Portfolio Performance Evaluation

McGraw-Hill/Irwin

Introduction

- Single period return
  - Total return
  - Definition of cash flow
- Multiple period return FV and PV
- Two common ways to measure average portfolio return:

  1. Dollar-weighted returns
  2. Time-weighted returns

- Returns must be adjusted for risk.

Dollar- and Time-Weighted Returns

Dollar-weighted returns

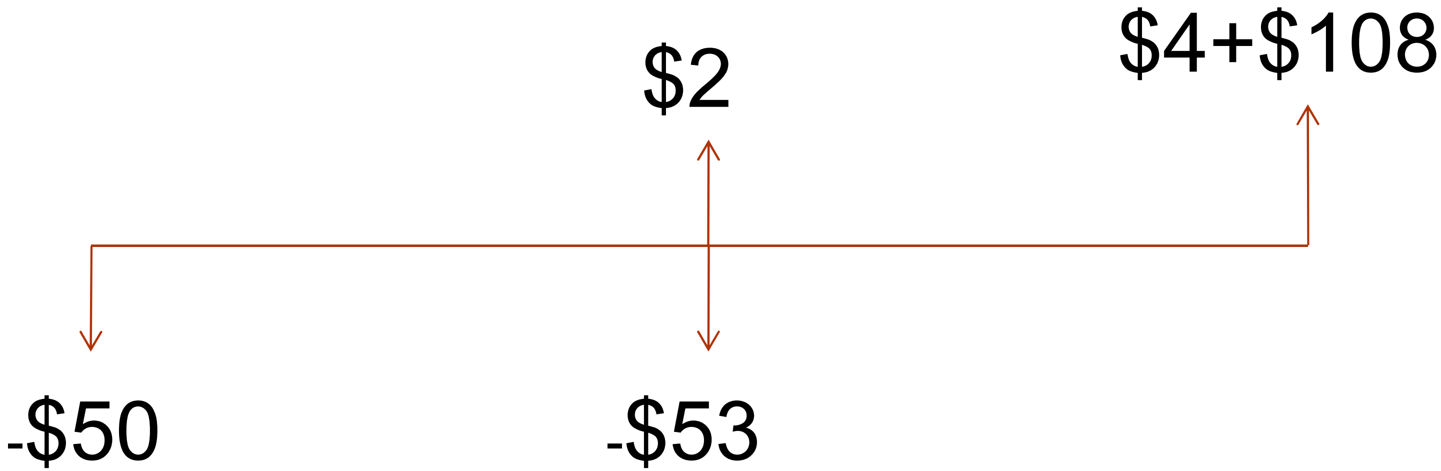
- Internal rate of return considering the cash flow from or to investment
- Returns are weighted by the amount invested in each period:

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \dots \frac{C_n}{(1+r)^n}$$

Example of Multiperiod Returns

Time	Outlay
0	$50 to purchase first share
1	$53 to purchase second share a year later
	Proceeds
1	$2 dividend from initially purchased share
2	$4 dividend from the 2 shares held in the second year, plus $108 received from selling both shares at $54 each

Dollar-Weighted Return



Dollar-weighted Return (IRR):

$$-50 = \frac{-51}{(1+r)^1} + \frac{112}{(1+r)^2}$$

$$r = 7.117\%$$

Dollar- and Time-Weighted Returns

<u>Time-weighted returns</u>

- The geometric average is a time-weighted average.
- Each period's return has equal weight.

$$(1+r_G)^n = (1+r_1)(1+r_2)\ldots(1+r_n)$$

Time-Weighted Return

$$r_1 = \frac{53 - 50 + 2}{50} = 10\%$$

$$r_2 = \frac{54 - 53 + 2}{53} = 5.66\%$$

$$r_G = [\,(1.1)\,(1.0566)\,]^{1/2} - 1 = 7.81\%$$

Averaging Returns

Arithmetic Mean:

$$\bar{r} = \sum_{t=1}^{n} \frac{r_t}{n}$$

Example:

(.10 + .0566) / 2 = 7.83%

Geometric Mean:

$$\bar{r} = \left[\prod_{t=1}^{n} (1 + r_t) \right]^{1/n} - 1$$

Example:

[(1.1) (1.0566)]$^{1/2}$ - 1

= 7.808%

Geometric Average

The arithmetic average provides unbiased estimates of the expected return of the stock. Use this to forecast returns in the next period.

