LECTURE 11

Introduction to Econometrics

Autocorrelation

November 29, 2016

On previous lectures

- We discussed the specification of a regression equation
- **Specification** consists of choosing:
  1. correct independent variables
  2. correct functional form
  3. correct form of the stochastic error term
- We talked about the choice of independent variables and their functional form
- We started to talk about the form of the error term - we discussed heteroskedasticity

On today's lecture

- We will finish the discussion of the form of the error term by talking about **autocorrelation** (or **serial correlation**)
- We will learn
  - what is the nature of the problem
  - what are its consequences
  - how it is diagnosed
  - what are the remedies available

Nature of autocorrelation

- Observations of the error term are correlated with each other
$$Cov(\varepsilon_i, \varepsilon_j) \neq 0 \quad , \quad i \neq j$$
- Violation of one of the classical assumptions
- Can exist in any data in which the order of the observations has some meaning - most frequently in time-series data
- Particular form of autocorrelation - $AR(p)$ process:

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \ldots + \rho_p \varepsilon_{t-p} + u_t$$

  - u_t is a classical (not autocorrelated) error term
  - ρ_k are autocorrelation coefficients (between -1 and 1)

Examples of pure autocorrelation

- Distribution of the error term has autocorrelation nature
- First order autocorrelation

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + u_t$$

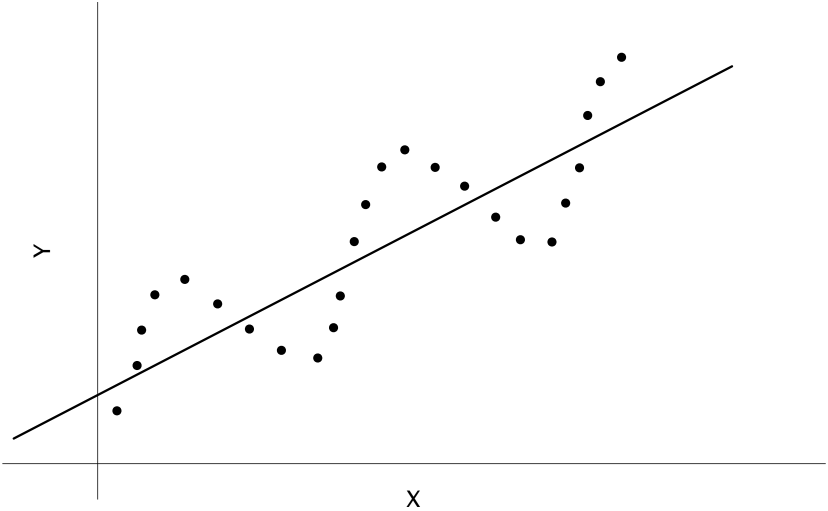
  - positive serial correlation: ρ_1 is positive
  - negative serial correlation: ρ_1 is negative
  - no serial correlation: ρ_1 is zero
  - positive autocorrelation very common in time series data
  - e.g.: a shock to GDP persists for more than one period
- Seasonal autocorrelation (in quarterly data)

$$\varepsilon_t = \rho_4 \varepsilon_{t-4} + u_t$$

Examples of impure autocorrelation

- Autocorrelation caused by specification error in the equation:
  - omitted variable
  - incorrect functional form
- How can misspecification cause autocorrelation in the error term?
  - Recall that the error term includes the omitted variables, nonlinearities, measurement error, and the classical error term.
  - If we omit a serially correlated variable, it is included in the error term, causing the autocorrelation problem.
- Impure autocorrelation can be corrected by better choice of specification (as opposed to pure autocorrelation).

AUTOCORRELATION



Consequences of autocorrelation

1. Estimated coefficients ($\widehat{\beta}$) remain unbiased and consistent

2. Standard errors of coefficients ($s.e.(\widehat{\beta})$) are biased (inference is incorrect)
   - serially correlated error term causes the dependent variable to fluctuate in a way that the OLS estimation procedure attributes to the independent variable
   - Serial correlation typically makes OLS underestimate the standard errors of coefficients
   - therefore we find t scores that are incorrectly too high

$\Rightarrow$ The same consequences as for the heteroskedasticity

Durbin-Watson test for autocorrelation

- Used to determine if there is a first-order serial correlation by examining the residuals of the equation
- Assumptions (criteria for using this test):
  - The regression includes the intercept
  - If autocorrelation is present, it is of $AR(1)$ type:

$$\varepsilon_t = \rho \varepsilon_{t-1} + u_t$$

  - The regression does not include a lagged dependent variable

Durbin-Watson test for autocorrelation

- Durbin-Watson d statistic (for T observations):

$$d = \frac{\sum_{t=2}^{T} (e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2} \approx 2(1 - \widehat{\rho})$$

where $\widehat{\rho}$ is the autocorrelation coefficient

- Values:
  1. Extreme positive serial correlation: $d \approx 0$
  2. Extreme negative serial correlation: $d \approx 4$
  3. No serial correlation: $d \approx 2$

