for the 1980–1987 period was 80.8 percent. They trace this increase to several sources, notably higher interest income (4.5 percentage points), higher transfers (2.2 percentage points), and higher after-tax cash from takeovers (0.6 percentage points).[14]

Using aggregate data, Hatsopoulos, Krugman, and Poterba [1989] estimate a high marginal propensity to consume out of current cash flow. Coupling this result with the higher cash flow levels, they are able to explain most of the savings decline in the 1980s. However, they do not explain *why* consumers should have such a high propensity to consume out of cash flow. The golden eggs model complements their analysis by providing a model that explains the high MPC. However, note that the golden eggs model can only explain the high MPC out of cash flow; the model cannot explain why the cash flow was high in the first place. Hence, application of the golden eggs model may only relabel the puzzle, changing it from a consumption puzzle to a cash flow puzzle.

The golden eggs model suggests a second explanation for the low level of savings during the past decade. The 1980s was a period of rapid expansion in the U. S. consumer credit market. Increasing access to *instantaneous* credit has reduced the effectiveness of commitment devices like illiquid assets. The golden eggs model predicts that the elimination of commitment devices would lower the level of capital accumulation. I will show that if the credit market were to become sufficiently sophisticated that consumers could instantaneously borrow against their illiquid assets, then the steady state capital-output ratio would fall. I calibrate this fall at the end of the subsection.

The rapid expansion of the U. S. consumer credit market provides the starting point for the argument summarized in the previous paragraph. One example of the expansion in instantaneous credit has been the growth in credit cards.[15] In 1970 only 16 percent of all U. S. families had a third party credit card (e.g., prede-

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14. Offsetting falls in cash flow occurred in labor income (−0.3 percentage points), noninterest capital income in disposable income (−2.0 percentage points), and taxes (−2.1 percentage points).

15. Another important development in the U. S. credit market has been the expanded use of home equity lines of credit. Before the mid-1980s home equity lines of credit were almost unheard of. By 1993–1994 8.3 percent of homeowners had a home equity line of credit. See Canner and Luckett [1994], p. 572.

cessors of current cards like Visa and MasterCard). By 1989 54 percent had one.[16] During this same period credit card acceptance by retailers also increased dramatically. Large retailers did not accept credit cards during the 1970s. In 1979 J. C. Penney broke ranks with its competitors and became the first major retailer to accept third party credit cards. By the end of the 1980s almost all large retailers accepted third party cards. The growth of ATMs (automatic teller machines) augmented the impact of the credit card expansion by enabling credit cardholders to readily receive cash advances. Regional and national ATM networks first began to form in the late 1970s and early 1980s.[17] Altogether these developments led to an explosion in revolving credit, which is principally composed of credit card debt. From 1970 to 1995, revolving credit grew from 3.7 percent to 36.3 percent of total consumer credit.[18] No single year stands out as the date at which most consumers experienced a sharp increase in their personal access to instantaneous credit. However, it is safe to say that by the mid-1980s most families had a third party credit card, and this card could be used in most large retail stores, or could be used in ATMs to receive cash advances to make purchases in the stores that still did not accept credit cards. Together these observations suggest that the mid-1980s represents the first time that a representative U. S. family had instantaneous access to consumer credit, or could rapidly apply for such access.

Introducing *instantaneous* credit into the golden eggs model dramatically changes the equilibrium analysis. (Recall that the original model had credit which could be accessed with a one-period delay.) In the original model consumption was bounded above by cash on hand:

$$c_t \leq y_t + R_t x_{t-1}.$$

With instantaneous access to credit, consumption is now constrained to lie below the sum of cash on hand and the value of all credit lines that can be instantaneously set up or are already set

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16. See Canner and Luckett [1992], p. 656.
17. See Mandell [1990] for a short history of the credit card industry.
18. Consumer credit includes automobile loans, revolving credit, "other" installment credit, and noninstallment credit. "Other" installment credit includes "mobile home loans and all other installment loans not included in automobile or revolving credit, such as loans for education, boats, trailers, or vacations. These loans may be secured or unsecured. Noninstallment credit is credit scheduled to be repaid in a lump sum, including single-payment loans, charge accounts, and service credit" [*Economic Report of the President* 1996, Table B-73].

up. I assume that the value of these existing and potential credit lines is approximately equal to the value of the illiquid assets held by the consumer. Hence, consumption is now constrained by

$$c_t \leq y_t + R_t x_{t-1} + R_t z_{t-1}.$$

In all other ways the model remains the same.

PROPOSITION 5. Consider the general equilibrium economy analyzed above, but now assume that consumers can instantaneously borrow against their illiquid asset. This economy is equivalent to one in which there is no illiquid asset (i.e., x is the only asset). In such an economy there exists a unique steady state, and in that steady state,

(4) $$\exp(\rho g) = \beta \delta R + (1 - \beta) \delta \exp(g).$$

COROLLARY. In the steady state characterized in Proposition 5 the capital-output ratio is less than the steady state capital-output ratio in the economy with the commitment technology.

