In Chapter 11, you learned some tricks that allow you to use techniques you already know for studying intertemporal choice. Here you will learn some similar tricks, so that you can use the same methods to study risk taking, insurance, and gambling.

One of these new tricks is similar to the trick of treating commodities at different dates as different commodities. This time, we invent new commodities, which we call *contingent commodities*. If either of two events A or B could happen, then we define one contingent commodity as *consumption if A happens* and another contingent commodity as *consumption if B happens*. The second trick is to find a budget constraint that correctly specifies the set of contingent commodity bundles that a consumer can afford.

This chapter presents one other new idea, and that is the notion of von Neumann-Morgenstern utility. A consumer's willingness to take various gambles and his willingness to buy insurance will be determined by how he feels about various combinations of contingent commodities. Often it is reasonable to assume that these preferences can be expressed by a utility function that takes the special form known as *von Neumann-Morgenstern utility*. The assumption that utility takes this form is called the *expected utility hypothesis*. If there are two events, 1 and 2 with probabilities π_1 and π_2, and if the contingent consumptions are c_1 and c_2, then the von Neumann-Morgenstern utility function has the special functional form, $U(c_1, c_2) = \pi_1 u(c_1) + \pi_2 u(c_2)$. The consumer's behavior is determined by maximizing this utility function subject to his budget constraint.

You are thinking of betting on whether the Cincinnati Reds will make it to the World Series this year. A local gambler will bet with you at odds of 10 to 1 against the Reds. You think the probability that the Reds will make it to the World Series is $\pi = .2$. If you don't bet, you are certain to have \$1,000 to spend on consumption goods. Your behavior satisfies the expected utility hypothesis and your von Neumann-Morgenstern utility function is $\pi_1 \sqrt{c_1} + \pi_2 \sqrt{c_2}$.

The contingent commodities are *dollars if the Reds make the World Series* and *dollars if the Reds don't make the World Series*. Let c_W be your consumption contingent on the Reds making the World Series and c_{NW} be your consumption contingent on their not making the Series. Betting on the Reds at odds of 10 to 1 means that if you bet \$x on the Reds, then if the Reds make it to the Series, you make a net gain of \$10x, but if they don't, you have a net loss of \$x. Since you had \$1,000 before betting, if you bet \$x on the Reds and they made it to the Series, you would have $c_W = 1,000 + 10x$ to spend on consumption. If you bet \$x on the Reds and they didn't make it to the Series, you would lose \$x, and you would have $c_{NW} = 1,000 - x$. By increasing the amount \$x that

you bet, you can make c_W larger and c_{NW} smaller. (You could also bet against the Reds at the same odds. If you bet \$$x$ against the Reds and they fail to make it to the Series, you make a net gain of $.1x$ and if they make it to the Series, you lose \$$x$. If you work through the rest of this discussion for the case where you bet against the Reds, you will see that the same equations apply, with x being a negative number.) We can use the above two equations to solve for a budget equation. From the second equation, we have $x = 1,000 - c_{NW}$. Substitute this expression for x into the first equation and rearrange terms to find $c_W + 10c_{NW} = 11,000$, or equivalently, $.1c_W + c_{NW} = 1,100$. (The same budget equation can be written in many equivalent ways by multiplying both sides by a positive constant.)

Then you will choose your contingent consumption bundle (c_W, c_{NW}) to maximize $U(c_W, c_{NW}) = .2\sqrt{c_W} + .8\sqrt{c_{NW}}$ subject to the budget constraint, $.1c_W + c_{NW} = 1,100$. Using techniques that are now familiar, you can solve this consumer problem. From the budget constraint, you see that consumption contingent on the Reds making the World Series costs $1/10$ as much as consumption contingent on their not making it. If you set the marginal rate of substitution between c_W and c_{NW} equal to the price ratio and simplify the resulting expression, you will find that $c_{NW} = .16c_W$. This equation, together with the budget equation implies that $c_W = \$4,230.77$ and $c_{NW} = \$676.92$. You achieve this bundle by betting \$323.08 on the Reds. If the Reds make it to the Series, you will have $\$1,000 + 10 \times 323.08 = \$4,230.80$. If not, you will have \$676.92. (We rounded the solutions to the nearest penny.)

12.1 (0) In the next few weeks, Congress is going to decide whether or not to develop an expensive new weapons system. If the system is approved, it will be very profitable for the defense contractor, *General Statics*. Indeed, if the new system is approved, the value of stock in *General Statics* will rise from \$10 per share to \$15 a share, and if the project is not approved, the value of the stock will fall to \$5 a share. In his capacity as a messenger for Congressman Kickback, Buzz Condor has discovered that the weapons system is much more likely to be approved than is generally thought. On the basis of what he knows, Condor has decided that the probability that the system will be approved is $3/4$ and the probability that it will not be approved is $1/4$. Let c_A be Condor's consumption if the system is approved and c_{NA} be his consumption if the system is not approved. Condor's von Neumann-Morgenstern utility function is $U(c_A, c_{NA}) = .75 \ln c_A + .25 \ln c_{NA}$. Condor's total wealth is \$50,000, all of which is invested in perfectly safe assets. Condor is about to buy stock in *General Statics*.

(a) If Condor buys x shares of stock, and if the weapons system is approved, he will make a profit of \$5 per share. Thus the amount he can consume, contingent on the system being approved, is $c_A = \$50,000 + 5x$. If Condor buys x shares of stock, and if the weapons system is not approved, then he will make a loss of \$_____ per share. Thus the amount

he can consume, contingent on the system not being approved, is $c_{NA} =$

_____ .

(b) You can solve for Condor's budget constraint on contingent commodity bundles (c_A, c_{NA}) by eliminating x from these two equations. His budget constraint can be written as _____ $c_A +$ _____ $c_{NA} = 50,000$.

(c) Buzz Condor has no moral qualms about trading on inside information, nor does he have any concern that he will be caught and punished. To decide how much stock to buy, he simply maximizes his von Neumann-Morgenstern utility function subject to his budget. If he sets his marginal rate of substitution between the two contingent commodities equal to their relative prices and simplifies the equation, he finds that $c_A/c_{NA} =$

_____ (Reminder: Where a is any constant, the derivative of $a \ln x$ with respect to x is a/x.)

(d) Condor finds that his optimal contingent commodity bundle is

$(c_A, c_{NA}) =$ _____ To acquire this contingent commodity bundle, he must buy _____ shares of stock in *General Statics*.

