LECTURE 2

Introduction to Econometrics

INTRODUCTION TO LINEAR REGRESSION ANALYSIS I.

October 6, 2017

Previous lecture...

- **Introduction, organization, review of statistical background**

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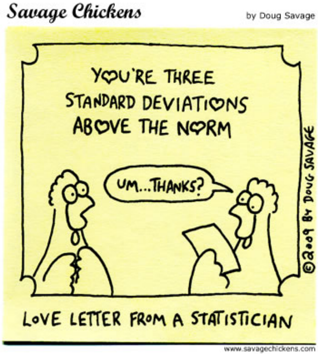

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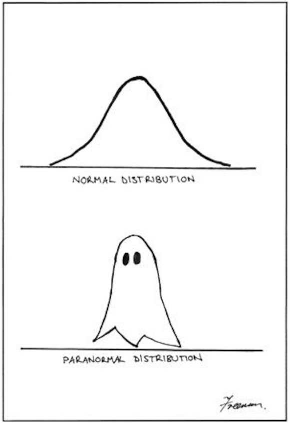

Warm-up exercise

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- **Variance:** $Var[X] = E\left[(X - E\left[X\right])^2\right] = E[X^2] - (E[X])^2$

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  - Sampling and estimation

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- Readings:
  - Studenmund, A. H., Using Econometrics: A Practical Guide, Chapters 1, 2.1, 17.2, 17.3
  - Wooldridge, J. M., Introductory Econometrics: A Modern Approach, Chapters 2.1, 2.2

SAMPLING

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- Examples: medical experiments, opinion polls

Random sampling vs selection bias

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RANDOM SAMPLING VS SELECTION BIAS

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  - Example: accident records of people who buy collision insurance

Exercise 1

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- Is this survey unbiased?

Estimation

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- **Estimate**: the specific value of the estimator that is obtained on a specific sample

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- An estimator is **efficient** if the variance of its sampling distribution is the smallest possible

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- When she compares indicators of profitability, such as ROA and ROE, between the domestic and foreign-owned firms, she finds significantly better outcomes for foreign-owned firms.
- She concludes that FDI increase firms' profitability. Is this conclusion correct?

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- We estimate: $Q = 31.50 - 0.73P + 0.11P_s + 0.23Y$

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  - Ordinary Least Squares (OLS)
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- We will perform estimation on different types of data

DATA USED IN ECONOMETRICS

Data used in econometrics

cross-section	repeated cross-section
sample of units (eg. firms, individuals) taken at a given point in time	several independent samples of units (eg. firms, individuals) taken at different points in time

time-series	panel data
observations of variable(s) in different points in time	time series for each cross-sectional unit in the data set

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- Annual social security or tax records of individual workers

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5. Interpretation of results

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- We call q dependent variable and c explanatory variable

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- Find the value of parameters β_1 (slope) and β_0 (intercept)

