Problem Set 4: Inefficiencies

1. Consider the equilibrium of the Robinson Crusoe economy with consumption taxation that we discussed in class. Change the setting by assuming, instead, that there is no consumption taxation and that the consumer's labor income is taxed at the flat rate of τ instead. As in class, assume that the *equilibrium* tax revenue is refunded to the consumer.

   (a) Illustrate the resulting equilibrium, its inefficiency, and the deviation of the equilibrium allocation from the Pareto efficient allocation.

   (b) How do your conclusions differ from the conclusions we drew in class for the case of consumption taxation?

   (c) What conclusions would you draw about the performance of the European economy relative to the US economy given that both income and consumption taxation is higher in Europe?

   (d) Intuitively explain how taxation disrupts the coordinating function of the price system and hence results in economic inefficiency.

2. A government is considering whether to institute mandatory vaccination program against an infectious disease. Absent a vaccination, an individual will catch the disease with a chance $p(x)$, with $p' > 0$, $p'' < 0$, where x is the fraction of individuals who have not been vaccinated. But getting a vaccination is a hassle, it hurts (ouch!), and sometimes it gives you a mild disease. Individuals have linear utility functions over wealth w, the chance p of getting the disease, and the total cost of vaccination v. That is, $u(w, p, v) = w - p\delta - v$. There is a continuum of individuals of measure 1 and $v \sim U[0, 1]$ in the population.

   (a) Characterize who gets vaccinated in a laissez-faire (competitive) equilibrium (assume interior solutions in all cases).

   (b) Characterize who gets vaccinated in the Pareto-optimal outcome, and find taxes that decentralize this outcome.

   (c) Rank the policies available to the government: mandatory vaccination, laissez-faire, and an optimal tax.

2. Consider a small Vysocina town named Jihlava where too many people are named Libor and Alena. There are altogether 10 Libors and 10 Alenas. Each Libor likes playing a guitar and singing country music outdoors. On the other hand, Alena's, who generally prefer opera, despise country music. In particular, let Libor $\#k$ ($k = 1, .., 10$) derive enjoyment of monetary value $110k \ln(1 + x_k)$ from playing at the volume level x_k. The minimum volume level is zero (silence), but each Libor can really max things up to the volume level of 1000. Jihlava is such a small town that the effect of volume is additive. For every Libor $\#k$, there is a corresponding Alena $\#k$ ($k = 1, .., 10$) who is hurt by the amount $k(x_1 + ... + x_{10})$ by the totality of singing. Suppose all the utilities are expressed in monetary terms.

   (a) How much playing do Libors do if they are unaware of Alenas' feelings?

(b) Now assume that Alenas speak up and go to the Town Council, which imposes an efficient individual-specific tax on music production. What is this tax? Is it uniform across different Libors?

(c) Now mayor Ilir gets swung into power in a major election upset, and being a true libertarian, he instantly abolishes the tax and insists that all Libors and Alenas cut a deal among themselves. Ilir insists that if no deal is cut, then no music is allowed. What deal will Libors and Alenas cut? Up to how much will each Libor be willing pay to have the deal? At least how much will each Alena need to get in order to allow the deal? Depict your answer in a diagram of marginal costs and benefits.

(d) Due to a clandestine operation of an underground country music movement, mayor Ilir is thrown out of office in a reactionary coup led by the pro-country mayor Olex, who then allows country music to be played at free will. However, he allows for the possibility of Libors striking a deal with Alenas and enforces any resulting contract. How is the resulting total volume level and the wealth of Libors and Alenas affected?

3. Consider a two-consumer economy in which there are two types of goods: a private good and a public good. The two consumers have identical preferences given by

$$u(x_i, g) = x_i + 4 \ln g,$$

where x_i is the consumer i's consumption of the private good, $i = 1, 2$, and g is the quantity of the public good. The total amount of the public good g is determined by the sum of the individual contributions, i.e., $g = g_1 + g_2$. Each consumer is endowed with 10 units of the private good that can be converted into the public good in a 1-for-1 fashion. That is, each consumer faces a budget constraint of the form

$$x_i + g_i \leq 10.$$

(a) What is the Pareto-efficient amount of the public good?

(b) Now suppose the public good is provided by private contributions. Also suppose that in equilibrium, each consumer treats the contribution level of the other consumer as given (i.e., exhibits Nash behavior). What is the equilibrium amount of the public good and how much does each consumer contribute?

(c) Compare this equilibrium amount with the Pareto-efficient amount. Do they differ? If yes, explain why.

(d) True or false: The amount of the public good provided by private contributions could be increased by reallocating 2 units of endowment from consumer 2 to consumer 1.