12.2 (0) Willy owns a small chocolate factory, located close to a river that occasionally floods in the spring, with disastrous consequences. Next summer, Willy plans to sell the factory and retire. The only income he will have is the proceeds of the sale of his factory. If there is no flood, the factory will be worth $500,000. If there is a flood, then what is left of the factory will be worth only $50,000. Willy can buy flood insurance at a cost of $.10 for each $1 worth of coverage. Willy thinks that the probability that there will be a flood this spring is $1/10$. Let c_F denote the contingent commodity *dollars if there is a flood* and c_{NF} denote *dollars if there is no flood*. Willy's von Neumann-Morgenstern utility function is $U(c_F, c_{NF}) = .1\sqrt{c_F} + .9\sqrt{c_{NF}}$.

(a) If he buys no insurance, then in each contingency, Willy's consumption will equal the value of his factory, so Willy's contingent commodity bundle

will be $(c_F, c_{NF}) =$ _____ .

(b) To buy insurance that pays him $\$x$ in case of a flood, Willy must pay an insurance premium of $.1x$. (The insurance premium must be paid whether or not there is a flood.) If Willy insures for $\$x$, then if there is a flood, he gets $\$x$ in insurance benefits. Suppose that Willy has contracted for insurance that pays him $\$x$ in the event of a flood. Then after paying

his insurance premium, he will be able to consume $c_F =$ _____ If Willy has this amount of insurance and there is no flood, then he will

be able to consume $c_{NF} =$ _____ .

(c) You can eliminate x from the two equations for c_F and c_{NF} that you found above. This gives you a budget equation for Willy. Of course there are many equivalent ways of writing the same budget equation, since multiplying both sides of a budget equation by a positive constant yields an equivalent budget equation. The form of the budget equation in which the "price" of c_{NF} is 1 can be written as $.9c_{NF}+$_____ $c_F =$

_____.

(d) Willy's marginal rate of substitution between the two contingent commodities, *dollars if there is no flood* and *dollars if there is a flood*, is $MRS(c_F, c_N F) = -\frac{.1\sqrt{c_{NF}}}{.9\sqrt{c_F}}$. To find his optimal bundle of contingent commodities, you must set this marginal rate of substitution equal to the

number _____ Solving this equation, you find that Willy will choose

to consume the two contingent commodities in the ratio_____.

(e) Since you know the ratio in which he will consume c_F and c_{NF}, and you know his budget equation, you can solve for his optimal consumption

bundle, which is $(c_F, c_{NF})=$_____ Willy will buy an in-

surance policy that will pay him _____ if there is a flood. The

amount of insurance premium that he will have to pay is_____.

12.3 (0) Clarence Bunsen is an expected utility maximizer. His preferences among contingent commodity bundles are represented by the expected utility function

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1\sqrt{c_1} + \pi_2\sqrt{c_2}.$$

Clarence's friend, Hjalmer Ingqvist, has offered to bet him $1,000 on the outcome of the toss of a coin. That is, if the coin comes up heads, Clarence must pay Hjalmer $1,000 and if the coin comes up tails, Hjalmer must pay Clarence $1,000. The coin is a fair coin, so that the probability of heads and the probability of tails are both 1/2. If he doesn't accept the bet, Clarence will have $10,000 with certainty. In the privacy of his car dealership office over at Bunsen Motors, Clarence is making his decision. (Clarence uses the pocket calculator that his son, Elmer, gave him last Christmas. You will find that it will be helpful for you to use a calculator too.) Let Event 1 be "coin comes up heads" and let Event 2 be "coin comes up tails."

(a) If Clarence accepts the bet, then in Event 1, he will have _____

dollars and in Event 2, he will have _____ dollars.

(b) Since the probability of each event is $1/2$, Clarence's expected utility for a gamble in which he gets c_1 in Event 1 and c_2 in Event 2 can be described by the formula _____ Therefore Clarence's expected utility if he accepts the bet with Hjalmer will be _____ (Use that calculator.)

(c) If Clarence decides not to bet, then in Event 1, he will have _____ dollars and in Event 2, he will have _____ dollars. Therefore if he doesn't bet, his expected utility will be_____.

(d) Having calculated his expected utility if he bets and if he does not bet, Clarence determines which is higher and makes his decision accordingly.

Does Clarence take the bet?_____.

12.4 (0) It is a slow day at Bunsen Motors, so since he has his calculator warmed up, Clarence Bunsen (whose preferences toward risk were described in the last problem) decides to study his expected utility function more closely.

(a) Clarence first thinks about really *big* gambles. What if he bet his entire $10,000 on the toss of a coin, where he loses if heads and wins if tails? Then if the coin came up heads, he would have 0 dollars and if it came up tails, he would have $20,000. His expected utility if he took the bet would be _____ , while his expected utility if he didn't take the bet would be _____ Therefore he concludes that he would not take such a bet.

(b) Clarence then thinks, "Well, of course, I wouldn't want to take a chance on losing all of my money on just an ordinary bet. But, what if somebody offered me a really good deal. Suppose I had a chance to bet where if a fair coin came up heads, I lost my $10,000, but if it came up tails, I would win $50,000. Would I take the bet? If I took the bet, my expected utility would be _____ If I didn't take the bet, my expected utility would be _____ Therefore I should _____ the bet."

(c) Clarence later asks himself, "If I make a bet where I lose my $10,000 if the coin comes up heads, what is the smallest amount that I would have to win in the event of tails in order to make the bet a good one for me to take?" After some trial and error, Clarence found the answer. You, too, might want to find the answer by trial and error, but it is easier to

find the answer by solving an equation. On the left side of your equation, you would write down Clarence's utility if he doesn't bet. On the right side of the equation, you write down an expression for Clarence's utility if he makes a bet such that he is left with zero consumption in Event 1 and x in Event 2. Solve this equation for x. The answer to Clarence's question is where $x = 10,000$. The equation that you should write is

_____ The solution is $x =$_____.

(d) Your answer to the last part gives you two points on Clarence's indifference curve between the contingent commodities, money in Event 1 and money in Event 2. (Poor Clarence has never heard of indifference curves or contingent commodities, so you will have to work this part for him, while he heads over to the Chatterbox Cafe for morning coffee.) One of these points is where money in both events is \$10,000. On the graph below, label this point A. The other is where money in Event 1 is zero

and money in Event 2 is _____ On the graph below, label this point B.