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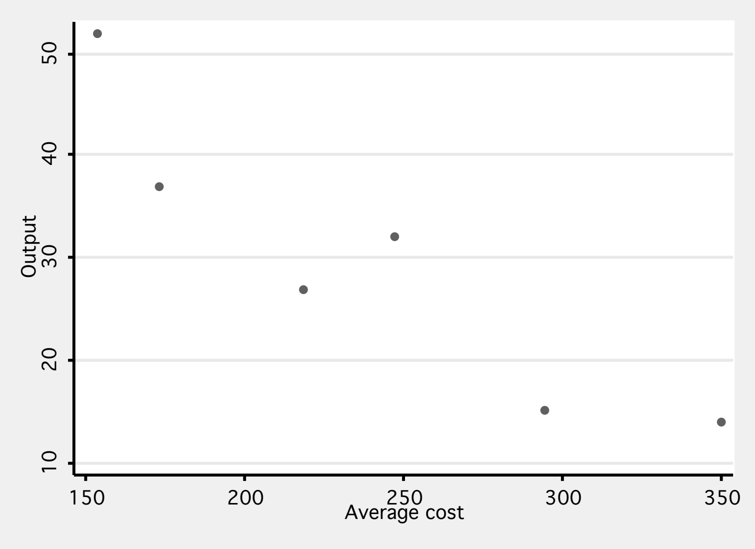
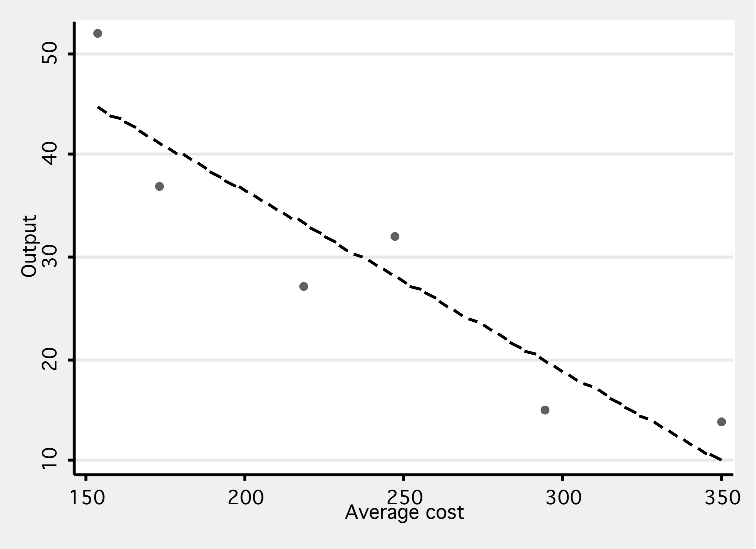
- Really: investigate a sample of firms
  - We need a random (unbiased) sample of firms
- Collect data:

Firm	1	2	3	4	5	6
q	15	32	52	14	37	27
c	294	247	153	350	173	218

Example - Data



Example - Estimation



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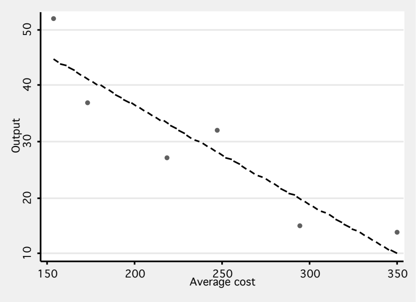

OLS method:

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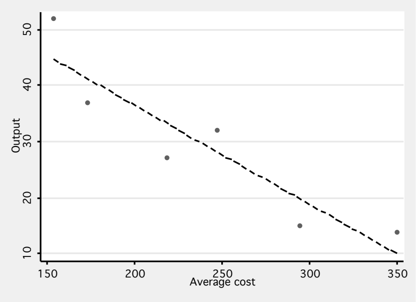

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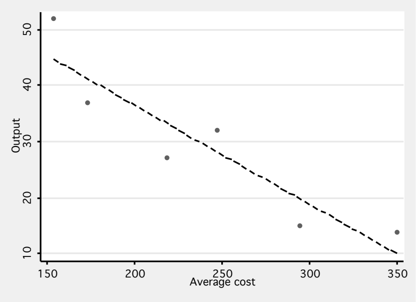

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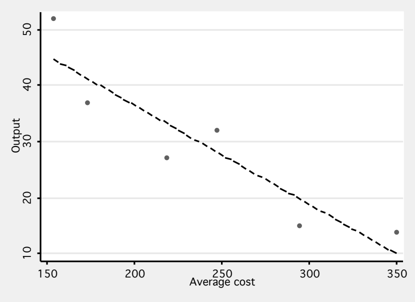

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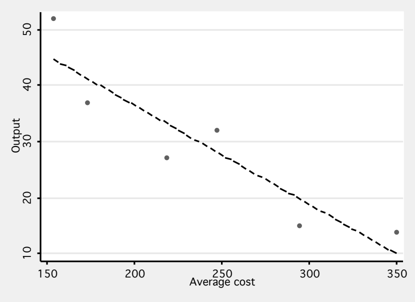

