# Problem Set 2: Producer Theory

1. Consider a production function that exhibits constant returns to scale.

   **(a)** Show that both the conditional factor demand function and the cost function are linear (homogeneous of degree one) in output. That is,

   $$z_i^C(w,q) = qz_i^C(w,1)$$

   and

   $$c(w,q) = qc(w,1).$$

   Hint: Show, by contradiction, that if $z$ is the cost-minimizing input choice for output level 1, then $qz$ is the cost-minimizing input choice for output level $q$.

   **(b)** Graph the marginal and average cost curves.

   **(c)** Given a particular price of the output and the inputs, how much will a firm using such production technology produce?

2. Consider a production function that exhibits decreasing returns to scale.

   **(a)** Show that the cost function is strictly convex in output. Hint: Show, by contradiction, that $c(w, \alpha q) > \alpha c(w,q)$ for any $\alpha > 1$. This is equivalent to the cost function being strictly convex in output.

   **(b)** Graph the marginal and average cost curves.

   **(c)** Given a particular price of the output and the inputs, how much will a firm using such production technology produce?

3. Consider a production function that exhibits increasing returns to scale.

   **(a)** Show that the cost function is strictly concave in output. Hint: Show, by contradiction, that $c(w, \alpha q) < \alpha c(w,q)$ for any $\alpha > 1$. This is equivalent to the cost function being strictly concave in output.

   **(b)** Graph the marginal and average cost curves.

   **(c)** Given a particular price of the output and the inputs, how much will a firm using such production technology produce?

   **(d)** Explain why a production technology with increasing returns to scale is inconsistent with price-taking.

4. Is it possible to have decreasing returns to scale separately in each input, and yet have increasing returns to scale overall?

5. Is it possible to have diminishing marginal productivity in each input and yet have increasing returns to scale overall?

6. This question concerns the relation between decreasing returns to scale and concavity of the production function.

(a) Suppose a production function exhibits decreasing returns to scale. Does that imply that the production function is concave? If yes, provide a proof. If not, provide a counterexample and examine the same question in the one input, one output case.

(b) Suppose a production function is *strictly* concave. Does that imply that the production function exhibits decreasing returns to scale? If yes, provide a proof. If not, provide a counterexample.

7. This question concerns the relation between increasing returns to scale and convexity of the production function.

   (a) Suppose a production function exhibits increasing returns to scale. Does that imply that the production function is convex? If yes, provide a proof. If not, provide a counterexample and examine the same question in the one input, one output case.

   (b) Suppose a production function is *strictly* convex. Does that imply that the production function exhibits increasing returns to scale? If yes, provide a proof. If not, provide a counterexample.

8. Consider a one output, multiple input scenario. Prove that the production set is convex if and only if the production function is concave.

9. In the production function with one input and one output, the elasticity of output supply with respect to the output price is the negative of the elasticity of output supply with respect to the input price.

10. Consider a two-input production function that employs capital ($K$) and labor ($L$) as the two inputs. Find the supply function, input demand functions, profit function, conditional factor demand functions, and the cost function for the following production functions (assume that both the output and the input prices are strictly positive):

    (a) (Leontieff, i.e. perfect complements) $f(K,L) = [min\{aK, bL\}]^{\alpha}$ , $\alpha \in (0,1)$, $a, b > 0$;

    (b) (Linear, i.e. perfect substitutes) $f(K,L) = [aK + bL]^{\alpha}$ , $\alpha \in (0,1)$, $a, b > 0$;

    (c) (Cobb-Douglas) $f(K,L) = K^{\alpha}L^{\beta}$, $\alpha, \beta > 0$.