The geometric average is less than the arithmetic average and this difference increases with the volatility of returns.

The geometric average is also called the time-weighted average (as opposed to the dollar weighted average), because it puts equal weights on each return.

Adjusting Returns for Risk

- The simplest and most popular way to adjust returns for risk is to compare the portfolio's return with the returns on a comparison universe.
- The comparison universe is a benchmark composed of a group of funds or portfolios with similar risk characteristics, such as growth stock funds or high-yield bond funds.

Figure 24.1 Universe Comparison

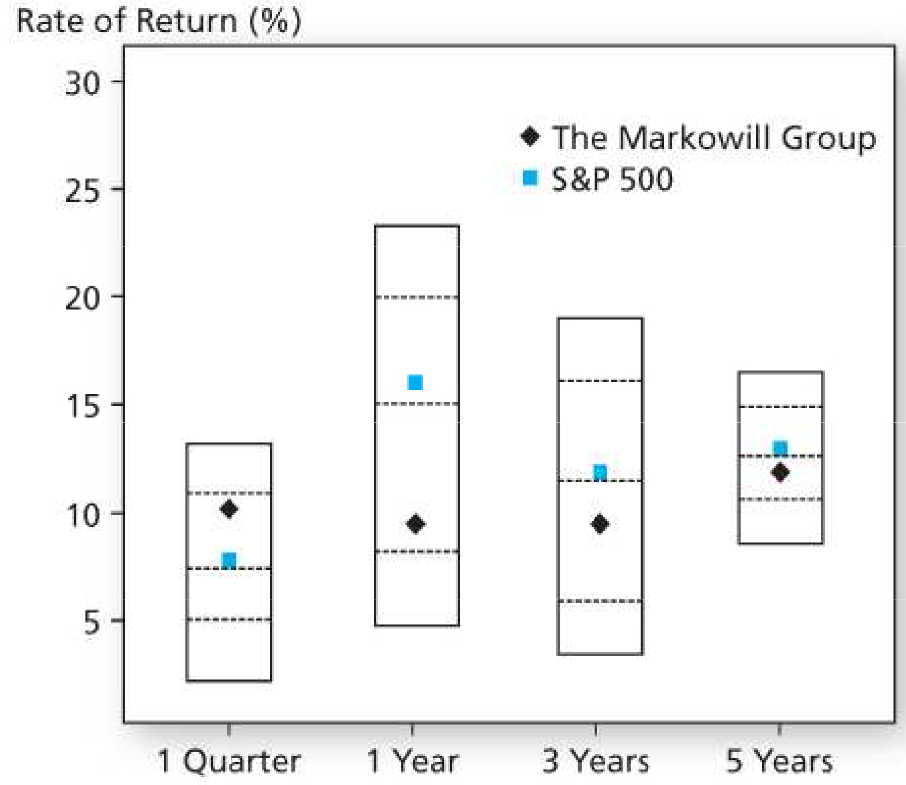


Figure 24.1 Universe comparison. Periods ending December 31, 2010

Risk Adjusted Performance: Sharpe

1) **Sharpe Index**

$$\frac{(\overline{r_P} - \overline{r_f})}{\sigma_P}$$

$\overline{r_p}$ = Average return on the portfolio

$\overline{r_f}$ = Average risk free rate

σ_p = Standard deviation of portfolio return

Risk Adjusted Performance: Treynor

2) Treynor Measure

$$\frac{(\overline{r_P} - \overline{r_f})}{\beta_P}$$

$\overline{r_p}$ = Average return on the portfolio

$\overline{r_f}$ = Average risk free rate

β_p = Weighted average beta for portfolio

Risk Adjusted Performance: Jensen

3) Jensen's Measure

$$\alpha_P = \overline{r}_P - \left[\overline{r}_f + \beta_P(\overline{r}_M - \overline{r}_f)\right]$$

α_p = Alpha for the portfolio

$\overline{r}_p$ = Average return on the portfolio

β_p = Weighted average Beta

$\overline{r}_f$ = Average risk free rate

$\overline{r}_m$ = Average return on market index portfolio

Information Ratio

Information Ratio = $\alpha_p / \sigma(e_p)$

The information ratio divides the alpha of the portfolio by the nonsystematic risk.