Table I reports the magnitude of the reduction in steady state capital that occurs when financial innovation moves an economy from a golden eggs financial technology to a new financial technology in which it is possible to instantaneously borrow against the illiquid asset (i.e., when financial innovation eliminates the illiquidity that makes partial commitment possible). The entries of Table I are derived in five steps. First, I assume that the U. S. economy has historically been a golden eggs economy, with $\alpha = 0.36$, $d = 0.08$, $g = 0.02$, and $K/Y = 3$. Second, I calibrate preference parameters ρ and δ based on equation (2) (the steady state equation in golden eggs economies) and the competitive equilibrium condition, $r = \alpha Y / K - d = (0.36)\frac{1}{3} - 0.08$. (Recall that equation (2) is independent of β.) These equations jointly imply that

(5) $$\exp(\rho(0.02)) = \delta(1.04).$$

Third, I take the set of preference parameter values derived in step 2 (i.e., defined in equation (5)) and plug that set into equation (4), the new steady state equation (i.e., the steady state equation associated with the economy in which consumers can instantaneously borrow against their illiquid assets). This yields the following "constrained" steady state equation that holds in the new economy:

TABLE I
STEADY STATE INTEREST RATES AND CAPITAL-OUTPUT RATIOS IN ECONOMIES WITH
AND WITHOUT PARTIAL COMMITMENT

	With commitment (i.e., no instantaneous credit)		Without commitment (i.e., instantaneous credit)	
	r	$\dfrac{K}{Y}$	r	$\dfrac{K}{Y}$
$\beta = 0.2$	0.040	3.00	0.119	1.81
$\beta = 0.4$	0.040	3.00	0.070	2.40
$\beta = 0.6$	0.040	3.00	0.053	2.70
$\beta = 0.8$	0.040	3.00	0.045	2.88
$\beta = 1.0$	0.040	3.00	0.040	3.00

$$(6) \qquad 1.04 = \beta R + (1 - \beta)\exp(.02).$$

Note that this constrained steady state relationship is independent of ρ and δ, and depends exclusively on β and R. Fourth, I vary β, the only free preference parameter in the constrained new steady state equation, and calculate the corresponding capital-output ratios (again using the competitive equilibrium relationship $r = \alpha(Y/K) - d$). Fifth, I compare these new capital-output ratios with the historical capital-output ratio.

Note that when commitment is available, the steady state interest rate and capital-output ratio are independent of the value of β (see Proposition 1). Now consider an example of a transition from a commitment economy to an economy without commitment. For a β value of 0.6, elimination of the commitment technology raises the steady state real interest rate 1.3 percentage points. This corresponds to a reduction in the capital-output ratio of 0.3.

These results should be compared with actual U. S. experience during the 1980s and 1990s. The model predicts that capital accumulation should have fallen at the same time that consumers gained access to instantaneous credit (approximately the mid-1980s). All measures of capital accumulation show a marked downturn that starts in the 1980s and continues into the 1990s. For example, U. S. personal savings as a percent of disposable personal income fell from an average of 7.3 percent from 1946–1984, to an average of 5.3 percent from 1985–1994. The 1985–1994 period had the lowest average saving rate of any ten-year

span in the postwar period.[19] The ratio of national net worth to gross national product (i.e., the U. S. capital-output ratio) fell from an average of 3.2 from 1946–1984, to 2.8 in 1994. The 1994 value is the low point for the series in the postwar period.[20]

F. Welfare Analysis of Financial Innovation

The introduction of instantaneous credit increases consumers' choice sets. Standard economic models imply that this development might lower levels of capital accumulation, but would *raise* consumer welfare. Yet, in the United States, policy-makers and pundits are concerned that instantaneous credit is somehow bad for consumers.

The golden eggs framework provides a formal model of the *costs* of financial innovation. By enabling the consumer to instantaneously borrow against illiquid assets, financial innovation eliminates the possibility for partial commitment. This has two effects on the welfare of the current self. First, the current self no longer faces a self-imposed liquidity constraint and can therefore consume more in its period of control. Second, future selves are also no longer liquidity constrained and may also consume at a higher rate out of the wealth stock that they inherit. The first effect makes the current self better off. The second effect makes the current self worse off (since the current self would like to constrain the consumption of future selves). Under most parameterizations the impact of the second effect dominates, and the welfare of the current self is reduced.