Money in Event 2 ($\times 1,000$)

Money in Event 1 ($\times 1,000$)
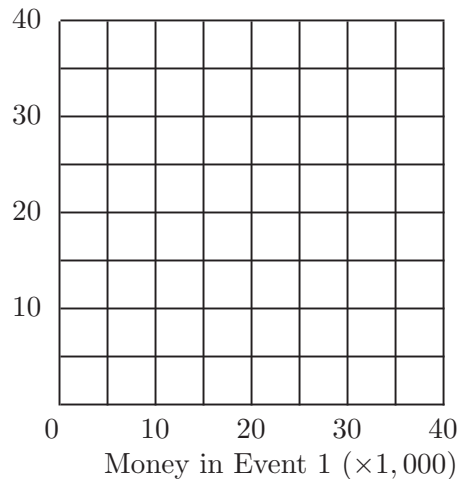
(e) You can quickly find a third point on this indifference curve. The coin is a fair coin, and Clarence cares whether heads or tails turn up only because that determines his prize. Therefore Clarence will be indifferent between two gambles that are the same except that the assignment of prizes to outcomes are reversed. In this example, Clarence will be indifferent between point B on the graph and a point in which he gets zero if

Event 2 happens and _____ if Event 1 happens. Find this point on the Figure above and label it C.

(f) Another gamble that is on the same indifference curve for Clarence as not gambling at all is the gamble where he loses $5,000 if heads turn

up and where he wins ———————— dollars if tails turn up. (Hint: To solve this problem, put the utility of not betting on the left side of an equation and on the right side of the equation, put the utility of having $10,000 − \$5,000$ in Event 1 and $10,000 + x$ in Event 2. Then solve the resulting equation for x.) On the axes above, plot this point and label it D. Now sketch in the entire indifference curve through the points that you have labeled.

12.5 (0) Hjalmer Ingqvist's son-in-law, Earl, has not worked out very well. It turns out that Earl likes to gamble. His preferences over contingent commodity bundles are represented by the expected utility function

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1^2 + \pi_2 c_2^2.$$

(a) Just the other day, some of the boys were down at Skoog's tavern when Earl stopped in. They got to talking about just how bad a bet they could get him to take. At the time, Earl had $100. Kenny Olson shuffled a deck of cards and offered to bet Earl $20 that Earl would not cut a spade from the deck. Assuming that Earl believed that Kenny wouldn't cheat, the probability that Earl would win the bet was 1/4 and the probability that Earl would lose the bet was 3/4. If he won the bet, Earl would have

———————— dollars and if he lost the bet, he would have ———————— dollars.

Earl's expected utility if he took the bet would be ———————— , and his

expected utility if he did not take the bet would be ———————— Therefore he refused the bet.

(b) Just when they started to think Earl might have changed his ways, Kenny offered to make the same bet with Earl except that they would bet $100 instead of $20. What is Earl's expected utility if he takes that

bet? ———————— Would Earl be willing to take this bet?————————

——.

(c) Let Event 1 be the event that a card drawn from a fair deck of cards is a spade. Let Event 2 be the event that the card is not a spade. Earl's preferences between income contingent on Event 1, c_1, and income contingent

on Event 2, c_2, can be represented by the equation ———————————— Use blue ink on the graph below to sketch Earl's indifference curve passing through the point $(100, 100)$.

Money in Event 2

Money in Event 1
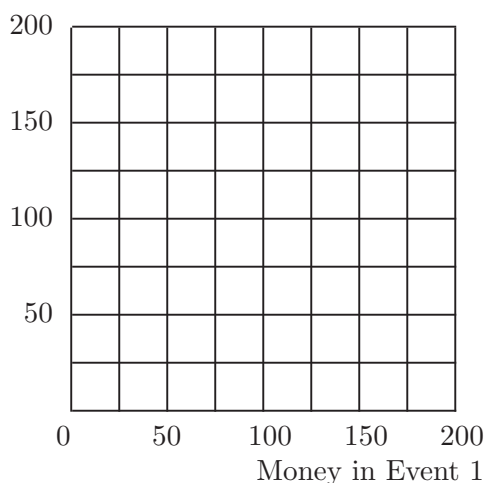
(d) On the same graph, let us draw Hjalmer's son-in-law Earl's indifference curves between contingent commodities where the probabilities are different. Suppose that a card is drawn from a fair deck of cards. Let Event 1 be the event that the card is black. Let event 2 be the event that the card drawn is red. Suppose each event has probability 1/2. Then Earl's preferences between income contingent on Event 1 and income contingent on Event 2 are represented by the formula _____ On the graph, use red ink to show two of Earl's indifference curves, including the one that passes through $(100, 100)$.

12.6 (1) Sidewalk Sam makes his living selling sunglasses at the boardwalk in Atlantic City. If the sun shines Sam makes \$30, and if it rains Sam only makes \$10. For simplicity, we will suppose that there are only two kinds of days, sunny ones and rainy ones.

(a) One of the casinos in Atlantic City has a new gimmick. It is accepting bets on whether it will be sunny or rainy the next day. The casino sells dated "rain coupons" for \$1 each. If it rains the next day, the casino will give you \$2 for every rain coupon you bought on the previous day. If it doesn't rain, your rain coupon is worthless. In the graph below, mark Sam's "endowment" of contingent consumption if he makes no bets with the casino, and label it E.

C_r

C_s
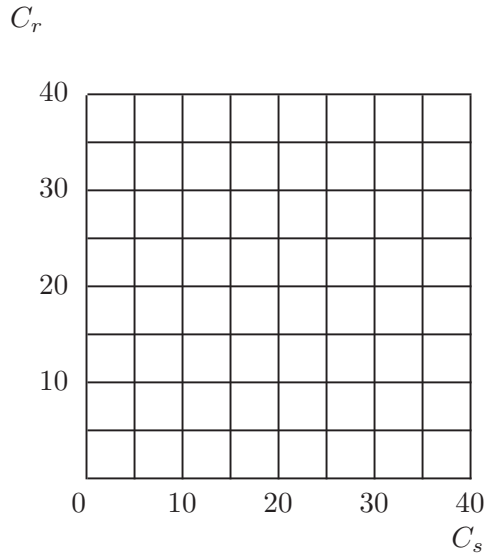
(b) On the same graph, mark the combination of consumption contingent on rain and consumption contingent on sun that he could achieve by buying 10 rain coupons from the casino. Label it A.

(c) On the same graph, use blue ink to draw the budget line representing all of the other patterns of consumption that Sam can achieve by buying rain coupons. (Assume that he can buy fractional coupons, but not negative amounts of them.) What is the slope of Sam's budget line at points above and to the left of his initial endowment?_____.