Using the Durbin-Watson test

1. Estimate the equation by OLS, save the residuals
2. Calculate the d statistic
3. Determine the sample size T and the number of explanatory variables (excluding the intercept!) k'
4. Find the upper critical value d_U and the lower critical value d_L for T and k' in statistical tables
5. Evaluate the test as one-sided or two-sided (see next slides)

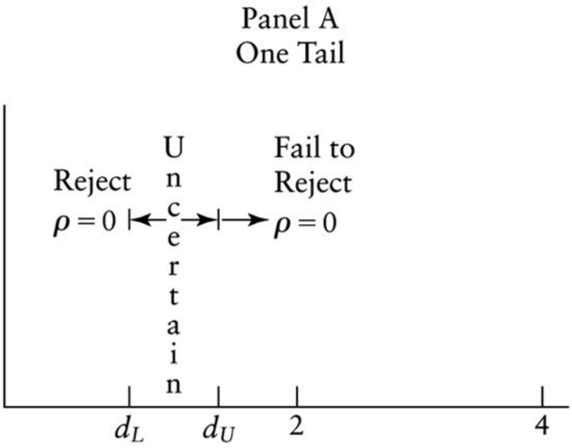
One-sided Durbin-Watson test

- For cases when we consider only positive serial correlation as an option
- Hypothesis:

$$H_0: \ \rho \leq 0 \quad \text{(no positive serial correlation)}$$
$$H_A: \ \rho > 0 \quad \text{(positive serial correlation)}$$

- Decision rule:
  - if $d < d_L$ reject H_0
  - if $d > d_U$ do not reject H_0
  - if $d_L \leq d \leq d_U$ inconclusive

DURBIN-WATSON CRITICAL VALUES FOR ONE-SIDED TEST



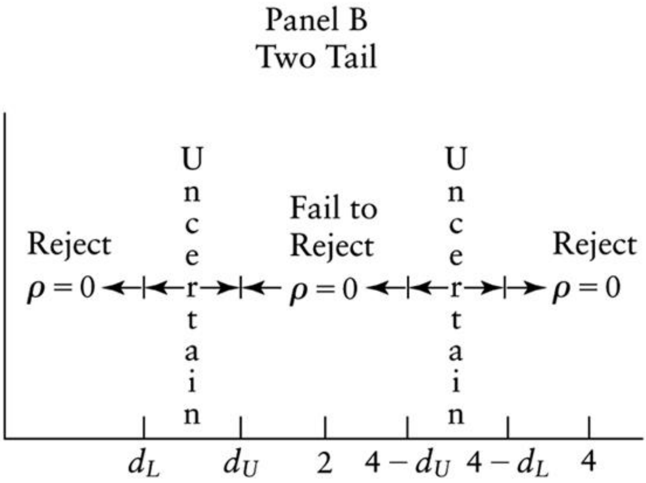
Two-sided Durbin-Watson test

- For cases when we consider both signs of serial correlation
- Hypothesis:

$$\begin{aligned} H_0 &: \ \rho = 0 \quad \text{(no serial correlation)} \\ H_A &: \ \rho \neq 0 \quad \text{(serial correlation)} \end{aligned}$$

- Decision rule:
  - if $d < d_L$ reject H_0
  - if $d > 4 - d_L$ reject H_0
  - if $d > d_U$ do not reject H_0
  - if $d < 4 - d_U$ do not reject H_0
  - otherwise inconclusive

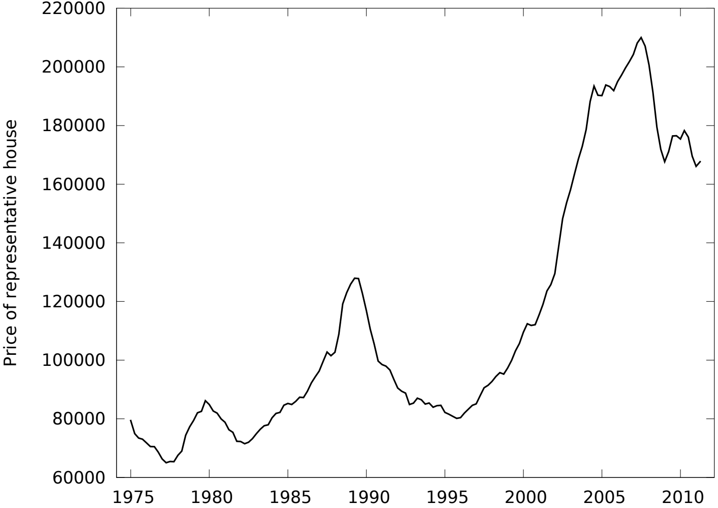
DURBIN-WATSON CRITICAL VALUES FOR TWO-SIDED TEST



Example

- Estimating housing prices in the UK
- Quarterly time series data on prices of a representative house in UK (in £)
- Explanatory variable: GDP (in billions of £)
- Time span: 1975 Q1 - 2011 Q2
- All series are seasonally adjusted and in real prices (i.e. adjusted for inflation)