Formally, I measure the welfare loss by calculating the minimum one-time payment—paid to a representative consumer—which would induce the representative consumer to switch from an infinite horizon golden eggs economy to an infinite horizon instantaneous credit economy. (Using the notation of Section II, the hypothetical payment that induces indifference is made during subperiod 2 of time t, and the indifference is from the perspective of self t.) I assume that the representative consumer starts in

19. National Income and Product Accounts, Table 2.1, Bureau of Economic Analysis, U. S. Department of Commerce.

20. National net worth is calculated from Tables B.11 and B.109 in *Balance Sheets for the U. S. Economy 1945–94*, Board of Governors of the Federal Reserve System. National net worth represents the sum of lines 1 and 30 from Table B.11 added to the difference between lines 43 and 42 from Table B.109. Gross national product is calculated by the Bureau of Economic Analysis, U. S. Department of Commerce.

the steady state of the golden eggs economy; this steady state is characterized in Proposition 1. The representative agent remains in that steady state if she remains in the golden eggs economy. By contrast, if the representative agent switches to the instantaneous credit economy (i.e., if she switches to the economy in which it is possible to instantaneously borrow against illiquid assets), then the new economy asymptotically converges to the steady state characterized in Proposition 5. The starting point for this convergence is the golden eggs steady state capital stock augmented (depleted) by a payment at time period one.

I calibrate this exercise with $\alpha = 0.36$, $d = 0.08$, $g = 0.02$, $\delta = 0.98$ and $\rho = 1$, and I assume that the golden eggs steady state is characterized by the historical capital-output ratio $K/Y = 3$. Note that these values are consistent with the steady state equation for the golden eggs economy (see Proposition 1).

The convergence path for the economy in which instantaneous borrowing is possible is characterized by a nonstandard Euler equation derived in Laibson [1996]:

(7) $$u'(c_t) = R_{t+1}\delta u'(c_{t+1})[\lambda(\beta - 1) + 1],$$

where λ is given by[21]

(8) $$\lambda = \frac{1 - \delta}{1 - \delta(1 - \beta)}.$$

Note that when $\beta = 1$ this Euler equation reduces to the standard case. To solve for the convergence path conditional on a starting level of financial wealth, it is necessary to search for the unique sequence $\{c_t, R_t\}_{t=1}^\infty$ that is 1) consistent with the nonstandard Euler equation given above; 2) consistent with the dynamic budget constraint; 3) consistent with the capital market competitive equilibrium condition; and 4) consistent with asymptotic convergence to the steady state characterized in Proposition 5. Identifying this sequence can be reduced to a one-dimensional search over values of c_1: start with a guess of the equilibrium value of c_1; combine this value of c_1 with the dynamic budget constraint and the competitive equilibrium conditions to generate R_2; use the nonstandard Euler equation to calculate c_2 as a function of c_1 and R_2; iterate the last two steps to generate a sequence $\{c_t, R_t\}_{t=1}^\infty$ that can be checked for asymptotic steady state convergence; if the sequence does not converge, start with a new guess for c_1.

This algorithm provides a way of calculating the convergence

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21. The derivation for λ uses the calibration assumption $\rho = 1$.

TABLE II
PAYMENTS TO INDUCE INDIFFERENCE
BETWEEN GOLDEN EGGS ECONOMY
AND INSTANTANEOUS CREDIT ECONOMY

	Payment as percent of output
$\beta = 0.2$	69.6
$\beta = 0.4$	29.5
$\beta = 0.6$	9.0
$\beta = 0.8$	1.6
$\beta = 1.0$	0.0

path given any level of initial financial wealth in the instantaneous credit economy. Once this has been done, it is straightforward to calculate the level of initial financial wealth in the instantaneous credit economy that induces indifference with the level of initial financial wealth in the golden eggs economy. The payment level is the difference between these two financial wealth levels. The payment level is reported in Table II, where it is normalized by the level of output at time of payment.[22] Note that a positive payment implies that the consumer needs compensation to induce her to willingly switch to the instantaneous credit scenario. Hence, if payment were withheld, the consumer would be worse off in the instantaneous credit scenario. Table II reports these normalized payments for a range of β values.

Note that when $\beta = 1$ there is no welfare loss. When $\beta = 1$, the consumer's preferences are not dynamically inconsistent, and the consumer has no need to constrain her future selves. By contrast, for the other cases, ($\beta \in \{0.2, 0.4, 0.6, 0.8\}$) the consumer is made worse off by financial innovation. Being able to borrow against illiquid assets is welfare reducing. However, note that this is not always the case. For β values sufficiently close to zero, the consumer is made better off by being able to splurge almost all of her financial wealth immediately. However, for the range of reasonable β values reported in Table II, the consumer is always made worse off by financial innovation.

Of course the costs of financial innovation explored above may be offset by unmodeled gains, like being able to consume in unforeseen emergencies (which are ruled out in the deterministic framework of this paper). The point of this subsection is to dem-

22. Note that output at time of payment is the same under the two scenarios, since output at any given period is determined by capital put aside in the previous period.

onstrate that there are potentially important costs that accompany those other well-known benefits of extra liquidity.

V. EVALUATION AND EXTENSIONS

I have analyzed the consumption problem of a dynamically inconsistent decision-maker who has access to a crude commitment mechanism. The model helps to explain many of the empirical puzzles in the consumption literature, notably consumption-income tracking and asset-specific MPCs. However, the model has several drawbacks that suggest four important areas to pursue extensions.