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Minimize the (vertical) distance between data points and regression line

$\Downarrow$

Minimize the sum of squared deviations

TERMINOLOGY

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \ \ldots \text{regression line}$$

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β_1 … slope parameter ($\widehat{\beta_1}$ … estimate of this parameter)

Ordinary Least Squares

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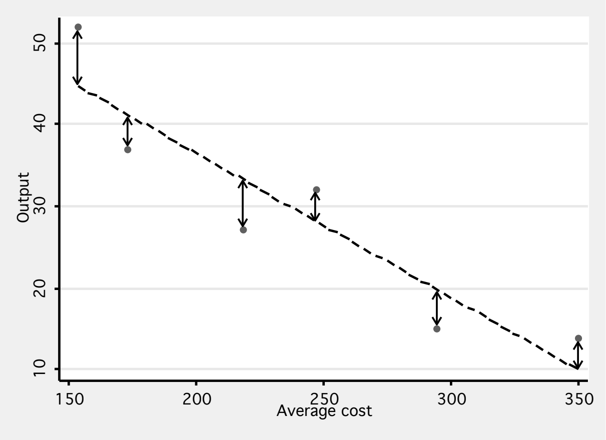

Ordinary Least Squares - principle

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$$(y_i - \beta_0 - \beta_1 x_i)^2$$

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- Find $\widehat{\beta}_0$ and $\widehat{\beta}_1$ such that they minimize this sum

$$\left[\widehat{\beta}_0, \widehat{\beta}_1\right] = \underset{\beta_0, \beta_1}{\operatorname{argmin}} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

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ORDINARY LEAST SQUARES - DERIVATION

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- FOC:

$$\frac{\partial}{\partial \beta_0} \quad : \qquad -2 \sum_{i=1}^{n} \left(y_i - \widehat{\beta}_0 - \widehat{\beta}_1 x_i\right) = 0$$

$$\frac{\partial}{\partial \beta_1} \quad : \qquad -2 \sum_{i=1}^{n} x_i \left(y_i - \widehat{\beta}_0 - \widehat{\beta}_1 x_i\right) = 0$$

Ordinary Least Squares - derivation

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Ordinary Least Squares - derivation

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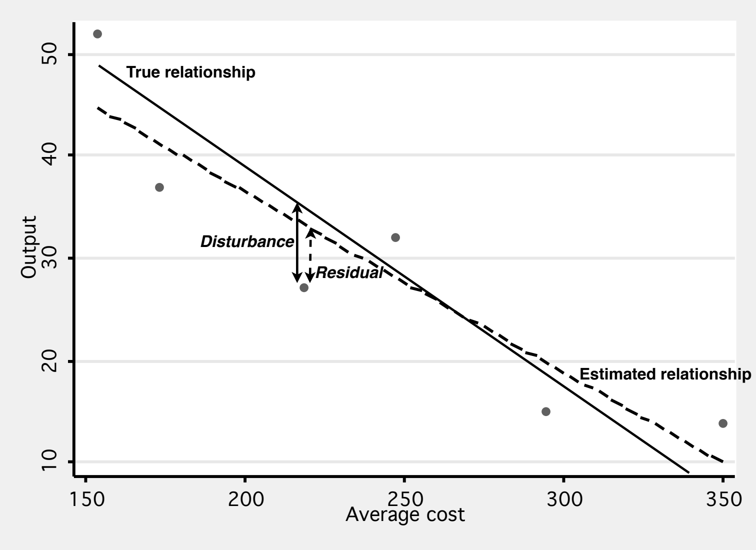
Residual vs. disturbance



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- Over data:

Firm	1	2	3	4	5	6
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c	294	247	153	350	173	218

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