Nonsystematic risk could, in theory, be eliminated by diversification.

M^2 Measure

- Developed by Modigliani and Modigliani
- Create an adjusted portfolio (P*)that has the same standard deviation as the market index.
- Because the market index and *P** have the same standard deviation, their returns are comparable:

$$M^2 = r_{P*} - r_M$$

M^2 Measure: Example

Managed Portfolio: return = 35% standard deviation = 42%

Market Portfolio: return = 28% standard deviation = 30%

T-bill return = 6%

P* Portfolio:

30/42 = .714 in P and (1-.714) or .286 in T-bills

The return on P* is (.714) (.35) + (.286) (.06) = 26.7%

Since this return is less than the market, the

Figure 24.2 M^2 of Portfolio *P*

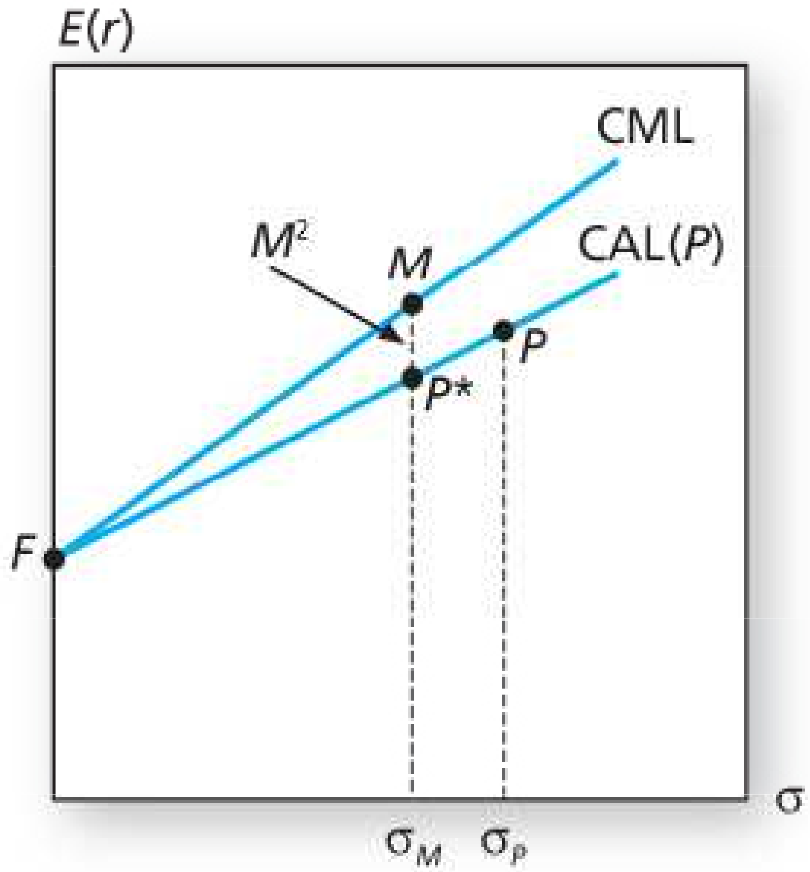


Figure 24.2 M^2 of portfolio *P*

Which Measure is Appropriate?

<u>It depends on investment assumptions</u>

1) If the portfolio represents the entire risky investment , then use the Sharpe measure.

2) If the portfolio is one of many combined into a larger investment fund, use the Jensen α or the Treynor measure. The Treynor measure is appealing because it weighs excess returns against systematic risk.