First, the golden eggs model does not explain how consumers accumulate assets in the first place. Note that consumption is always greater than labor income on the equilibrium path. However, this is less of a problem than it might first appear. Although there is evidence that individuals often consume less than they earn in labor income, most of this saving is nondiscretionary (e.g., pension contributions, life-insurance payments, mortgage payments, and other payments to creditors). Bringing such "nondiscretionary savings" into the model can be done very simply. For example, the consumer could elect to take on a 30-period mortgage obligation at time zero, represented by a mortgage payment of m for the next 30 periods. Then the consumer's cash flow at time $t \leq 30$ would be $y_t + R_t x_{t-1} - m$, which would be less than y_t if m were greater than $R_t x_{t-1}$. A related way to model nondiscretionary savings would be to let the consumer set x_{t-1} itself less than zero, (e.g., $\underline{x} \leq x_{t-1}$, where $\underline{x} < 0$).

A second problem associated with the model is the anomalous prediction that consumers will always face a binding self-imposed liquidity constraint. For example, the golden eggs model predicts that after making their consumption choice, consumers should have no liquid funds left in their bank accounts. This prediction contradicts many consumers' experiences. However, this problem can be readily addressed by introducing a precautionary savings motive for holding liquidity. For example, consider a continuous-time analog of the golden eggs model, and assume that instantaneous liquidity needs arrive with some hazard rate. Then in equilibrium the consumer will only rarely completely exhaust her liquidity.

A third problem with the golden eggs model is that some con-

sumers may not need to use external commitment devices (like illiquid assets) to achieve self-control. Consumers may have internal self-control mechanisms, like "will power" and "personal rules." In Laibson [1994a] I analyze an infinite-horizon consumption/savings game with no external commitment technology and find a multiplicity of Pareto-rankable equilibria. I interpret this multiplicity as a potential model for self-control and willpower. However, this approach raises problematic and as yet unresolved equilibrium selection problems. More work is needed to develop theoretically robust models of internal self-control mechanisms, and to empirically validate such models.

The fourth problem with the golden eggs model is that some consumers may have access to an array of "social" commitment devices that are far richer than the simple illiquid asset proposed in this essay. In Laibson [1994b] I analyze the problem of a consumer who can use social systems like marriage, work, and friendship to achieve personal commitment. Future work should try to identify the most important mechanisms that consumers use to overcome the self-control problems induced by hyperbolic preferences.

APPENDIX

Theorem 1 is proved with four intermediate lemmas. These lemmas apply to the game described in Theorem 1.

LEMMA 1. Let s be a resource-exhausting element of the joint strategy space S. Assume that s satisfies P1–P4. Then for all histories h_t, strategy s implies that $c_t \geq y_t$.

Proof of Lemma 1. Use induction to prove result. Fix a period t and feasible history, h_t. Let $s \mid h_t = \{c_{t+\tau}^A, x_{t+\tau}^A, z_{t+\tau}^A\}_{\tau=0}^{T-t}$. Assume that $c_{t+\tau} \geq y_{t+\tau} \ \forall \ \tau \geq 1$. By P1, $u'(c_t) \geq \max_{\tau \in \{1, \ldots, T-t\}} \beta\delta^\tau (\Pi_{i=1}^\tau R_{t+i}) u'(c_{t+\tau})$. If this inequality is strict, then P2 implies that $c_t \geq y_t$. So WLOG assume that $u'(c_t) = \max_{\tau \geq 1} \beta\delta^\tau (\Pi_{i=1}^\tau R_{t+i}) u'(c_{t+\tau})$:

$$u'(c_t) = \max_{\tau \in \{1,\ldots,T-t-1\}} \beta\delta^\tau \left(\prod_{i=1}^\tau R_{t+i}\right) u'(c_{t+\tau}) \quad \text{by assumption}$$

$$\leq \max_{\tau \in \{1,\ldots,T-t-1\}} \beta\delta^\tau \left(\prod_{i=1}^\tau R_{t+i}\right) u'(y_{t+\tau}) \quad \text{by assumption}$$

$$\leq u'(y_t) \quad \text{by A1.}$$

So $u'(c_t) \leq u'(y_t)$, and hence $c_t \geq y_t$. After confirming that $c_T \geq y_T$ (by resource exhaustion), the proof is completed by applying a standard induction argument.

LEMMA 2. Let s^A and s^B be resource-exhausting elements of the joint strategy space S. Assume that s^A and s^B satisfy P1–P4. Let $\{c_t^A, x_t^A, z_t^A\}_{t=1}^T$ and $\{c_t^B, x_t^B, z_t^B\}_{t=1}^T$ be the respective paths of actions generated by s^A and s^B. Fix a particular value of t, and assume $c_{t+\tau}^A \geq c_{t+\tau}^B \ \forall \ \tau \geq 1$, with $c_{t+\tau}^A > c_{t+\tau}^B$ for at least one $\tau \geq 1$. Then $c_t^A \geq c_t^B$.