(d) Suppose that the casino also sells sunshine coupons. These tickets also cost $1. With these tickets, the casino gives you $2 if it doesn't rain and nothing if it does. On the graph above, use red ink to sketch in the budget line of contingent consumption bundles that Sam can achieve by buying sunshine tickets.

(e) If the price of a dollar's worth of consumption when it rains is set equal to 1, what is the price of a dollar's worth of consumption if it shines?_____.

12.7 (0) Sidewalk Sam, from the previous problem, has the utility function for consumption in the two states of nature

$$u(c_s, c_r, \pi) = c_s^{1-\pi} c_r^{\pi},$$

where c_s is the dollar value of his consumption if it shines, c_r is the dollar value of his consumption if it rains, and π is the probability that it will rain. The probability that it will rain is $\pi = .5$.

(a) How many units of consumption is it optimal for Sam to consume conditional on rain?_____.

(b) How many rain coupons is it optimal for Sam to buy?_____.

12.8 (0) Sidewalk Sam's brother Morgan von Neumanstern is an expected utility maximizer. His von Neumann-Morgenstern utility function for wealth is $u(c) = \ln c$. Sam's brother also sells sunglasses on another beach in Atlantic City and makes exactly the same income as Sam does. He can make exactly the same deal with the casino as Sam can.

(a) If Morgan believes that there is a 50% chance of rain and a 50% chance of sun every day, what would his expected utility of consuming (c_s, c_r) be?_____.

(b) How does Morgan's utility function compare to Sam's? Is one a monotonic transformation of the other?_____

_____.

(c) What will Morgan's optimal pattern of consumption be? Answer: Morgan will consume _____ on the sunny days and _____ on the rainy days. How does this compare to Sam's consumption?_____

_____.

12.9 (0) Billy John Pigskin of Mule Shoe, Texas, has a von Neumann-Morgenstern utility function of the form $u(c) = \sqrt{c}$. Billy John also weighs about 300 pounds and can outrun jackrabbits and pizza delivery trucks. Billy John is beginning his senior year of college football. If he is not seriously injured, he will receive a $1,000,000 contract for playing professional football. If an injury ends his football career, he will receive a $10,000 contract as a refuse removal facilitator in his home town. There is a 10% chance that Billy John will be injured badly enough to end his career.

*(a)* What is Billy John's expected utility?_____

_____.

(b) If Billy John pays p for an insurance policy that would give him $1,000,000 if he suffered a career-ending injury while in college, then he would be sure to have an income of $1,000,000 - p$ no matter what happened to him. Write an equation that can be solved to find the largest price that Billy John would be willing to pay for such an insurance policy.

_____.

(c) Solve this equation for p._____.

12.10 (1) You have $200 and are thinking about betting on the Big Game next Saturday. Your team, the Golden Boars, are scheduled to play their traditional rivals the Robber Barons. It appears that the going odds are 2 to 1 against the Golden Boars. That is to say if you want to bet $10 on the Boars, you can find someone who will agree to pay you $20 if the Boars win in return for your promise to pay him $10 if the Robber Barons win. Similarly if you want to bet $10 on the Robber Barons, you can find someone who will pay you $10 if the Robber Barons win, in return for your promise to pay him $20 if the Robber Barons lose. Suppose that you are able to make as large a bet as you like, either on the Boars or on the Robber Barons so long as your gambling losses do not exceed $200. (To avoid tedium, let us ignore the possibility of ties.)

(a) If you do not bet at all, you will have $200 whether or not the Boars win. If you bet $50 on the Boars, then after all gambling obligations are settled, you will have a total of _____ dollars if the Boars win and

_____ dollars if they lose. On the graph below, use blue ink to draw a line that represents all of the combinations of "money if the Boars win" and "money if the Robber Barons win" that you could have by betting from your initial $200 at these odds.

Money if the Boars lose

Money if the Boars win
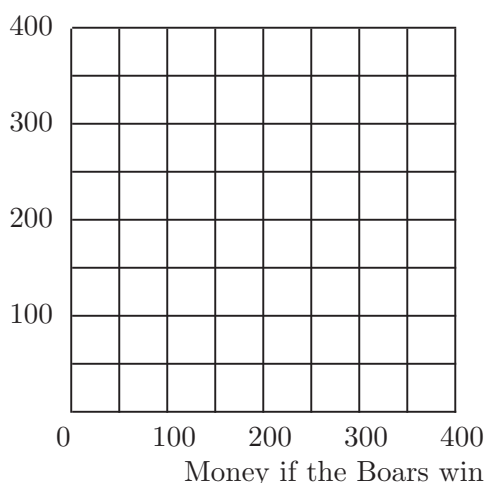
(b) Label the point on this graph where you would be if you did not bet at all with an *E*.

(c) After careful thought you decide to bet \$50 on the Boars. Label the point you have chosen on the graph with a *C*. Suppose that after you have made this bet, it is announced that the star Robber Baron quarterback suffered a sprained thumb during a tough economics midterm examination and will miss the game. The market odds shift from 2 to 1 against the Boars to "even money" or 1 to 1. That is, you can now bet on either team and the amount you would win if you bet on the winning team is the same as the amount that you would lose if you bet on the losing team. You cannot cancel your original bet, but you can make new bets at the new odds. Suppose that you keep your first bet, but you now also bet \$50 on the Robber Barons at the new odds. If the Boars win, then after you collect your winnings from one bet and your losses from the other,

how much money will you have left? ——————— If the Robber Barons win, how much money will you have left after collecting your winnings

and paying off your losses?————————————————————.

(d) Use red ink to draw a line on the diagram you made above, showing the combinations of "money if the Boars win" and "money if the Robber Barons win" that you could arrange for yourself by adding possible bets at the new odds to the bet you made before the news of the quarterback's misfortune. On this graph, label the point *D* that you reached by making the two bets discussed above.

12.11 (2) The *certainty equivalent* of a lottery is the amount of money you would have to be given with certainty to be just as well-off with that lottery. Suppose that your von Neumann-Morgenstern utility function over lotteries that give you an amount x if Event 1 happens and y if Event 1 does not happen is $U(x, y, \pi) = \pi\sqrt{x} + (1 - \pi)\sqrt{y}$, where π is the probability that Event 1 happens and $1 - \pi$ is the probability that Event 1 does not happen.

(a) If $\pi = .5$, calculate the utility of a lottery that gives you \$10,000 if

Event 1 happens and \$100 if Event 1 does not happen.————————————

————————————————————————————————————.

(b) If you were sure to receive \$4,900, what would your utility be? ——————
(Hint: If you receive \$4,900 with certainty, then you receive \$4,900 in both events.)