Table 24.1 Portfolio Performance

Table 24.1

Portfolio performance

	Portfolio *P*	Portfolio *Q*	Market
Beta	.90	1.60	1.0
Excess return $(\bar{r} - \bar{r}_f)$	11%	19%	10%
Alpha*	2%	3%	0

*Alpha = Excess return − (Beta × Market excess return)

$= (\bar{r} - \bar{r}_f) - \beta(\bar{r}_M - \bar{r}_f) = \bar{r} - [\bar{r}_f + \beta(\bar{r}_M - \bar{r}_f)]$

Is Q better than P?

Figure 24.3 Treynor's Measure

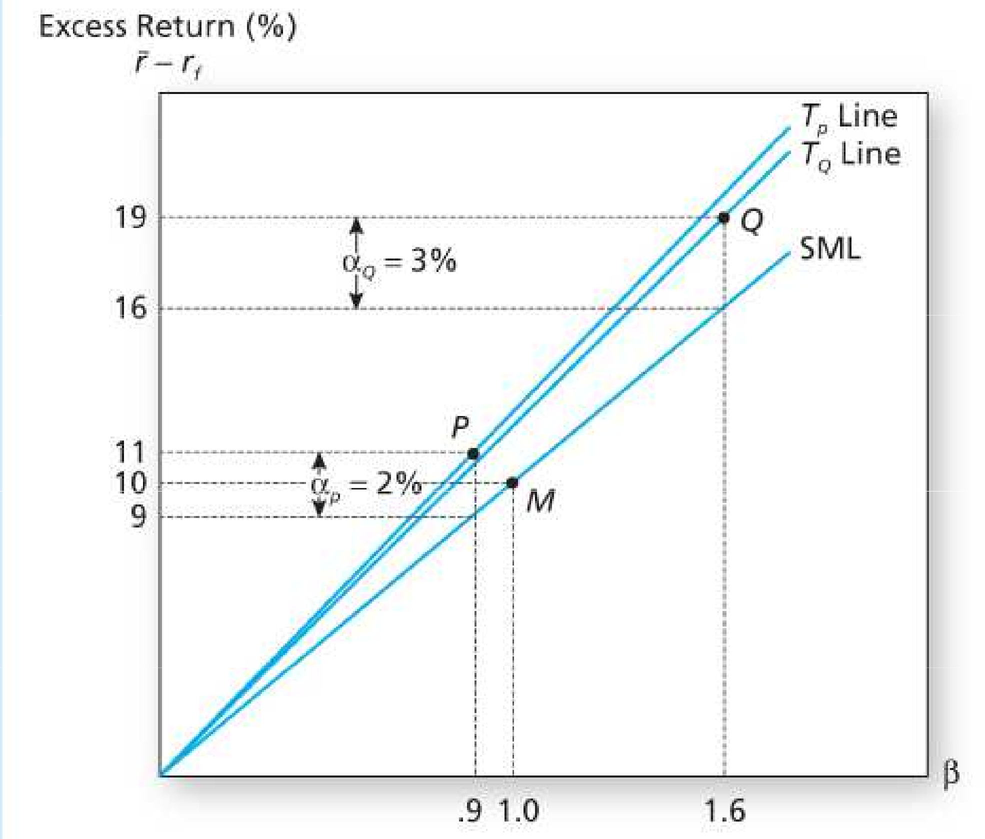


Figure 24.3 Treynor's measure

Table 24.3 Performance Statistics

Table 24.3

Performance statistics

	Portfolio *P*	Portfolio *Q*	Portfolio *M*
Sharpe's measure	0.45	0.51	0.19
M^2	2.19	2.69	0.00
SCL regression statistics			
Alpha	1.63	5.28	0.00
Beta	0.69	1.40	1.00
Treynor	4.00	5.40	1.63
T^2	2.37	3.77	0.00
$\sigma(e)$	1.95	8.98	0.00
Information ratio	0.84	0.59	0.00
R-SQR	0.91	0.64	1.00

Interpretation of Table 24.3

- If P or Q represents the entire investment, Q is better because of its higher Sharpe measure and better M^2.
- If P and Q are competing for a role as one of a number of subportfolios, Q also dominates because its Treynor measure is higher.
- If we seek an active portfolio to mix with an index portfolio, P is better due to its higher information ratio.