Proof of Lemma 2. By P1, $u'(c_t^A) \geq \max_{\tau \in \{1, \ldots, T-1\}} \beta \delta^\tau$ $(\Pi_{i=1}^\tau R_{t+i}) u'(c_{t+\tau}^A)$. If this is satisfied with equality, then

$$u'(c_t^B) \geq \max_{\tau \in \{1,\ldots,T-t-1\}} \beta \delta^\tau \left(\prod_{i=1}^\tau R_{t+i} \right) u'(c_{t+\tau}^B) \quad \text{by P1}$$

$$\geq \max_{\tau \in \{1,\ldots,T-t-1\}} \beta \delta^\tau \left(\prod_{i=1}^\tau R_{t+i} \right) u'(c_{t+\tau}^A) \quad \text{by assumption}$$

$$= u'(c_t^A) \quad \text{by assumption.}$$

Hence, $u'(c_t^B) \geq u'(c_t^A)$, implying that $c_t^A \geq c_t^B$. So WLOG assume that $u'(c_t^A) > \max_{\tau \in \{1, \ldots, T-t-1\}} \beta \delta^\tau (\Pi_{i=1}^\tau R_{t+i}) u'(c_{t+\tau}^A)$. By P2, $c_t^A = y_t + R_t x_{t-1}$. If $t = 1$, then $c_t^A \geq c_t^B$, since $c_1^B \leq y_1 + R_1 x_0 = c_1^A$. So WLOG assume that $t \geq 2$. If $x_{t-1}^B \geq x_{t-1}^A$, then $c_t^B \leq y_t + R_t x_{t-1}^B \leq y_t + R_t x_{t-1}^A = c_t^A$. So WLOG assume $x_{t-1}^B > x_{t-1}^A \geq 0$.

$$0 \quad < \quad \sum_{\tau=1}^{T-t} \left(\prod_{i=1}^\tau R_{t+i}^{-1} \right) (c_{t+\tau}^A - c_{t+\tau}^B) \quad \text{by assumption}$$

$$\leq \quad \sum_{\tau=1}^{T-t} \left(\prod_{i=1}^\tau R_{t+i}^{-1} \right) (c_{t+\tau}^A - y_{t+\tau}) \quad \text{by Lemma 1}$$

$$= \quad R_t(x_{t-1}^A + z_{t-1}^A) - c_t^A + y_t^A \quad \text{by res. exhaust.}$$

$$= \quad R_t z_{t-1}^A \quad \text{as } c_t^A = y_t + R_t x_{t-1}^A.$$

So $z_{t-1}^A > 0$, and,

$$u'(c_t^B) \geq \max_{\tau \in \{1,\ldots,T-t-1\}} \delta^\tau \left(\prod_{i=1}^\tau R_{t+i} \right) u'(c_{t+\tau}^B) \quad \text{by P3 and } x_{t-1}^B > 0$$

$$\geq \max_{\tau \in \{1,\ldots,T-t-1\}} \delta^\tau \left(\prod_{i=1}^\tau R_{t+i} \right) u'(c_{t+\tau}^A) \quad \text{by assumption}$$

$$\geq u'(c_t^A) \quad \text{by P4 and } z_{t-1}^A > 0.$$

Hence, $u'(c_t^B) \geq u'(c_t^A)$, implying that $c_t^A \geq c_t^B$.

LEMMA 3. Let s be a resource-exhausting element of the joint strategy space S. Assume that s satisfies P1–P4. Let $\{c_t, x_t, z_t\}_{t=1}^T$ represent the path of actions generated by s. Then $c_t = y_t + R_t x_{t-1} \,\forall\, t \geq 2$.

Proof of Lemma 3. Suppose that $c_t < y_t + R_t x_{t-1}$ for some $t \geq 2$ and look for a contradiction. By P1 and P2, $u'(c_t) = \max_{\tau \in \{1, \ldots, \ T-t-1\}} \beta \delta^\tau \left(\Pi_{i=1}^\tau R_{t+i}\right) u'(c_{t+\tau})$, so $u'(c_t) < \max_{\tau \in \{1, \ldots, T-t-1\}} \delta^\tau \left(\Pi_{i=1}^\tau R_{t+i}\right) u'(c_{t+\tau})$. Hence, by P3, $x_{t-1} = 0$. So $c_t < y_t$, which contradicts Lemma 1.

LEMMA 4. Let $\{c_t, x_t, z_t\}_{t=1}^T$ be a solution path to the following problem:

$$\max_{\{c_t, x_t, z_t\}_{t=1}^T} \quad u(c_1) + \beta \sum_{\tau=1}^{T-1} \delta^\tau u(c_{1+\tau})$$

subject to

$$x_t, z_t \geq 0 \quad \forall t \geq 1$$
$$c_t \leq y_t + R_t x_{t-1} \quad \forall t \geq 1$$
$$x_t + z_t = R_t(x_{t-1} + z_{t-1}) + y_t - c_t \quad \forall t \geq 1$$
$$x_0, z_0 \text{ fixed}$$
$$x_T = z_T = 0$$
$$\{c_t, x_t, z_t\}_{t=2}^T \text{ satisfies P1–P4.}$$

Then $\{c_t, x_t, z_t\}_{t=1}^T$ satisfies P1–P4.