(c) Given this utility function and $\pi = .5$, write a general formula for the certainty equivalent of a lottery that gives you \$$x$ if Event 1 happens and

\$$y$ if Event 1 does not happen.————————————————————————.

(d) Calculate the certainty equivalent of receiving $10,000 if Event 1 happens and $100 if Event 1 does not happen._____.

12.12 (0) Dan Partridge is a risk averter who tries to maximize the expected value of $\sqrt{c}$, where c is his wealth. Dan has $50,000 in safe assets and he also owns a house that is located in an area where there are lots of forest fires. If his house burns down, the remains of his house and the lot it is built on would be worth only $40,000, giving him a total wealth of $90,000. If his home doesn't burn, it will be worth $200,000 and his total wealth will be $250,000. The probability that his home will burn down is .01.

(a) Calculate his expected utility if he doesn't buy fire insurance._____

_____.

(b) Calculate the certainty equivalent of the lottery he faces if he doesn't buy fire insurance._____.

(c) Suppose that he can buy insurance at a price of $1 per $100 of insurance. For example if he buys $100,000 worth of insurance, he will pay $1,000 to the company no matter what happens, but if his house burns, he will also receive $100,000 from the company. If Dan buys $160,000 worth of insurance, he will be fully insured in the sense that no matter what happens his after-tax wealth will be_____.

(d) Therefore if he buys full insurance, the certainty equivalent of his wealth is _____ , and his expected utility is_____.

12.13 (1) Portia has been waiting a long time for her ship to come in and has concluded that there is a 25% chance that it will arrive today. If it does come in today, she will receive $1,600. If it does not come in today, it will never come and her wealth will be zero. Portia has a von Neumann-Morgenstern utility such that she wants to maximize the expected value of $\sqrt{c}$, where c is total wealth. What is the minimum price at which she will sell the rights to her ship?_____.

Supply and demand problems are bread and butter for economists. In the problems below, you will typically want to solve for equilibrium prices and quantities by writing an equation that sets supply equal to demand. Where the price received by suppliers is the same as the price paid by demanders, one writes supply and demand as functions of the same price variable, p, and solves for the price that equalizes supply and demand. But if, as happens with taxes and subsidies, suppliers face different prices from demanders, it is a good idea to denote these two prices by separate variables, p_s and p_d. Then one can solve for equilibrium by solving a system of two equations in the two unknowns p_s and p_d. The two equations are the equation that sets supply equal to demand and the equation that relates the price paid by demanders to the net price received by suppliers.

The demand function for commodity x is $q = 1,000 - 10p_d$, where p_d is the price paid by consumers. The supply function for x is $q = 100 + 20p_s$, where p_s is the price received by suppliers. For each unit sold, the government collects a tax equal to half of the price paid by consumers. Let us find the equilibrium prices and quantities. In equilibrium, supply must equal demand, so that $1,000 - 10p_d = 100 + 20p_s$. Since the government collects a tax equal to half of the price paid by consumers, it must be that the sellers only get half of the price paid by consumers, so it must be that $p_s = p_d/2$. Now we have two equations in the two unknowns, p_s and p_d. Substitute the expression $p_d/2$ for p_s in the first equation, and you have $1,000 - 10p_d = 100 + 10p_d$. Solve this equation to find $p_d = 45$. Then $p_s = 22.5$ and $q = 550$.

16.1 (0) The demand for yak butter is given by $120 - 4p_d$ and the supply is $2p_s - 30$, where p_d is the price paid by demanders and p_s is the price received by suppliers, measured in dollars per hundred pounds. Quantities demanded and supplied are measured in hundred-pound units.

(a) On the axes below, draw the demand curve (with blue ink) and the supply curve (with red ink) for yak butter.
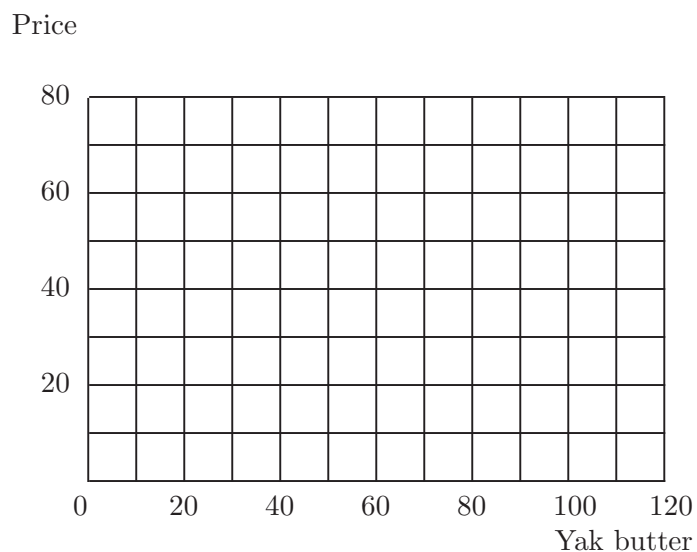
Price

(b) Write down the equation that you would solve to find the equilibrium

price._____.

(c) What is the equilibrium price of yak butter? _____ What is the

equilibrium quantity? _____ Locate the equilibrium price and quantity
on the graph, and label them p_1 and q_1.

(d) A terrible drought strikes the central Ohio steppes, traditional home-
land of the yaks. The supply schedule shifts to $2p_s - 60$. The demand
schedule remains as before. Draw the new supply schedule. Write down
the equation that you would solve to find the new equilibrium price of

yak butter._____.

(e) The new equilibrium price is _____ and the quantity is _____
Locate the new equilibrium price and quantity on the graph and label
them p_2 and q_2.

(f) The government decides to relieve stricken yak butter consumers and
producers by paying a subsidy of \$5 per hundred pounds of yak butter to
producers. If p_d is the price paid by demanders for yak butter, what is the

total amount received by producers for each unit they produce?_____

_____ When the price paid by consumers is p_d, how much yak butter

is produced?_____.

