Problem Set 3: General Equilibrium

1. Consider a two-consumer (a and b) and two-good economy. Let $u^a(x_1^a, x_2^a) = (x_1^a)^{1/3}(x_2^a)^{2/3}$, $\omega^a = (3, 1)$, $u^b(x_1^b, x_2^b) = (x_1^b)^{1/3}(x_2^b)^{2/3}$ and $\omega^b = (1, 1)$. Take good one as the numeraire.

   (a) Find the competitive equilibria. Graph then in the Edgeworth box.

   (b) Find the set of interior Pareto efficient allocations. Graph it in the Edgeworth box.

2. Robinson Crusoe lives on an island and derives utility $u(x, y) = \sqrt{xy}$ from goods x and y. Their respective production functions from the two factors of production, bamboo shoots b and coconuts c, are $x = [\min\{b, c\}]^2$ and $y = \sqrt{bc}$. Robinson is endowed with one unit of each of b and c.

   (a) Find the optimal allocation of b and c to their productive uses.

   (b) Can this outcome arise from a competitive allocation (i.e., if each good is produced by a price-taking profit-maximizing firm, and Robinson is a price-taking supplier of both factors and also the owner of both firms)? Explain.

3. Consider an economy with two consumers, two firms, and two commodities: time and consumption good. The consumers have to sleep 12 hours a day and hence each consumer is endowed with 12 hours of waking time and has preferences over consumption (x_1) and leisure (x_2) given by the utility function $u^1(x_1^1, x_2^1) = (x_1^1)^{3/4}(x_2^1)^{1/4}$ for consumer 1 and $u^2(x_1^2, x_2^2) = (x_1^2)^{1/4}(x_2^2)^{3/4}$ for consumer 2. Each firm produces the consumption good (y_1) out of labor ($-y_2$) using the production possibility set characterized by $y_1 - 2\sqrt{-y_2} \leq 0$. Each consumer owns one of the two firms.

   (a) Solve for the competitive equilibrium in this economy when the wage rate is normalized to 1 (including the price of consumption, consumption bundles and production plans). Do both consumers work in equilibrium?

   (b) Find the set of interior Pareto-efficient allocations in this economy. For simplicity, ignore the fact that in reality a person can consume no more than 24 hours of leisure per day.

4. Consider an exchange economy with $I + 1$ consumers and L goods. Let $(\omega_1^i, .., \omega_L^i)$ denote consumer i's initial endowment vector and u^i denote his utility function. Suppose (x, p) is a competitive equilibrium of this economy, where $x = (x^1, .., x^I, \omega^{I+1})$. That is, consumer $I+1$ consumes his initial endowment ω^{I+1} at the competitive equilibrium. Now consider the situation when consumer $I+1$ does not show up in the market. That is, consider the I-person exchange economy where the agents $i = 1, .., I$ have the same initial endowment ω^i and the utility function u^i as before. Is the allocation $(x^1, .., x^I)$ a competitive equilibrium allocation of this economy?

5. Consider an economy composed of $2I + 1$ consumers. Of these, I each own one right shoe and $I + 1$ each own a left shoe. Shoes are indivisible. Everyone has the same utility function given by $\min\{L, R\}$, where L and R are the quantities of left and right shoes consumed, respectively. Free disposal is not allowed.

(a) Show that any allocation of shoes that is matched (i.e., every individual consumes the same number of each kind) with the exception of one consumer who is consuming the remaining left shoe is Pareto efficient and conversely.

(b) Let p_L and p_R be the respective prices of the two kinds of shoes. Find the competitive equilibria of this economy.

6. Consider the following statements each of which may be true or false. If a statement is true, prove it. If it is false, give a counterexample. Your examples may be based on "pictures", but make sure they are clear and precise. You can assume that a competitive equilibrium exists and it is unique. You can also assume that agents have locally non-satiated and continuous preferences.

   (a) Consider two separate exchange economies. The competitive mechanism is used in both economies. Suppose these two economies merge. Then there is no agent who strictly prefers the initial competitive allocation to the competitive allocation under the merged economy. That is, no one suffers from "globalization".

   (b) Consider two separate exchange economies. The competitive mechanism is used in both economies. Suppose these two economies merge. Then there is at least one agent who is at least as well off under the competitive allocation of the merged economy as he/she is in the initial "autarkic" competitive allocation. That is, not everyone loses from "globalization".

   (c) Consider an exchange economy in which the competitive mechanism is used to determine the allocation. Suppose an agent destroys a part of his initial endowment before the market-clearing prices are determined. Then he necessarily suffers from this act. That is, he necessarily ends up with a bundle that is worse than what he would receive in the case he didn't destroy any of his endowment.