Proof of Lemma 4. The first step in the proof is to show that the solution set of the program above is a subset of the solution set of the program below.

$$\max_{\{c_t, x_t, z_t\}_{t=1}^T} \quad u(c_1) + \beta \sum_{\tau=1}^{T-1} \delta^\tau u(c_{1+\tau})$$

subject to

$$x_t, z_t \geq 0 \quad \forall t \geq 1$$
$$c_t \leq y_t + R_t x_{t-1} \quad \forall t \geq 1$$
$$x_t + z_t = R_t(x_{t-1} + z_{t-1}) + y_t - c_t \quad \forall t \geq 1$$
$$x_0, z_0 \text{ fixed}$$
$$x_T = z_T = 0$$
$$c_2 \geq y_2$$
$$c_t = y_t + R_t x_{t-1} \quad \forall t \geq 3.$$

Henceforth I will refer to these, respectively, as program I and program II. Note that program II is a convex program with linear

constraints, so the Kuhn-Tucker first-order conditions are necessary and sufficient for a global optimum. I will return to this fact later in the proof.

The following notation will be used to prove the lemma. Let Ω represent the set of all real vectors, $\omega = \{c_t, x_t, z_t\}_{t=1}^T$. Let $C_I \subset \Omega$ ($C_{II} \subset \Omega$) represent the subset of vectors in Ω which satisfy the constraints of program I (II). Let $C_I^* \subset \Omega$ ($C_{II}^* \subset \Omega$) represent the subset of vectors in Ω that are solutions to program I (II).

The first step in the proof is to show that $C_I \subset C_{II}$. Fix any $\omega \in C_I$, and let $\omega = \{c_t, x_t, z_t\}_{t=1}^T$. Note that the first five constraints of program I are identical to the first five constraints of program II. Also note that if $\{c_t, x_t, z_t\}_{t=2}^T$ satisfies P1–P4, then by Lemma 1, $c_2 \geq y_2$, and by Lemma 3, $c_t = y_t + R_t x_{t-1} \, \forall \, t \geq 3$. Hence, $\omega \in C_{II}$, implying that $C_I \subset C_{II}$.

The next step is to show $C_I^* \subset C_{II}^*$. Fix any $\omega \in C_I^*$. Fix any $\omega' \in C_{II}^*$, and let $\omega' = \{c_t, x_t, z_t\}_{t=1}^T$. Define $\hat{x}_1$ such that $c_2 = y_2 + R_2 \hat{x}_1$. Let ω'' be equivalent to ω' except that x_1 is replaced by $\hat{x}_1$, and z_1 is replaced by $\hat{z}_1 = z_1 - (\hat{x}_1 - x_1)$. Let $U(\omega)$ represent the value of the objective function evaluated at ω. Consider the following two properties of ω'': $\omega'' \in C_{II}$, $U(\omega') = U(\omega'')$. Recall that $\omega' \in C_{II}^*$. Then ω'' must also be an element of C_{II}^*. Hence ω'' must satisfy the Kuhn-Tucker conditions of program II (since the conditions are necessary and sufficient). Using the Kuhn-Tucker conditions and the definition of ω'', it is straightforward to show that $\omega'' \in C_I$. Note that $\omega'' \in C_{II}^*$ and $\omega \in C_I^* \subset C_I \subset C_{II}$ imply that $U(\omega'') \geq U(\omega)$. Note that $\omega \in C_I^*$ and $\omega'' \in C_I$ imply that $U(\omega'') \leq U(\omega)$. Hence, $U(\omega) = U(\omega'')$, which implies that $U(\omega) = U(\omega')$. So $\omega' \in C_{II}^*$ and $\omega \in C_I^* \subset C_I \subset C_{II}$ imply that $\omega \in C_{II}^*$. Hence, $C_I^* \subset C_{II}^*$.

I am now ready to complete the proof of the lemma. Let ω be a solution to program I, and let $\omega = \{c_t, x_t, z_t\}_{t=1}^T$. So $\{c_t, x_t, z_t\}_{t=2}^T$ satisfies P1–P4. Since $C_I^* \subset C_{II}^*$, ω must also satisfy the necessary and sufficient Kuhn-Tucker conditions of program II. Combining these constraints, it is straightforward to show that $\{c_t, x_t, z_t\}_{t=1}^T$ satisfies P1–P4. $\qquad\square$