(g) Write down an equation that can be solved for the equilibrium price paid by consumers, given the subsidy program. _____ What are the equilibrium price paid by consumers and the equilibrium quantity of yak butter now?_____.
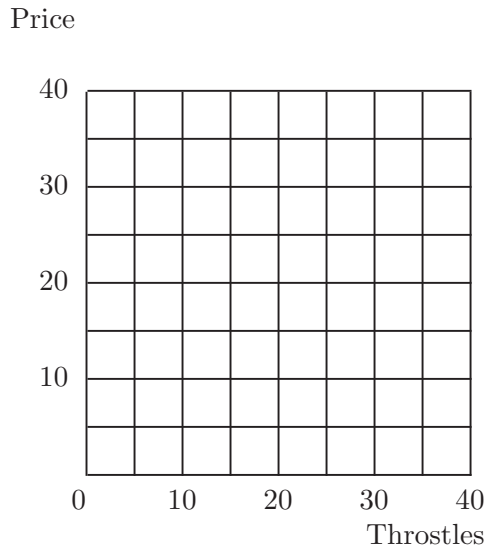
(h) Suppose the government had paid the subsidy to consumers rather than producers. What would be the equilibrium net price paid by consumers? _____ The equilibrium quantity would be_____.

16.2 (0) Here are the supply and demand equations for throstles, where p is the price in dollars:

$$D(p) = 40 - p$$

$$S(p) = 10 + p.$$

On the axes below, draw the demand and supply curves for throstles, using blue ink.

Price

(a) The equilibrium price of throstles is _____ and the equilibrium quantity is_____.

(b) Suppose that the government decides to restrict the industry to selling only 20 throstles. At what price would 20 throstles be demanded? _____ How many throstles would suppliers supply at that price? _____ At what price would the suppliers supply only 20 units?_____.

(c) The government wants to make sure that only 20 throstles are bought, but it doesn't want the firms in the industry to receive more than the minimum price that it would take to have them supply 20 throstles. One way to do this is for the government to issue 20 ration coupons. Then in order to buy a throstle, a consumer would need to present a ration coupon along with the necessary amount of money to pay for the good. If the ration coupons were freely bought and sold on the open market,

what would be the equilibrium price of these coupons?_____.

(d) On the graph above, shade in the area that represents the deadweight loss from restricting the supply of throstles to 20. How much is this expressed in dollars? (Hint: What is the formula for the area of a triangle?)

_____.

16.3 (0) The demand curve for ski lessons is given by $D(p_D) = 100 - 2p_D$ and the supply curve is given by $S(p_S) = 3p_S$.

(a) What is the equilibrium price? _____ What is the equilibrium

quantity?_____.

(b) A tax of \$10 per ski lesson is imposed on consumers. Write an equation that relates the price paid by demanders to the price received by suppliers.
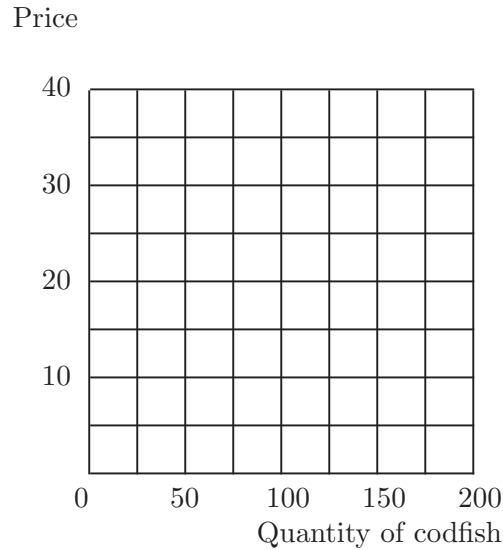
_____ Write an equation that states that supply equals

demand._____.

(c) Solve these two equations for the two unknowns p_S and p_D. With the

\$10 tax, the equilibrium price p_D paid by consumers would be _____

per lesson. The total number of lessons given would be_____.

(d) A senator from a mountainous state suggests that although ski lesson consumers are rich and deserve to be taxed, ski instructors are poor and deserve a subsidy. He proposes a \$6 subsidy on production while maintaining the \$10 tax on consumption of ski lessons. Would this policy have any different effects for suppliers or for demanders than a tax of \$4 per

lesson?_____.

16.4 (0) The demand curve for salted codfish is $D(P) = 200 - 5P$ and the supply curve $S(P) = 5P$.

(a) On the graph below, use blue ink to draw the demand curve and the supply curve. The equilibrium market price is _____ and the equilibrium quantity sold is_____.

Price


```
40 ┌─┬─┬─┬─┬─┬─┬─┬─┐
   ├─┼─┼─┼─┼─┼─┼─┼─┤
30 ├─┼─┼─┼─┼─┼─┼─┼─┤
   ├─┼─┼─┼─┼─┼─┼─┼─┤
20 ├─┼─┼─┼─┼─┼─┼─┼─┤
   ├─┼─┼─┼─┼─┼─┼─┼─┤
10 ├─┼─┼─┼─┼─┼─┼─┼─┤
   ├─┼─┼─┼─┼─┼─┼─┼─┤
   └─┴─┴─┴─┴─┴─┴─┴─┘
   0    50   100  150  200
```

Quantity of codfish

(b) A quantity tax of \$2 per unit sold is placed on salted codfish. Use red ink to draw the new supply curve, where the price on the vertical axis remains the price per unit paid by demanders. The new equilibrium price paid by the demanders will be _____ and the new price received by the suppliers will be _____ The equilibrium quantity sold will be

_____.

(c) The deadweight loss due to this tax will be _____ On your graph, shade in the area that represents the deadweight loss.

16.5 (0) The demand function for merino ewes is $D(P) = 100/P$, and the supply function is $S(P) = P$.

(a) What is the equilibrium price?_____.

(b) What is the equilibrium quantity?_____.

(c) An ad valorem tax of 300% is imposed on merino ewes so that the price paid by demanders is four times the price received by suppliers. What is the equilibrium price paid by the demanders for merino ewes now?

_____ What is the equilibrium price received by the suppliers for

merino ewes? _____ What is the equilibrium quantity?_____.

16.6 (0) Schrecklich and LaMerde are two justifiably obscure nineteenth-century impressionist painters. The world's total stock of paintings by Schrecklich is 100, and the world's stock of paintings by LaMerde is 150. The two painters are regarded by connoisseurs as being very similar in style. Therefore the demand for either painter's work depends both on its own price and the price of the other painter's work. The demand function for Schrecklichs is $D_S(P) = 200 - 4P_S - 2P_L$, and the demand function for LaMerdes is $D_L(P) = 200 - 3P_L - P_S$, where P_S and P_L are respectively the price in dollars of a Schrecklich painting and a LaMerde painting.

(a) Write down two simultaneous equations that state the equilibrium condition that the demand for each painter's work equals supply.

_____.

(b) Solving these two equations, one finds that the equilibrium price of

Schrecklichs is _____ and the equilibrium price of LaMerdes is_____.