EXAMPLE


Price of representative house
220000
200000
180000
160000
140000
120000
100000
80000
60000
1975
1980
1985
1990
1995
2000
2005
2010

Example

```
Model 1: OLS, using observations 1975:1-2011:2 (T = 146)
Dependent variable: house_price

             coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------
  const      −38409.8      6675.01      −5.754    5.04e-08 ***
  gdp           737.065      31.4846    23.41     6.09e-51 ***

Mean dependent var   113072.8   S.D. dependent var   43254.80
Sum squared resid    5.65e+10   S.E. of regression   19799.38
R-squared            0.791921   Adjusted R-squared   0.790476
F(1, 144)            548.0434   P-value(F)           6.09e-51
Log-likelihood      −1650.595   Akaike criterion     3305.191
Schwarz criterion    3311.158   Hannan-Quinn         3307.615
rho                  0.984890   Durbin-Watson        0.023930
```

Example

- We test for positive serial correlation:

$$\begin{aligned} H_0 &: \ \rho \leq 0 \quad \text{(no positive serial correlation)} \\ H_A &: \ \rho > 0 \quad \text{(positive serial correlation)} \end{aligned}$$

- One-sided DW critical values at 95% confidence for $T = 146$ and $k' = 1$ are:

$$d_L = 1.72 \quad \text{and} \quad d_U = 1.74$$

- Decision rule:
  - if $d < 1.72$ reject H_0
  - if $d > 1.74$ do not reject H_0
  - if $1.72 \leq d \leq 1.74$ inconclusive

- Since $d = 0.02 < 1.72$, we reject the null hypothesis of no positive serial correlation

Alternative approach to autocorrelation testing

- Suppose we suspect the stochastic error term to be $AR(p)$

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \ldots + \rho_p \varepsilon_{t-p} + u_t$$

- Since OLS is consistent even under autocorrelation, the residuals are consistent estimates of the stochastic error term

- Hence, it is sufficient to:

  1. Estimate the original model by OLS, save the residuals e_t
  2. Regress $e_t = \rho_1 e_{t-1} + \rho_2 e_{t-2} + \ldots + \rho_p e_{t-p} + u_t$
  3. Test if $\rho_1 = \rho_2 = \ldots = \rho_p = 0$ using the standard F-test

Back to example

```
Model 1: OLS, using observations 1975:1-2011:2 (T = 146)
Dependent variable: house_price

             coefficient   std. error   t-ratio   p-value
  --------------------------------------------------------
  const      −38409.8      6675.01      −5.754    5.04e-08 ***
  gdp           737.065      31.4846    23.41     6.09e-51 ***

Mean dependent var   113072.8   S.D. dependent var   43254.80
Sum squared resid    5.65e+10   S.E. of regression   19799.38
R-squared            0.791921   Adjusted R-squared   0.790476
F(1, 144)            548.0434   P-value(F)           6.09e-51
Log-likelihood      −1650.595   Akaike criterion     3305.191
Schwarz criterion    3311.158   Hannan-Quinn         3307.615
rho                  0.984890   Durbin-Watson        0.023930
```

Back to example

```
Model 2: OLS, using observations 1976:1-2011:2 (T = 142)
Dependent variable: e

             coefficient   std. error   t-ratio   p-value
  ---------------------------------------------------------
  e_1          1.75237     0.0843401    20.78     2.53e-44 ***
  e_2         -1.05900     0.168179     -6.297    3.79e-09 ***
  e_3          0.477195    0.168362      2.834    0.0053   ***
  e_4         -0.190822    0.0848111    -2.250    0.0260   **

Mean dependent var  -443.8153   S.D. dependent var   19823.71
Sum squared resid    7.22e+08   S.E. of regression   2287.633
R-squared            0.986973   Adjusted R-squared   0.986690
F(4, 138)            2613.852   P-value(F)           5.8e-129
Log-likelihood      -1297.869   Akaike criterion     2603.739
Schwarz criterion    2615.562   Hannan-Quinn         2608.543
rho                  0.006283   Durbin-Watson        1.967108
```

Remedy: White robust standard errors

- Note that autocorrelation does not lead to inconsistent estimates, only to incorrect inference - similar to heteroskedasticity problem
- We can keep the estimated coefficients, and only adjust the standard errors
- The White robust standard errors solve not only heteroskedasticity, but also serial correlation
- Note also that all derived results hold if the assumption $Cov(x, \varepsilon) = 0$ is not violated
  - First make sure the specification of the model is correct, only then try to correct for the form of an error term!

SUMMARY

- Autocorrelation does not lead to inconsistent estimates, but it makes the inference wrong (estimated coefficients are correct, but their standard errors are not)
- It can be diagnosed using
  - Durbin-Watson test
  - Analysis of residuals
- It can be remedied by
  - White robust standard errors
- Readings:
  - Studenmund, Chapter 9
  - Wooldridge, Chapter 12