Proof of Theorem 1. Suppose that there exist two resource-exhausting joint strategies, s^A, $s^B \in S$, that satisfy P1–P4. Fix any period t, and any feasible history h_t. Let $s^A \,|\, h_t \equiv \{c_{t+\tau}^A, x_{t+\tau}^A, z_{t+\tau}^A\}_{\tau=0}^{T-t}$, $s^B \,|\, h_t \equiv \{c_{t+\tau}^B, x_{t+\tau}^B, z_{t+\tau}^B\}_{\tau=0}^{T-t}$. By resource exhaustion and Lemma 2, $c_{t+\tau}^A = c_{t+\tau}^B \, \forall \, \tau \geq 0$. Hence, by Lemma 3, $x_{t+\tau}^A = x_{t+\tau}^B \, \forall \, \tau \geq 0$. This in turn implies that $z_{t+\tau}^A = z_{t+\tau}^B \, \forall \, \tau \geq 0$, as

a result of the savings constraints. Because the proof started with arbitary h_t, we can conclude that $s^A = s^B$ proving that there exists a unique resource-exhausting joint strategy, $s^* \in S$, that satisfies P1–P4. The second part of the theorem follows from this uniqueness result, Lemma 4, and a standard induction argument.   □

Proof of Proposition 1. If a steady state satisfies A1, then $\exp(\rho g) \geq R\delta$. Moreover, it is easy to construct a steady state at which $\exp(\rho g) = R\delta$. Suppose that there exists a steady state at which $\exp(\rho g) > R\delta$. Then by P1–P4, $\lim_{t \to \infty} z_t = \lim_{t \to \infty} x_t = 0$, which implies that no such steady state could exist.   □

Proof of Proposition 2. By P1, $u'(c_t) \geq \beta\delta^\tau (\Pi_{i=1}^\tau R_{t+i}) u'(c_{t+\tau}) \forall$ $t \geq 2, \tau \geq 0$. Suppose that this inequality is satisfied exactly for some t, τ pair. Then $x_{t-1} = 0$ by P3. Hence,

$$u'(y_t) \; = \; u'(c_t) \qquad\qquad \text{by Lemma 3}$$

$$= \; \beta\delta^\tau \left(\prod_{i=1}^\tau R_{t+i}\right) u'(c_{t+\tau}) \qquad \text{by assumption}$$

$$\leq \; \beta\delta^\tau \left(\prod_{i=1}^\tau R_{t+i}\right) u'(y_{t+\tau}) \qquad \text{by Lemma 3.}$$

But $u'(y_t) \leq \beta\delta^\tau (\Pi_{i=1}^\tau R_{t+i}) u'(y_{t+\tau})$ violates A1, (since A1 is assumed to hold strictly). So WLOG, assume that $u'(c_t) > \beta\delta^\tau$ $(\Pi_{i=1}^\tau R_{t+i}) u'(c_{t+\tau}) \forall t \geq 2, \tau \geq 0$. Hence, for sufficiently small $|\epsilon|$ > 0, $u'(c_t+\epsilon) > \beta\delta^\tau (\Pi_{i=1}^\tau R_{t+i}) u'(c_{t+\tau}) \forall t \geq 2, \tau \geq 0$. So by P3 and the uniqueness result of Theorem 1, in the subgame starting after any sufficiently small perturbation to the liquid asset stock, the equality $c_t = y_t + R_t x_{t-1}$ continues to hold, and hence, $\partial c_t/(\partial(R_t x_{t-1})) = 1$.   □

Proof of Proposition 3. WLOG, assume that $u'(c_t) > \beta\delta^\tau$ $(\Pi_{i=1}^\tau R_{t+i}) u'(c_{t+\tau}) \forall t \geq 2, \tau \geq 0$ (see Proof of Proposition 2). Hence, for sufficiently small $|\epsilon| > 0$, $u'(c_t) > \beta\delta^\tau (\Pi_{i=1}^\tau R_{t+i}) u' (c_{t+\tau} +$ $(\Pi_{i=1}^\tau R_i)\epsilon) \forall t \geq 2, \tau \geq 0$. So current consumption does not change when z_{t-1} is perturbed.   □

Proposition 4 is proved with two intermediate lemmas.

LEMMA 5. Fix the economy described in Proposition 4. On the equilibrium path of this game $u'(c_t) = (\delta R)^\tau u'(c_{t+\tau}) \forall t \geq 2$.

Proof of Lemma 5. Suppose that $u'(c_t) < (\delta R)^\tau u'(c_{t+\tau})$ for some $t \geq 2$. Then P3 implies $x_{t-1} = 0$, implying that

$$u'(y_t) \ = \ u'(c_t) \qquad\qquad \text{by Lemma 3}$$

$$< \ (\delta R)^\tau u'(c_{t+\tau}) \qquad \text{by assumption}$$

$$\leq \ (\delta R)^\tau u'(y_{t+\tau}) \qquad \text{by Lemma 3.}$$

Hence, $y_t^{-\rho} < (\delta R)^\tau y_{t+\tau}^{-\rho}$, implying that $\exp(\tau g) y_t > y_{t+\tau}$, which contradicts the assumptions of Proposition 4.