Performance Measurement for Hedge Funds

- When the hedge fund is optimally combined with the baseline portfolio (M) and active portfolio (H), the improvement in the Sharpe measure will be determined by its information ratio:

- $S_P^2 = S_M^2 + \left[\frac{\alpha_H}{\sigma(e_H)}\right]$^2

Performance Measurement with Changing Portfolio Composition

- We need a very long observation period to measure performance with any precision, even if the return distribution is stable with a constant mean and variance.
- What if the mean and variance are not constant? We need to keep track of portfolio changes.

Figure 24.4 Portfolio Returns

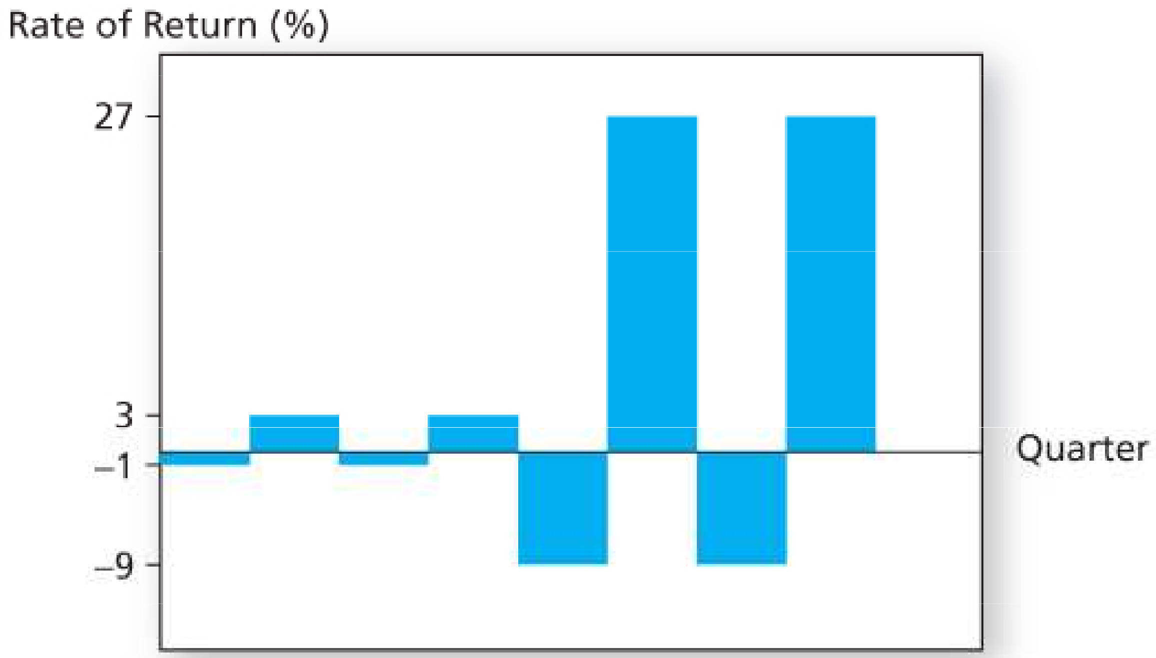


Figure 24.4 Portfolio returns. Returns in last four quarters are more variable than in the first four.

Style Analysis

- Introduced by William Sharpe
- Regress fund returns on indexes representing a range of asset classes.
- The regression coefficient on each index measures the fund's implicit allocation to that "style."
- R –square measures return variability due to style or asset allocation.
- The remainder is due either to security selection or to market timing.

Table 24.5 Style Analysis for Fidelity's Magellan Fund

Monthly returns on Magellan Fund over five year period.

Regression coefficient only positive for 3.

They explain 97.5% of Magellan's returns.

2.5 percent attributed to security selection within asset classes.

Style Portfolio	Regression Coefficient
T-Bill	0
Small Cap	0
Medium Cap	35
Large Cap	61
High P/E (growth)	5
Medium P/E	0
Low P/E (value)	0
Total	100
R-square	97.5

Figure 24.7 Fidelity Magellan Fund Cumulative Return Difference