LECTURE 5

Introduction to Econometrics

Hypothesis testing & Goodness of fit

October 20, 2017

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- We defined the concept of the *p*-value
- We explained what significance of a coefficient means

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Model 3: OLS, using observations 1-526
Dependent variable: wage

             coefficient   std. error   t-ratio   p-value
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  educ        0.644272     0.0538061    11.97     2.28e-29 ***
  exper       0.0700954    0.0109776     6.385    3.78e-10 ***

Mean dependent var   5.896103   S.D. dependent var   3.693086
Sum squared resid    5548.160   S.E. of regression   3.257044
R-squared            0.225162   Adjusted R-squared   0.222199
F(2, 523)            75.98998   P-value(F)           1.07e-29
Log-likelihood      −1365.969   Akaike criterion     2737.937
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- We will define the notion of the overall significance of a regression
- We will introduce a measure of the goodness of fit of a regression (R^2)
- Readings for this week:
  - Studenmund, Chapters 5.5 & 2.4
  - Wooldridge, Chapters 4 & 3

Testing multiple hypotheses

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- We will use an *F*-test

Restricted vs. unrestricted model

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$$y_i^* = \beta_0 + \beta_1 x_i^* + \varepsilon_i \quad ,$$

where $y_i^* = y_i - x_{i2}$ and $x_i^* = x_{i1} - x_{i2}$

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- The idea is thus to compare the residuals from the two models

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  - Calculate the sum of squared residuals in the two models
  - Test if the difference between the two sums is equal to zero (statistically)
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  - H_A: the difference is positive (residuals in the restricted model are bigger, restrictions do not hold)

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- Sum of squared residuals
  - $SSE = \sum_{i=1}^{n}(y_i - \widehat{y}_i)^2 = \sum_{i=1}^{n} e_i^2$

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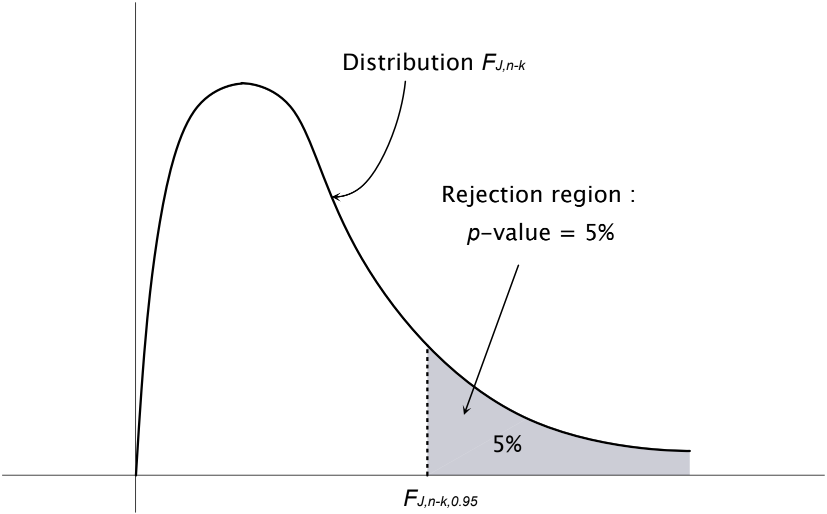
J	...	number of restrictions
n	...	number of observations
k	...	number of estimated coefficients (including intercept)

F-TEST



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- Under H_0, we obtained the restricted model

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- We construct the F-statistic $F = \frac{(SSE_R - SSE_U)/2}{SSE_U/(n-4)}$
- We find the critical value of the F distribution with 2 and $n - 4$ degrees of freedom at the 95% confidence level

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- We find the critical value of the F distribution with 2 and $n - 4$ degrees of freedom at the 95% confidence level
- If $F > F_{2,n-4,0.95}$, we reject the null hypothesis
  - we reject that the restrictions hold jointly

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- Usually, we are interested in knowing if the model has some explanatory power, i.e. if the independent variables indeed "explain" the dependent variable
- We test this using the F-test of the joint significance of all $(k-1)$ slope coefficients:

$$H_0 : \begin{cases} \beta_1 = 0 \\ \beta_2 = 0 \\ \vdots \\ \beta_{k-1} = 0 \end{cases} \quad \text{vs.} \quad H_A : \begin{cases} \beta_j \neq 0 \\ \\ \text{for at least one } j = 1, \ldots, k-1 \end{cases}$$

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  - Number of restrictions = $k-1$
  - This F-statistic and the corresponding p-value are part of the regression output

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- But how important are they in determining wages?
- How much of difference in wages between people is explained by differences in education and in experience?
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- This are the questions answered by the goodness of fit measure - R^2

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GOODNESS OF FIT - R^2

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- Denote:
  - $SST = \sum_{i=1}^{n} \left(y_i - \overline{y}_n\right)^2$ ... *Total Sum of Squares*
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- Define the measure of the goodness of fit:

$$R^2 = \frac{SSR}{SST} = \frac{\text{Explained variation in } y}{\text{Total variation in } y}$$

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  - $R^2 = 0.3$ means that the independent variables can explain 30% of the variation in the dependent variable
- Higher R^2 means better fit of the regression model (not necessarily a better model!)

Decomposing the variance

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- For models with intercept, R^2 can be rewritten using the decomposition of variance.
- Variance decomposition:

$$\sum_{i=1}^{n} \left(y_i - \overline{y}_n\right)^2 = \sum_{i=1}^{n} (\widehat{y}_i - \overline{y}_n)^2 + \sum_{i=1}^{n} e_i^2$$

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  - $SST = \sum_{i=1}^{n} \left(y_i - \overline{y}_n\right)^2 \ldots$ *Total Sum of Squares*

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- $SST = \sum_{i=1}^{n} \left(y_i - \overline{y}_n\right)^2$ ... *Total Sum of Squares*
- $SSR = \sum_{i=1}^{n} (\widehat{y}_i - \overline{y}_n)^2$ ... *Regression Sum of Squares*
- $SSE = \sum_{i=1}^{n} e_i^2$ ... *Sum of Squared Residuals*

VARIANCE DECOMPOSITION AND R^2

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- Intuition: total variation can be divided between the explained variation and the unexplained variation
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  - $y_i = \widehat{y}_i + e_i$
- We can rewrite R^2:

$$R^2 = \frac{SSR}{SST} = \frac{SST - SSE}{SST} = 1 - \frac{SSE}{SST}$$

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  - Models with more variables automatically have better fit.
- To deal with this problem, we define the *adjusted* R^2:

$$R^2_{adj} = 1 - \frac{\frac{SSE}{n-k}}{\frac{SST}{n-1}} \quad \left(\leq R^2\right)$$

(k is the number of coefficients including intercept)

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- This measure introduces a "punishment" for including more explanatory variables

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- Let us recall the F-statistic:

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$$F = \frac{(SSE_R - SSE_U)/J}{SSE_U/(n-k)} \sim F_{J,n-k}$$

- We can use the formula $R^2 = 1 - \frac{SSE}{SST}$ to rewrite the F-statistic in R^2 form:

$$F = \frac{(R_U^2 - R_R^2)/J}{(1 - R_U^2)/(n-k)} \sim F_{J,n-k}$$

- We can use this R^2 form of F-statistic under the condition that $SST_U = SST_R$ (the dependent variables in restricted and unrestricted models are the same)

Summary

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- We showed how restrictions are incorporated in regression models

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- We explained the idea of the F-test
- We defined the notion of the overall significance of a regression
- We introduced the measure or the goodness of fit - R^2
- We learned how total variation in the dependent variable can be decomposed