Alternatively, suppose that $u'(c_t) > (\delta R)^\tau u'(c_{t+\tau})$ for some $t \geq$ 2. Then,

$$u'(y_t) \ \geq \ u'(c_t) \qquad\qquad\qquad \text{by Lemma 1}$$

$$> \ (\delta R)^\tau u'(c_{t+\tau}) \qquad\qquad \text{by assumption}$$

$$= \ (\delta R)^\tau u'(y_{t+\tau} + R x_{t+\tau-1}) \qquad \text{by Lemma 3.}$$

Note that $u'(y_t) = (\delta R)^\tau u'(y_{t+\tau})$, follows from the assumptions. So the previous inequalities imply that $x_{t+\tau-1} > 0$, which together with P3 implies that $u'(c_{t+\tau}) \geq \sup_{n \geq 1}(\delta R)^n u'(c_{t+\tau+n})$. In addition, $x_{t+\tau-1} > 0$, implies that $z_{t-1} > 0$, which together with P4 implies that $u'(c_t) \leq \sup_{n \geq 1}(\delta R)^n u'(c_{t+n})$. So there exists a finite $\hat{t} \in \{t + 1, t + 2, \ldots, t + \tau - 1\}$, such that $u'(c_{\hat{t}}) > \sup_{n \geq 1}(\delta R)^n u'(c_{\hat{t}+n})$. Hence, by P4, $z_{\hat{t}-1} = 0$, contradicting the result that $x_{t+\tau-1} > 0$. $\square$

LEMMA 6. Fix the economy described in Proposition 4. On the equilibrium path of this game $x_{t+\tau} = \exp(\tau g) x_t$, $z_{t+\tau} = \exp(\tau g) z_t$, $\forall\, t \geq 1$.

Proof of Lemma 6. By Lemma 5, $u'(c_t) = (\delta R)^\tau u'(c_{t+\tau})\ \forall\, t \geq$ 2. Combining this with Lemma 3 implies that $u'(y_t + R x_{t-1}) = (\delta R)^\tau u'(y_{t+\tau} + R x_{t+\tau-1})$. The assumptions, $\exp(\rho g) = \delta R$ and $y_{t+\tau} = \exp(\tau g) y_t$, can be used to simplify the previous equation, yielding, $x_{t+\tau-1} = \exp(\tau g) x_{t-1}\ \forall\, t \geq 2$. Note that resource exhaustion and Lemma 3 together imply that $z_t = \sum_{i=1}^{\infty} R^{-i} x_{t+i}\ \forall\, t \geq 1$. So $z_{t+\tau} = \sum_{i=1}^{\infty} R^{-i} x_{t+\tau+i} = \sum_{i=1}^{\infty} R^{-i} \exp(\tau g) x_{t+i} = \exp(\tau g) z_t\ \forall\, t \geq 1.\square$

Proof of Proposition 4. To prove this proposition, I consider two games: an original game, and a perturbed game. The perturbed game is identical to the original game except that in the perturbed game illiquid assets are higher at time zero. Let Δa represent the difference between variable a in the perturbed game and variable a in the original game. Then Lemma 3 implies that

$$\Delta z_{t+\tau-1} = R^\tau (\Delta z_{t-1}) \left(1 - \frac{\Delta x_t}{\Delta(R z_{t-1})}\right) \cdots \left(1 - \frac{\Delta x_{t+\tau-1}}{\Delta(R z_{t+\tau-2})}\right)$$

for all $t \geq 2$. Hence,

$$1 - MPC_t^z = \lim_{\Delta z_0 \to 0}\left[\left(1 - \frac{\Delta x_t}{\Delta(Rz_{t-1})}\right) \cdots \left(1 - \frac{\Delta x_{t+\tau-1}}{\Delta(Rz_{t+\tau-2})}\right)\right]^{1/\tau}$$

$$= \lim_{\Delta z_0 \to 0}\left(\frac{\Delta z_{t+\tau-1}}{\Delta z_{t-1}}\right)^{1/\tau}\frac{1}{R}$$

$$= \frac{\exp(g)}{R}$$

$$= (\delta R^{1-\rho})^{1/\rho},$$

where the second to last equality follows from Lemma 6.   □

Proof of Proposition 5. Laibson [1996] analyzes the economy without the commitment technology. I show that the infinite horizon equilibrium which corresponds to the limit of the finite horizon equilibria is characterized by constant proportional consumption of the wealth stock, where wealth is defined as the sum of financial assets and the discounted value of future labor income. Let λ represent the coefficient of proportionality; I show that λ is given by

$$\lambda = 1 - [\delta R^{1-\rho}(\lambda(\beta - 1) + 1)]^{1/\rho}.$$

With proportional consumption the steady state condition is

$$R(1 - \lambda) = \exp(g).$$

Solving these equations to eliminate λ yields equation (4).   □

Proof of Corollary to Proposition 5. Let R^* represent the steady state gross interest rate in the economy with commitment. Recall Proposition 1: $\exp(\rho g) = \delta R^*$. Using Proposition 5, it follows that $R - R^* = (r - g)(1 - \beta) > 0$ as $r > g$ is required for the existence of a steady state.

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