(c) On the diagram below, draw a line that represents all combinations of prices for Schrecklichs and LaMerdes such that the supply of Schrecklichs equals the demand for Schrecklichs. Draw a second line that represents those price combinations at which the demand for LaMerdes equals the supply of LaMerdes. Label the unique price combination at which both markets clear with the letter E.

P_L
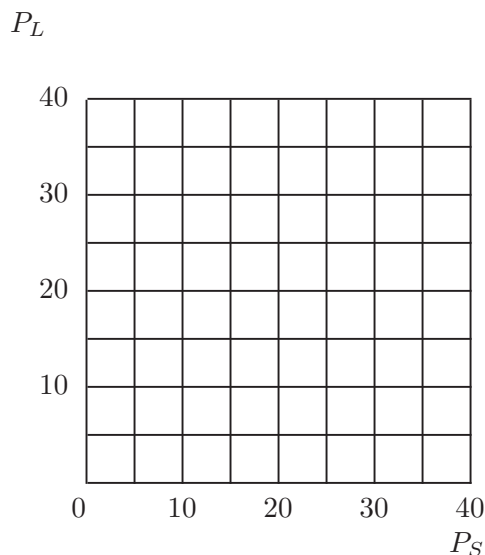
(d) A fire in a bowling alley in Hamtramck, Michigan, destroyed one of the world's largest collections of works by Schrecklich. The fire destroyed a total of 10 Schrecklichs. After the fire, the equilibrium price of Schreck-

lichs was ——————— and the equilibrium price of LaMerdes was————————.

(e) On the diagram you drew above, use red ink to draw a line that shows the locus of price combinations at which the demand for Schrecklichs equals the supply of Schrecklichs after the fire. On your diagram, label the new equilibrium combination of prices E'.

16.7 (0) The price elasticity of demand for oatmeal is constant and equal to -1. When the price of oatmeal is \$10 per unit, the total amount demanded is 6,000 units.

(a) Write an equation for the demand function. ——————————— Graph this demand function below with blue ink. (Hint: If the demand curve has a constant price elasticity equal to ϵ, then $D(p) = ap^\epsilon$ for some constant a. You have to use the data of the problem to solve for the constants a and ϵ that apply in this particular case.)

Price

Quantity (thousands)
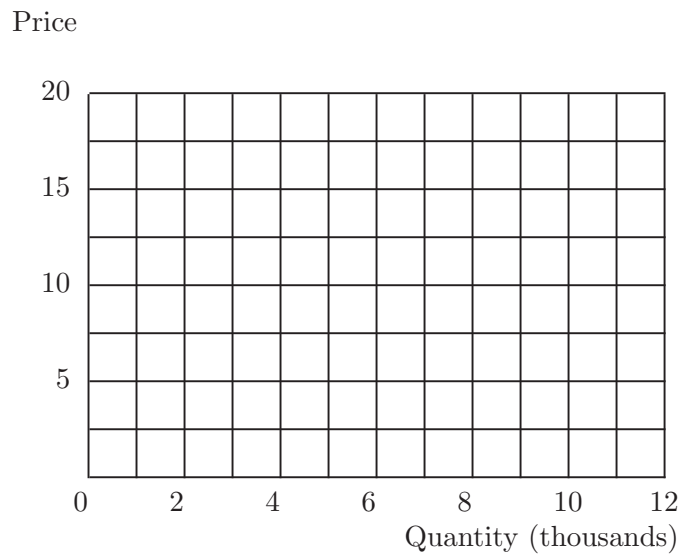
(b) If the supply is perfectly inelastic at 5,000 units, what is the equilib-

rium price? ——————— Show the supply curve on your graph and label the equilibrium with an E.

(c) Suppose that the demand curve shifts outward by 10%. Write down

the new equation for the demand function. ——————————— Suppose that the supply curve remains vertical but shifts to the right by 5%. Solve

for the new equilibrium price ——————— and quantity————————.

(d) By what percentage approximately did the equilibrium price rise?

_____ Use red ink to draw the new demand curve and the new supply curve on your graph.

(e) Suppose that in the above problem the demand curve shifts outward by $x\%$ and the supply curve shifts right by $y\%$. By approximately what percentage will the equilibrium price rise?_____.

16.8 (0) An economic historian* reports that econometric studies indicate for the pre–Civil War period, 1820–1860, the price elasticity of demand for cotton from the American South was approximately -1. Due to the rapid expansion of the British textile industry, the demand curve for American cotton is estimated to have shifted outward by about 5% per year during this entire period.

(a) If during this period, cotton production in the United States grew by 3% per year, what (approximately) must be the rate of change of the price of cotton during this period?_____.

(b) Assuming a constant price elasticity of -1, and assuming that when the price is $20, the quantity is also 20, graph the demand curve for cotton. What is the total revenue when the price is $20? _____

What is the total revenue when the price is $10?_____.

Price of cotton
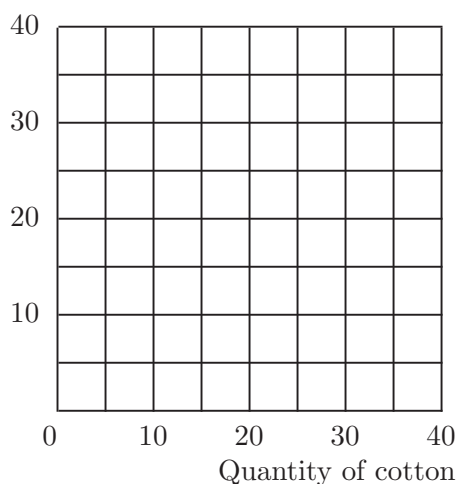
Quantity of cotton

(c) If the change in the quantity of cotton supplied by the United States is to be interpreted as a movement along an upward-sloping long-run supply curve, what would the elasticity of supply have to be? (Hint: From 1820 to 1860 quantity rose by about 3% per year and price rose by _____ % per year. [See your earlier answer.] If the quantity change is a movement along the long-run supply curve, then the long-run price elasticity must be what?)_____.

(d) The American Civil War, beginning in 1861, had a devastating effect on cotton production in the South. Production fell by about 50% and remained at that level throughout the war. What would you predict would be the effect on the price of cotton?_____

_____.

(e) What would be the effect on total revenue of cotton farmers in the South?_____

_____.

(f) The expansion of the British textile industry ended in the 1860s, and for the remainder of the nineteenth century, the demand curve for American cotton remained approximately unchanged. By about 1900, the South approximately regained its prewar output level. What do you think happened to cotton prices then?_____.

16.9 (0) The number of bottles of chardonnay demanded per year is $1,000,000 - 60,000P$, where P is the price per bottle (in U.S. dollars). The number of bottles supplied is $40,000P$.

