

GUESSING

- Guessing games embody situations where one's payoff depends on how well he or she can judge what other people think or expect. The classic example is the **Beauty Contest** game.
- The idea was originally developed by Keynes in his *General Theory of Employment, Interest and Money*. Here is a slightly simplified version of the game. Suppose that a newspaper or a web-site runs a beauty contest where photographs of n women are published. Each reader is asked to "vote" for the winner. One of the readers who voted for the ultimate winner is then selected at random and given a prize. Think about potential strategies one may utilize in this game:
 - Level 0 (naive strategy): vote for whom you like the most.
 - Level 1 (first level of sophistication): assume that others adopt level 0 behavior, so guess which woman is the most beautiful one and vote for that one.
 - Level 2 (second level of sophistication): assume that others adopt level 1 behavior, so guess which woman others guessed to be judged the most beautiful one and vote for that one.
 - ...
 - Level k
 - ...
 - One may also assume that behavior of other people is distributed across different levels of sophistication and optimally respond to that.
- Therefore in essence any woman may end up being the winner if readers believe that other readers believe that other readers believe... that she is the most beautiful one.
- The Beauty Contest is mostly a metaphor, though. This game in fact tries to attempt to capture trading in financial markets. Think about the stock market, for example. Even though everyone may agree on what the fundamental value of the share is (i.e., who the most beautiful woman is), the market price may be very different from it as long as all traders believe that other traders believe that other traders believe that... the price will stay away from the fundamental value for an extended period of time. This reasoning goes contrary to the **Efficient Market Hypothesis**.
- This idea has been formalized by **Moulin (1986)** as a so-called p -beauty-contest game. In this game, all participants simultaneously guess a number between 0 and 100. The winner is the participant whose number is closest to p times the average of all numbers submitted, where $p \in (0, 1)$. If there are multiple winners, the prize is shared equally among all of them. The unique Nash equilibrium of this game is for all players to guess 0.
- Now think about level- k behavior in this game:
 - Level 0: pick a random number uniformly between 0 and 100.
 - Level 1: mean of level 0 is 50. So guess $50p$.
 - Level 2: level 1 play $50p$, so optimally respond by playing $50p^2$.
 - ...
 - Level k : Optimally respond by playing $50p^k$.

- ...
- We observe that behavior converges to the Nash equilibrium as $k \rightarrow \infty$.
- repeated play - move toward Nash, introspection is not enough

Nagel (1995)

- Nagel (1995) implements the game proposed by Moulin experimentally for $p = 1/2$, $p = 2/3$ and $p = 4/3$ (in this case there are two Nash equilibria: everybody guessing 0 and everybody guessing 100). Each game is repeated 4 times.
- Results for the first period:

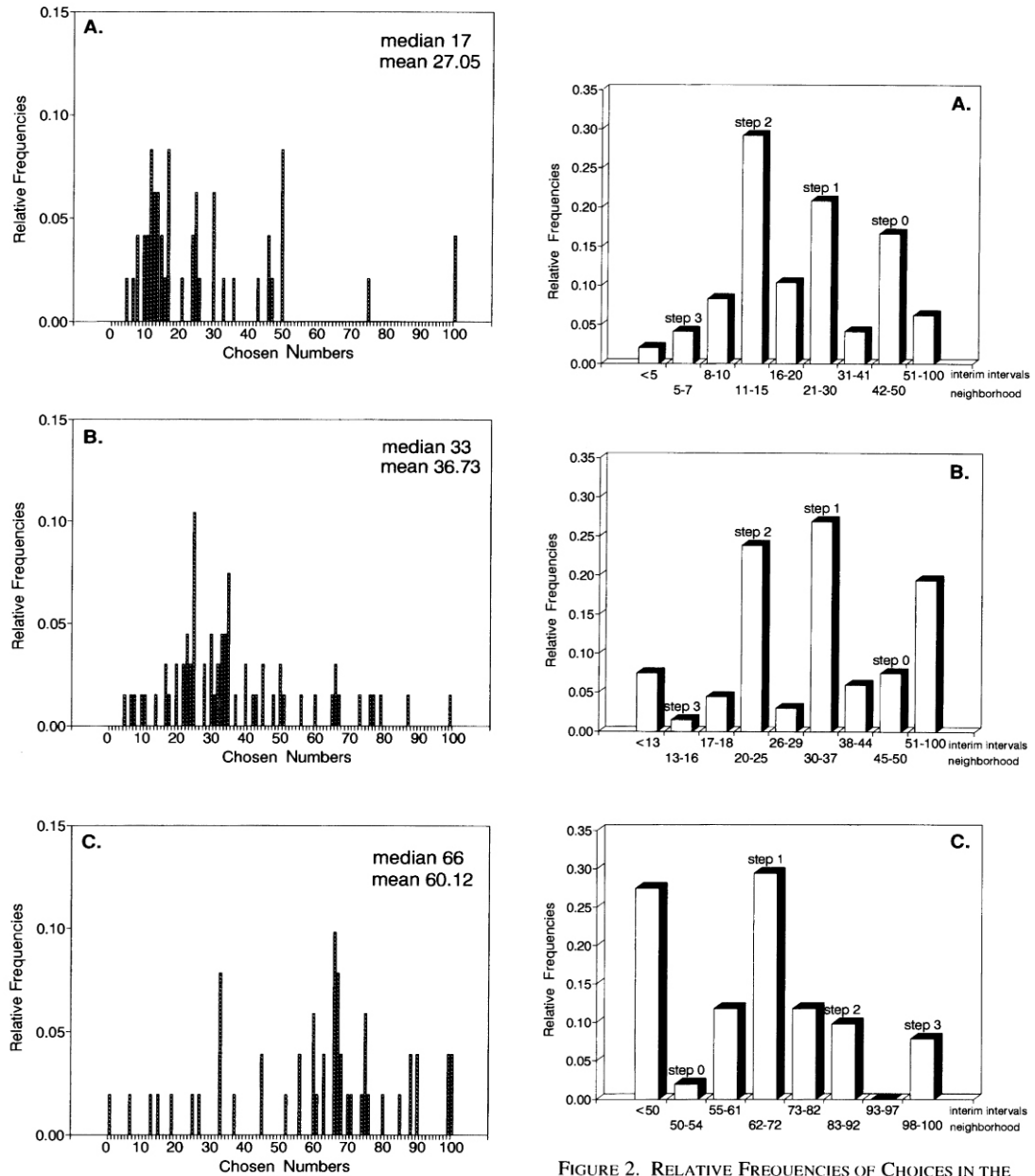


FIGURE 1. CHOICES IN THE FIRST PERIOD: A) SESSIONS 1-3 ($p = 1/2$); B) SESSIONS 4-7 ($p = 2/3$); C) SESSIONS 8-10 ($p = 4/3$)

FIGURE 2. RELATIVE FREQUENCIES OF CHOICES IN THE FIRST PERIOD ACCORDING TO THE INTERVAL CLASSIFICATION WITH REFERENCE POINT 50: A) SESSIONS 1-3 ($p = 1/2$); B) SESSIONS 4-7 ($p = 2/3$); C) SESSIONS 8-10 ($p = 4/3$)

- We observe that:

1. Higher p shifts the distribution of guesses to the right.
 2. For $p = 1/2$ and $p = 2/3$, near-level 1 and near-level 2 behavior account for almost 50 percent of guesses in the first period (a bit less for $p = 4/3$). In any of the sessions, only about 6% to 10% of guesses are level 3 or higher.
- In rounds 2-4, most subjects decrease their choices over time, suggesting the presence of learning. Furthermore, if the mean from the previous period is taken as the starting reference point for k -level rationality, then modal choices are around level 2 and the majority of observations remain below level 3.