(a) What is the equilibrium price? _____ What is the equilibrium quantity?_____.

(b) Suppose that the government introduces a new tax such that the wine maker must pay a tax of $5 per bottle for every bottle that he produces. What is the new equilibrium price paid by consumers? _____ What is the new price received by suppliers? _____ What is the new equilibrium quantity?_____.

16.10 (0) The inverse demand function for bananas is $P_d = 18 - 3Q_d$ and the inverse supply function is $P_s = 6 + Q_s$, where prices are measured in cents.

(a) If there are no taxes or subsidies, what is the equilibrium quantity?

_____ What is the equilibrium market price?_____.

(b) If a subsidy of 2 cents per pound is paid to banana growers, then in equilibrium it still must be that the quantity demanded equals the quantity supplied, but now the price received by sellers is 2 cents higher than the price paid by consumers. What is the new equilibrium quantity?

_____ What is the new equilibrium price received by suppliers?

_____ What is the new equilibrium price paid by demanders?

_____.

(c) Express the change in price as a percentage of the original price.

_____ If the cross-elasticity of demand between bananas and apples is $+.5$, what will happen to the quantity of apples demanded as a consequence of the banana subsidy, if the price of apples stays constant?

(State your answer in terms of percentage change.)_____.

16.11 (1) King Kanuta rules a small tropical island, Nutting Atoll, whose primary crop is coconuts. If the price of coconuts is P, then King Kanuta's subjects will demand $D(P) = 1,200 - 100P$ coconuts per week for their own use. The number of coconuts that will be supplied per week by the island's coconut growers is $S(p) = 100P$.

(a) The equilibrium price of coconuts will be _____ and the equilibrium quantity supplied will be_____.

(b) One day, King Kanuta decided to tax his subjects in order to collect coconuts for the Royal Larder. The king required that every subject who consumed a coconut would have to pay a coconut to the king as a tax. Thus, if a subject wanted 5 coconuts for himself, he would have to purchase 10 coconuts and give 5 to the king. When the price that is received by the sellers is p_S, how much does it cost one of the king's subjects to get an extra coconut for himself?_____.

(c) When the price paid to suppliers is p_S, how many coconuts will the king's subjects demand for their own consumption? (Hint: Express p_D in terms of p_S and substitute into the demand function.)_____

_____.

(d) Since the king consumes a coconut for every coconut consumed by the subjects, the total amount demanded by the king and his subjects is twice the amount demanded by the subjects. Therefore, when the price received by suppliers is p_S, the total number of coconuts demanded per week by Kanuta and his subjects is_____.

(e) Solve for the equilibrium value of p_S_____ , the equilibrium total number of coconuts produced_____ , and the equilibrium total number of coconuts consumed by Kanuta's subjects._____.

(f) King Kanuta's subjects resented paying the extra coconuts to the king, and whispers of revolution spread through the palace. Worried by the hostile atmosphere, the king changed the coconut tax. Now, the shopkeepers who sold the coconuts would be responsible for paying the tax. For every coconut sold to a consumer, the shopkeeper would have to pay one coconut to the king. This plan resulted in _____ coconuts being sold to the consumers. The shopkeepers got _____ per coconut after paying their tax to the king, and the consumers paid a price of _____ per coconut.

16.12 (1) On August 29, 2005, Hurricane Katrina caused severe damage to oil installations in the Gulf of Mexico. Although this damage could eventually be repaired, it resulted in a substantial reduction in the short run supply of gasoline in the United States. In many areas, retail gasoline prices quickly rose by about 30% to an average of $3.06 per gallon.

Georgia governor Sonny Perdue suspended his state's 7.5 cents-a-gallon gas tax and 4% sales tax on gasoline purchases until Oct. 1. Governor Perdue explained that, "I believe it is absolutely wrong for the state to reap a tax windfall in this time of urgency and tragedy." Lawmakers in several other states were considering similar actions.

Let us apply supply and demand analysis to this problem. Before the hurricane, the United States consumed about 180 million gallons of gasoline per day, of which about 30 million gallons came from the Gulf of Mexico. In the short run, the supply of gasoline is extremely inelastic and is limited by refinery and transport capacity. Let us assume that the daily short run supply of gasoline was perfectly inelastic at 180 million gallons before the storm and perfectly inelastic at 150 million gallons after the storm. Suppose that the demand function, measured in millions of gallons per day, is given by $Q = 240 - 30P$ where P is the dollar price, including tax, that consumers pay for gasoline.

(a) What was the market equilibrium price for gasoline before the hurricane? _____ After the hurricane?_____.

(b) Suppose that both before and after the hurricane, a government tax of 10 cents is charged for every gallon of gasoline sold in the United States. How much money would *suppliers* receive *per gallon* of gasoline before the hurricane? _____ After the hurricane?_____.

(c) Suppose that after the hurricane, the federal government removed the gas tax. What would then be the equilibrium price paid by consumers?

_____ How much money would suppliers receive per gallon of gaso-

line? _____ How much revenue would the government lose per day

by removing the tax? _____ What is the net effect of removing

the tax on gasoline prices?_____ Who are the gainers

and who are the losers from removing the tax?_____

_____.

(d) Suppose that after the hurricane, the ten-cent tax is removed in some states but not in others. The states where the tax is removed consti-tute just half of the demand in the United States. Thus the demand schedule in each half of the country is $Q = 120 - 15P$ where P is the price paid by consumers in that part of the country. Let P^* be the equilibrium price for consumers in the part of the country where the tax is removed. In equilibrium, suppliers must receive the same price per gallon in all parts of the country. Therefore the equilibrium price for con-sumers in states that keep the tax must be $P^* + \$0.10$. In equilibrium it must be that the total amount of gasoline demanded in the two parts of the country equals the total supply. Write an equation for total de-

mand as a function of P^*. _____ Set demand equal to supply and solve for the price paid by consumers in the states

that remove the tax _____ and for the price paid by consumers in

states that do not remove the tax. _____ How much money

do suppliers receive per gallon of gasoline sold in every state? _____

_____ How does the tax removal affect daily gasoline consumption in

each group of states?_____

_____

_____.

(e) If half of the states remove the gasoline tax, as described above, some groups will be better off and some worse off than they would be if the tax were left in place. Describe the gains or losses for each of the following groups.

Consumers in the states that remove the tax_____

_____.

Consumers in other states_____.

Gasoline suppliers_____.

Governments of the states that remove the tax_____

_____.

Governments of states that do not remove the tax_____

_____.